

# When Can We Incrementally Prove Computations of Arbitrary Depth?

**Matteo Campanelli**

Offchain Labs  
University of Tartu, Estonia

[matteo@offchainlabs.com](mailto:matteo@offchainlabs.com)  
[www.binarywhales.com](http://www.binarywhales.com)



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A joint work with  
Dario Fiore and Mahak Pancholi  
(IMDEA Software Institute)

# This Talk in a Nutshell

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A study of the security of  
Incrementally Verifiable Computation (IVC) under  
the lens of the depth of the proven computation.

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the lens of the depth of the proven computation.

**Our main motivation:**  
how can we prove security (or insecurity)  
when we move beyond constant depth?

# Succinct Cryptographic Proofs (SNARKs)



Server (Prover)



Client (Verifier)

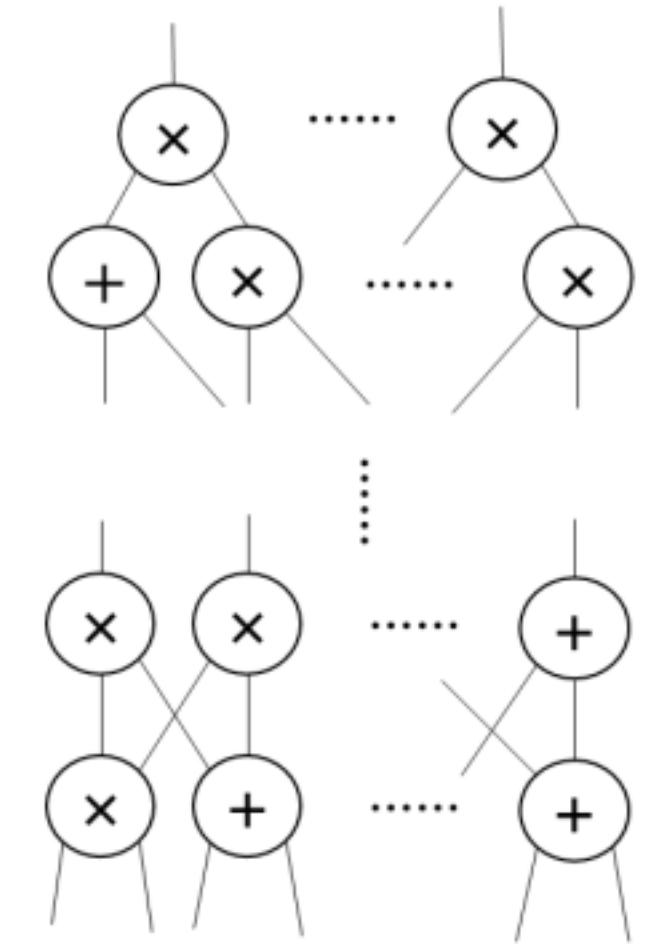
# Succinct Cryptographic Proofs (SNARKs)



Server (Prover)



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Some program  $F$

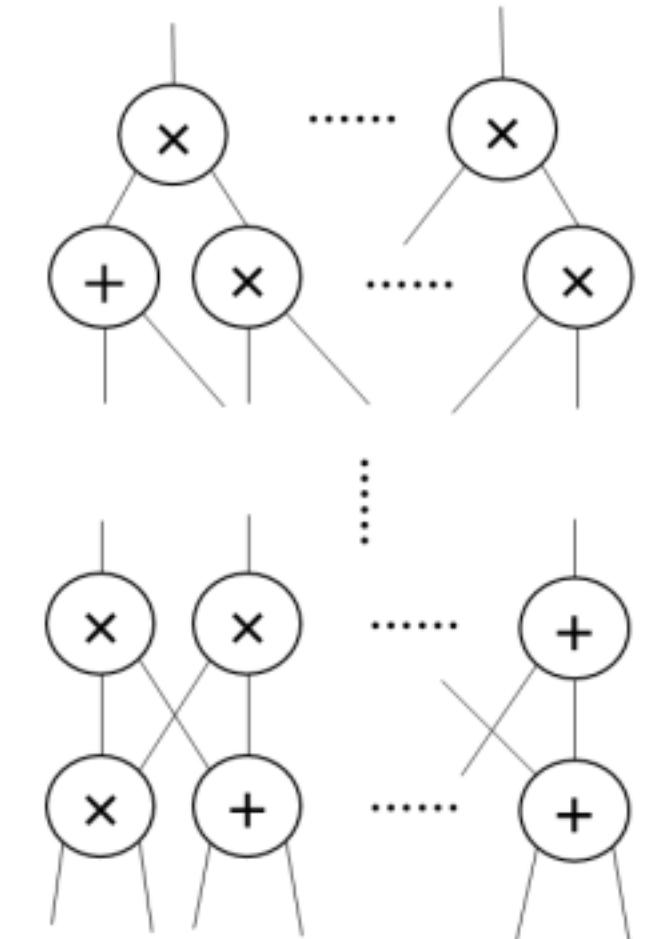
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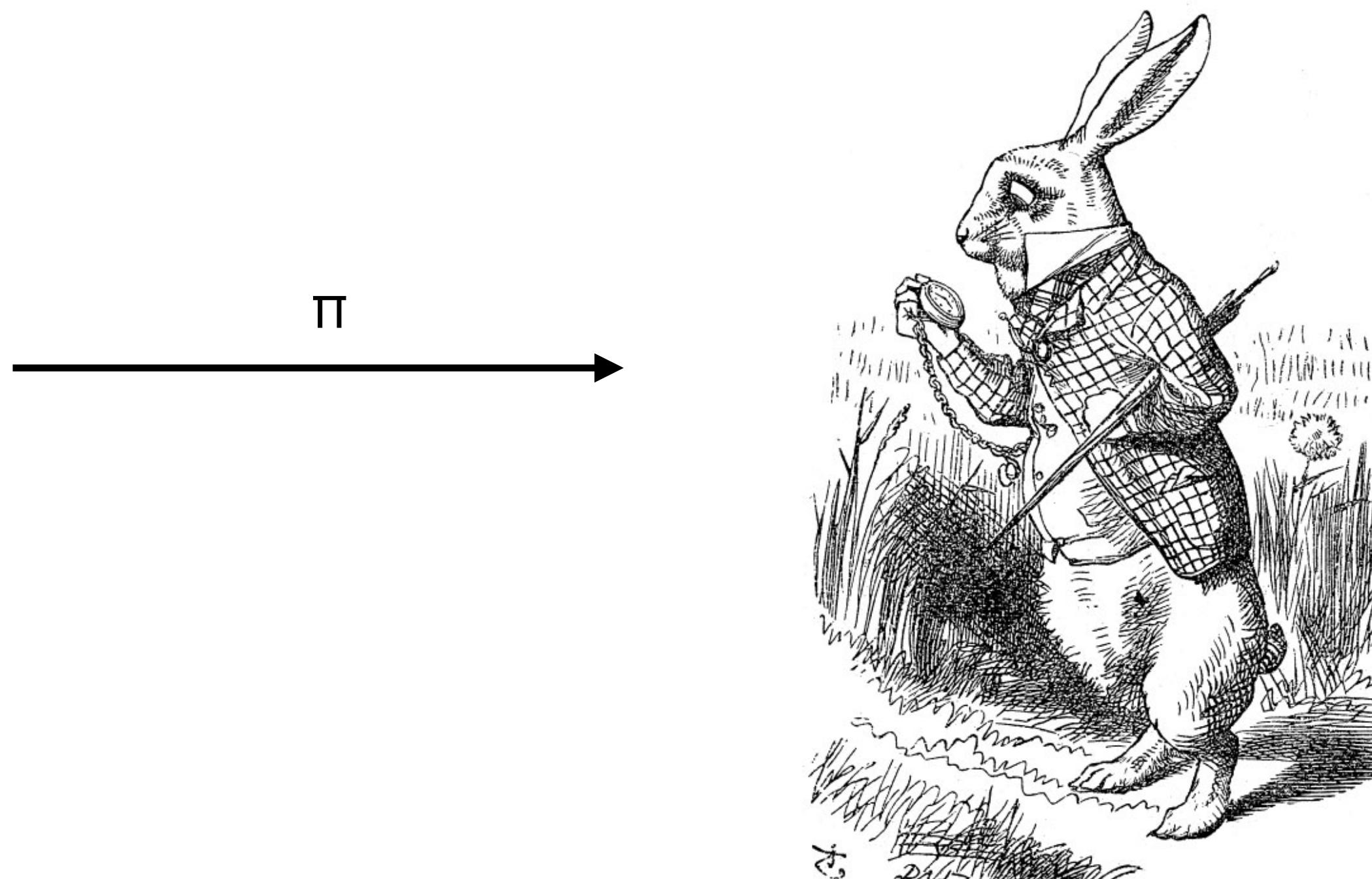
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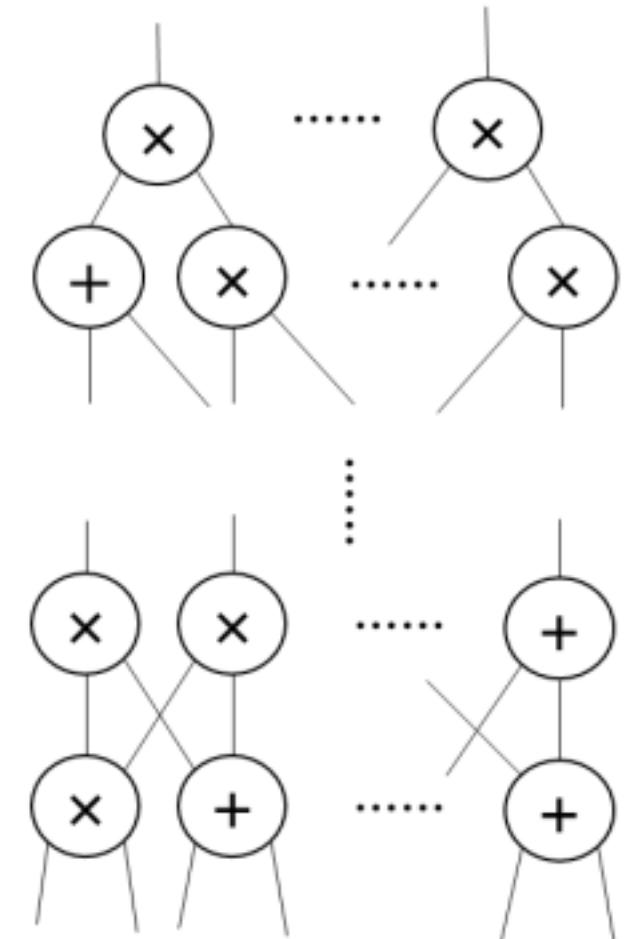


Server (Prover)



Client (Verifier)

$\Pi$



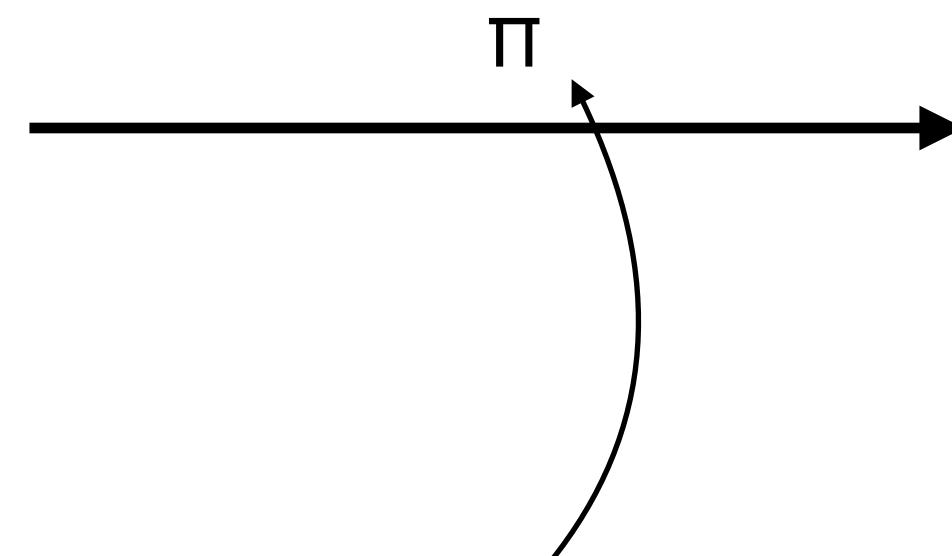
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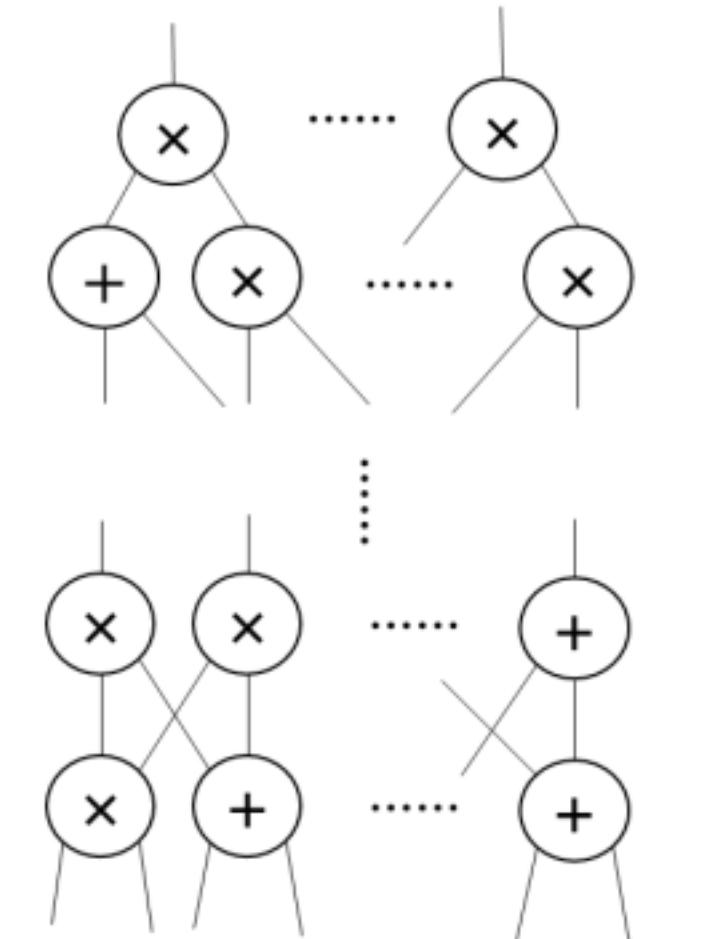
Server (Prover)



Proof that statement is true



Client (Verifier)



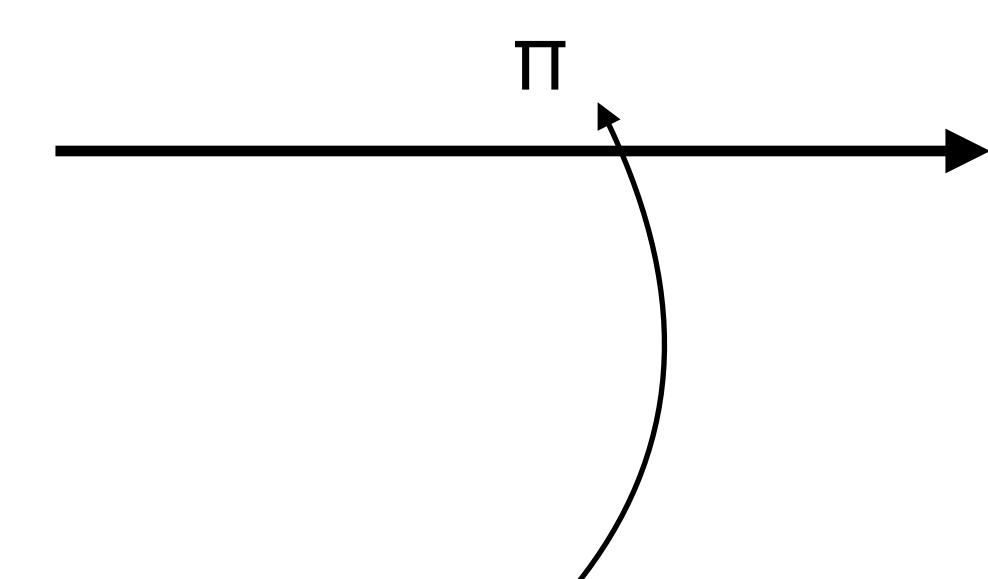
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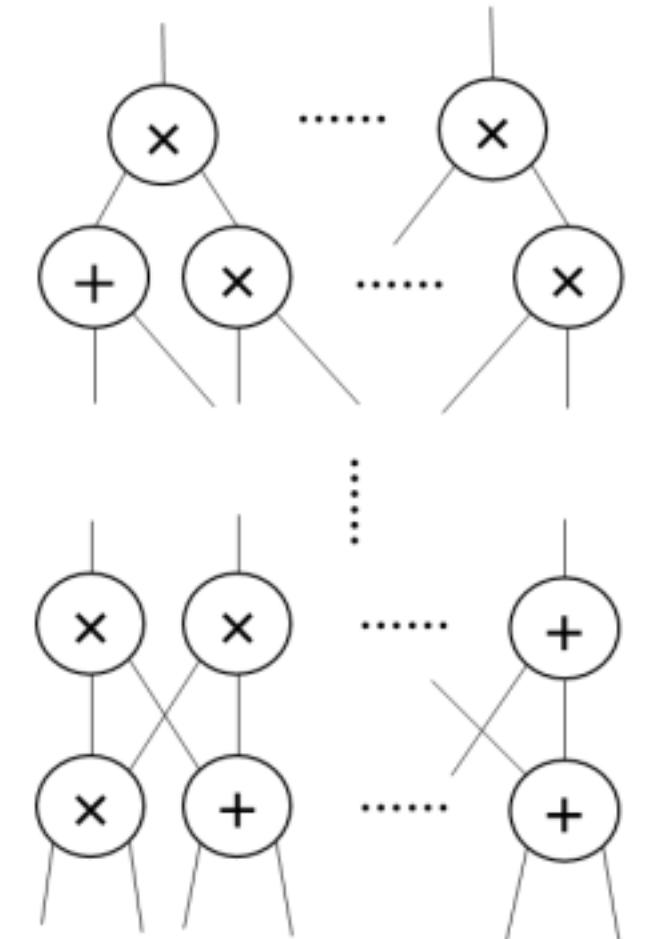


Proof that statement is true



Client (Verifier)

Verify( $x, \pi$ )



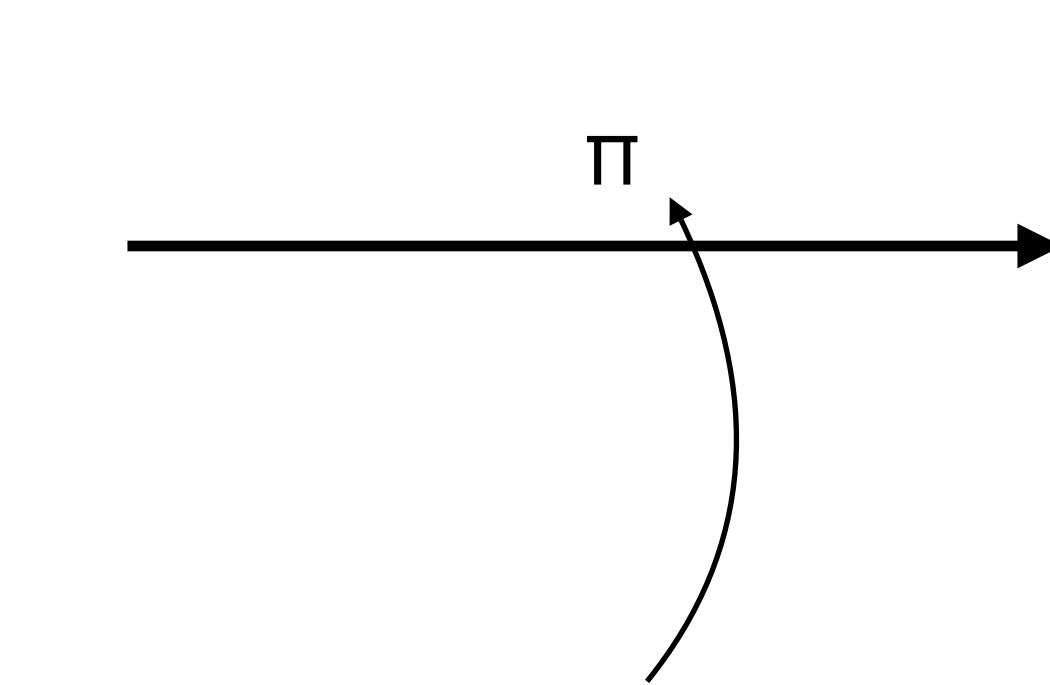
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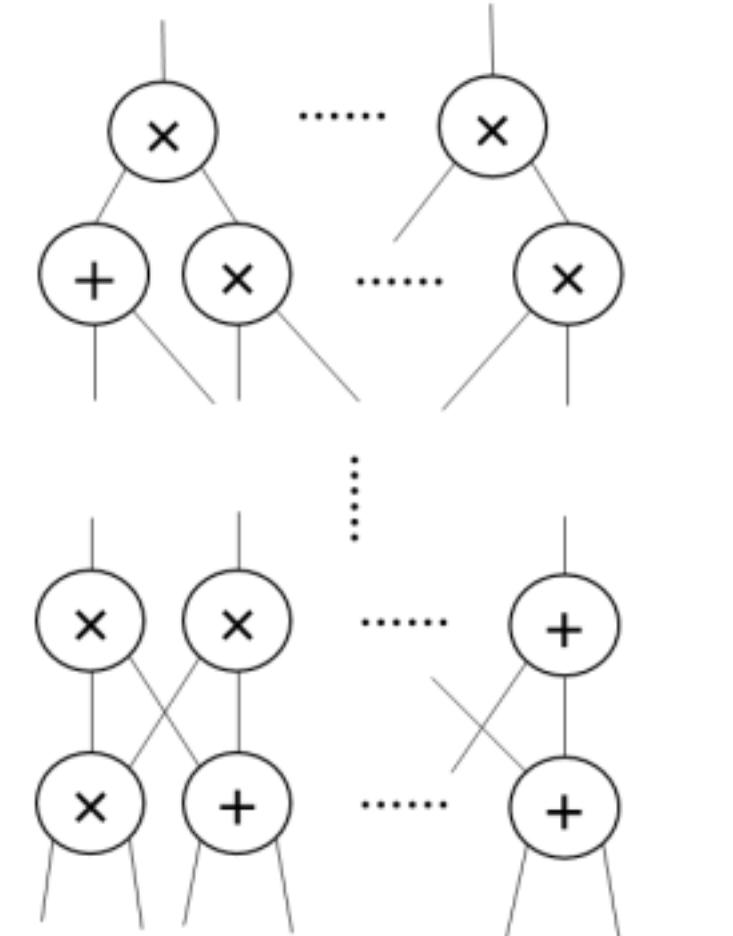


Proof that statement is true



Client (Verifier)

Verify( $x, \pi$ )



Some program  $F$

Common requirement: **Succinctness**  
( $\pi$  is very small; Verify is very fast)

Client would like to know  
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# Limitations of Traditional “Monolithic” Proofs

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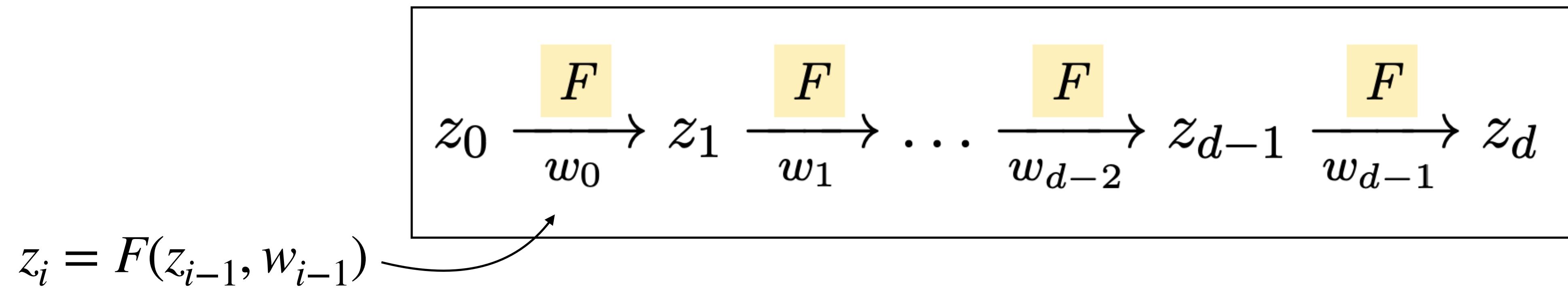
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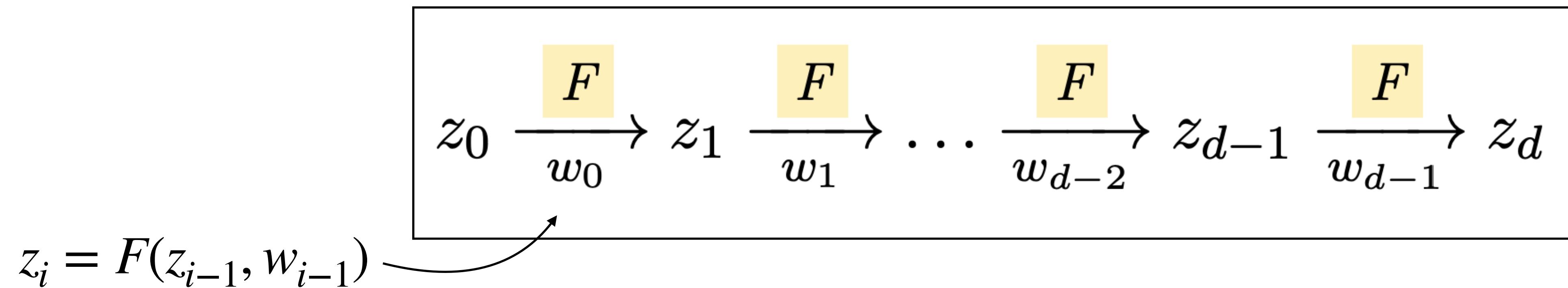
# Limitations of Traditional “Monolithic” Proofs

- Large memory requirements
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- No pipelining
  - Must finish the computation before starting proving
  - Cannot take advantage of incremental computations (next slide)

# Incremental Computations

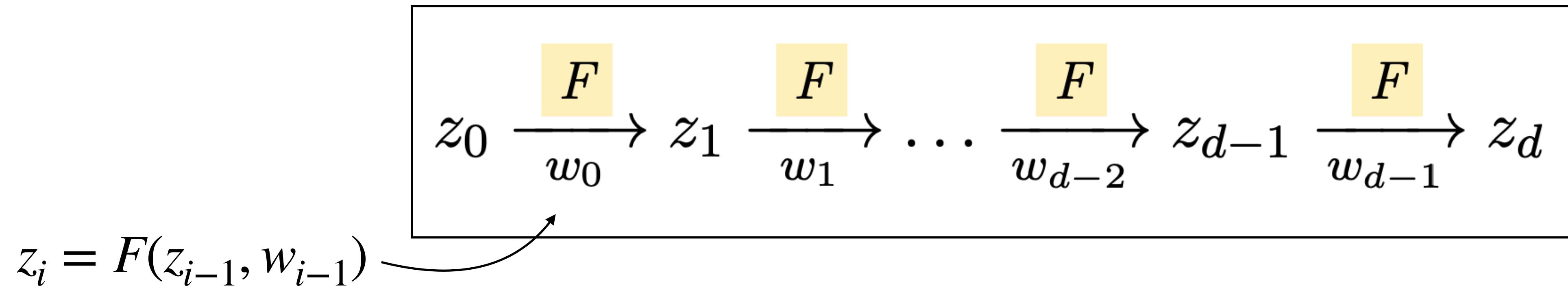


# Incremental Computations



**Examples of natural applications:**

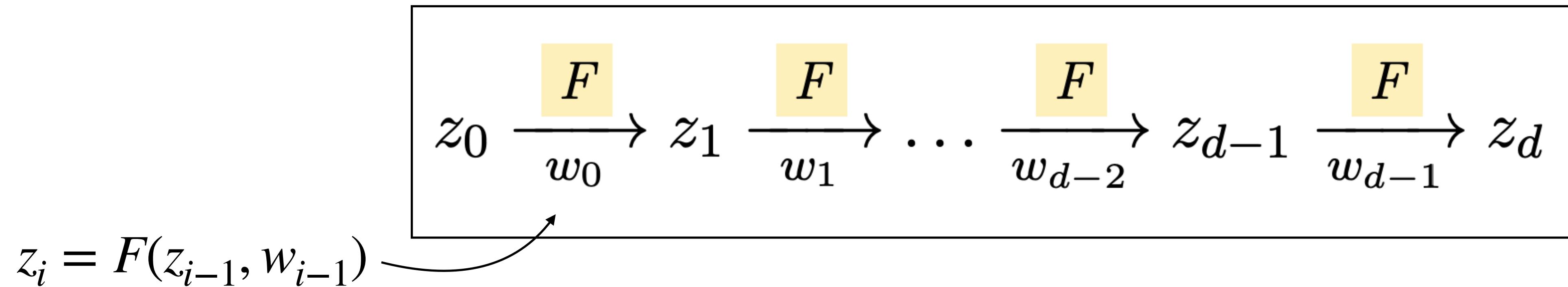
# Incremental Computations



**Examples of natural applications:**

- Streaming algorithms

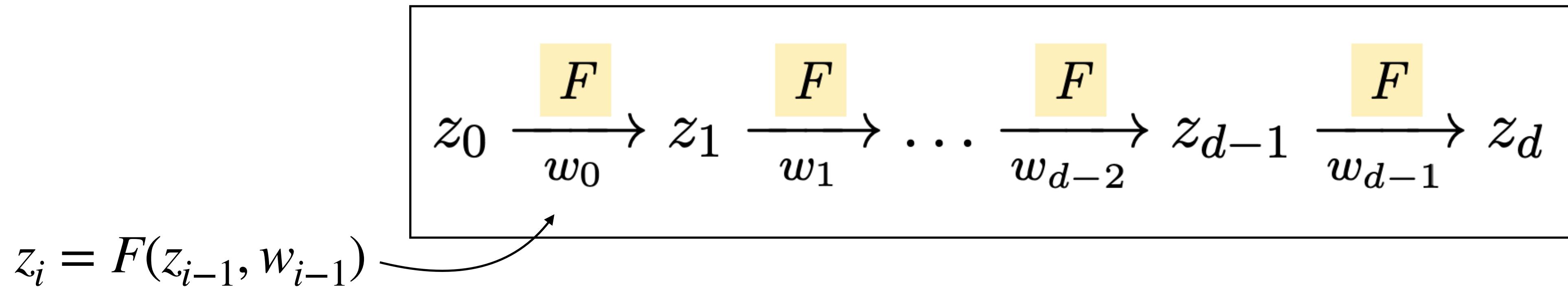
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## Examples of natural applications:

- Streaming algorithms
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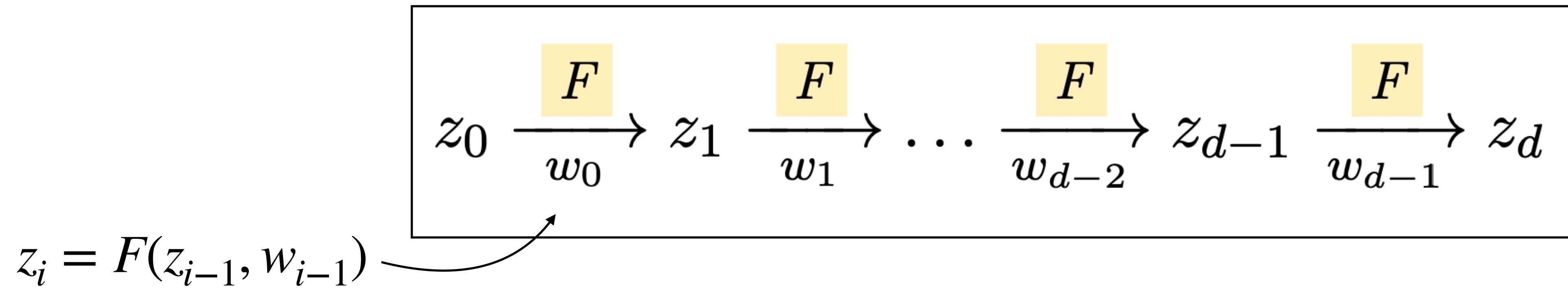
# Incremental Computations



## Examples of natural applications:

- Streaming algorithms
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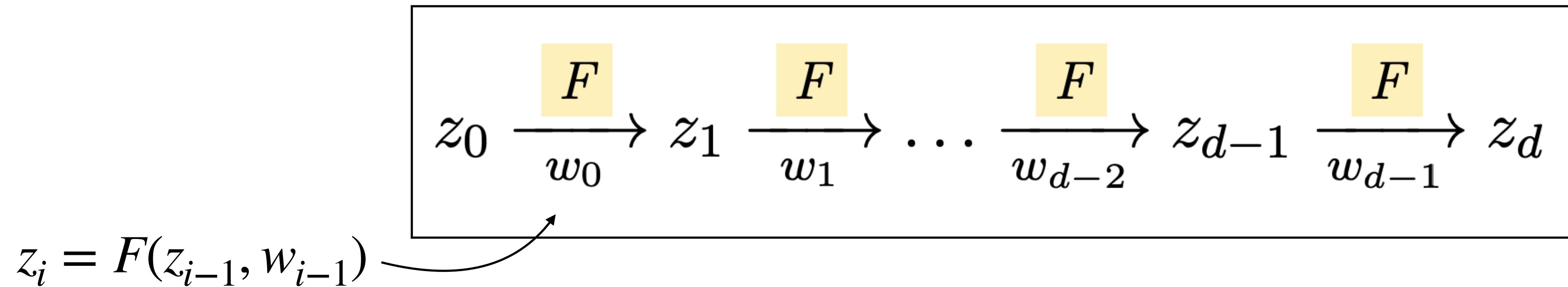
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## Examples of natural applications:

- Streaming algorithms
- RAM computations
- Verifiable Delay Functions (VDF)
- Round functions in symmetric primitives

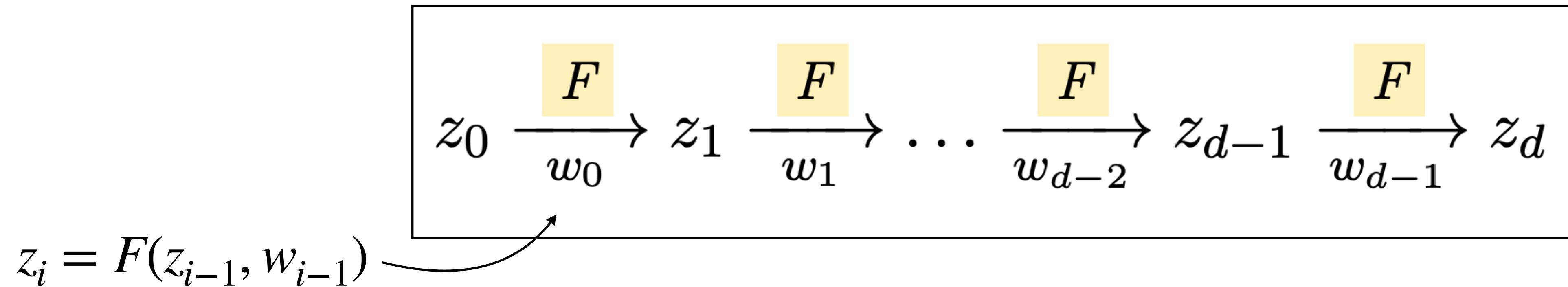
# Incremental Computations



## Examples of natural applications:

- Streaming algorithms
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- Verifiable Delay Functions (VDF)
- Round functions in symmetric primitives
- Recurrent neural networks

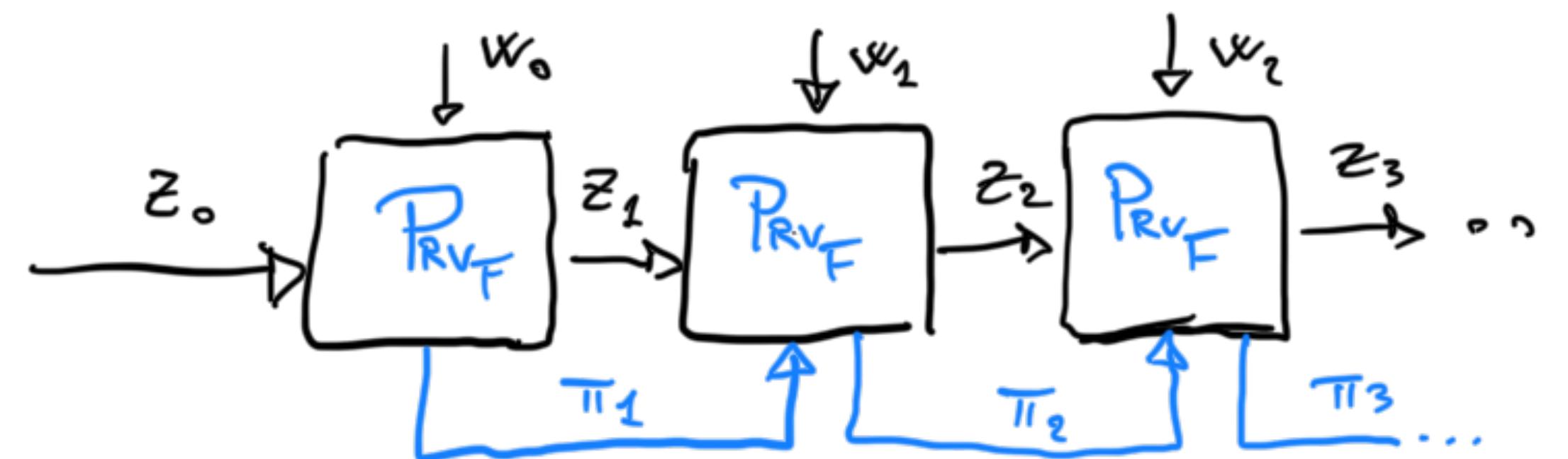
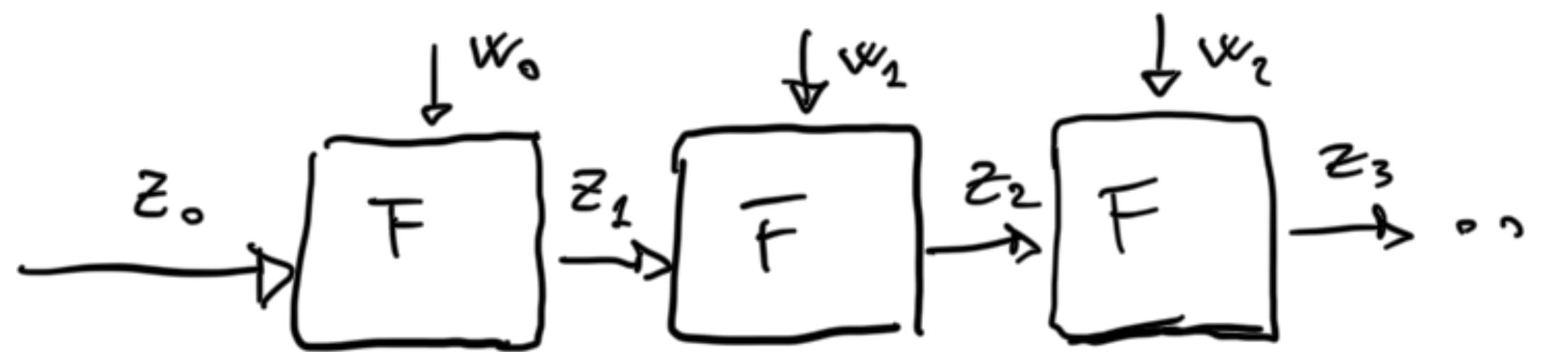
# Incremental Computations



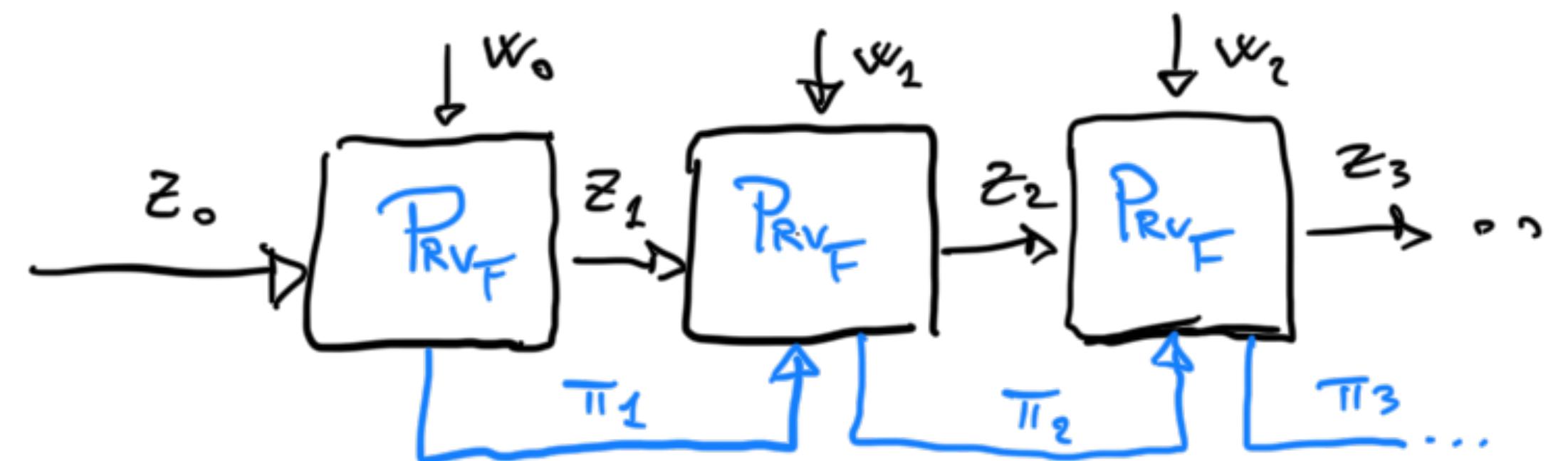
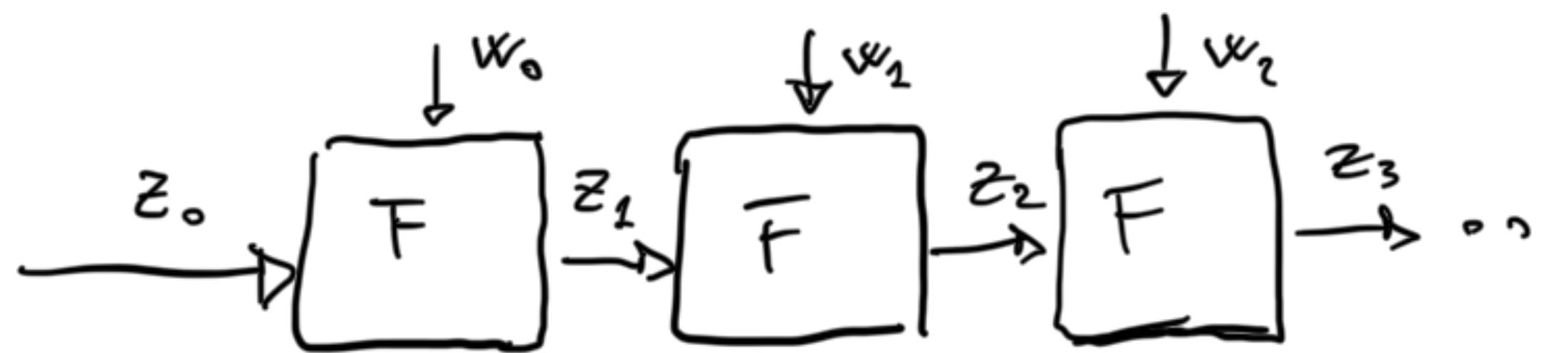
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# Incrementally Verifiable Computations (IVC)

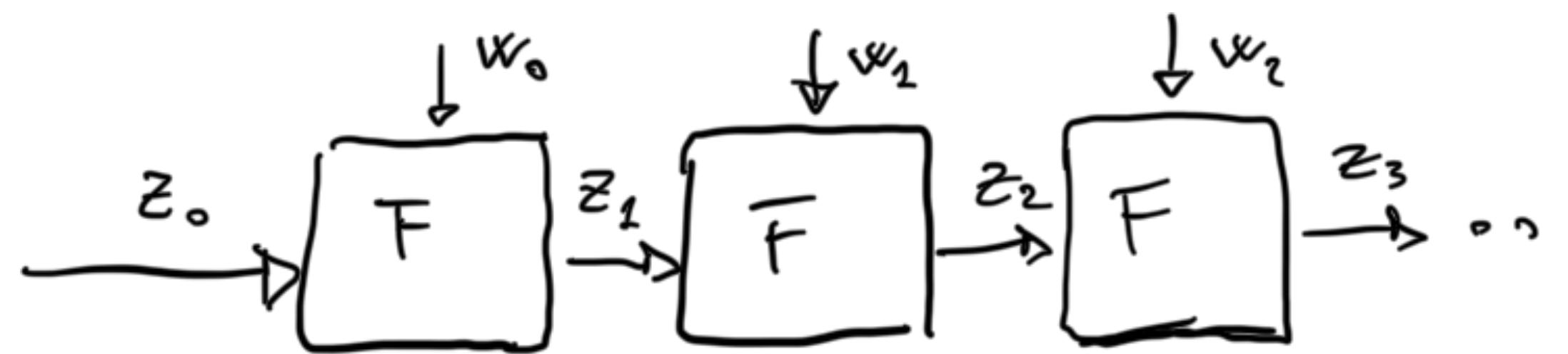


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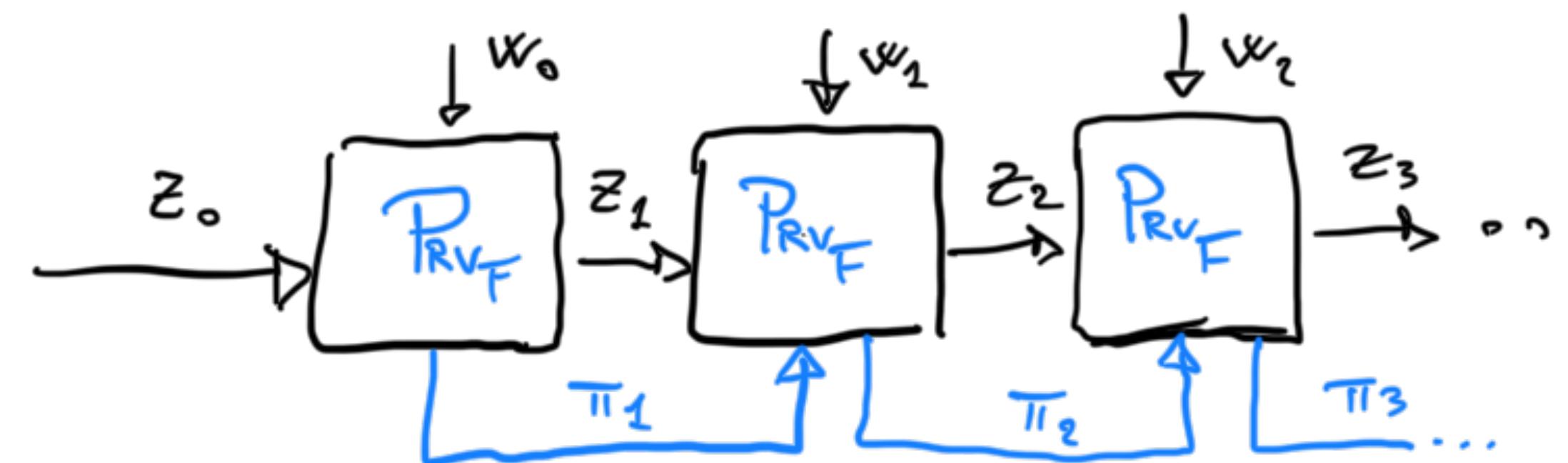


Proof size should be sublinear in the # of steps (the *depth* of the computation)

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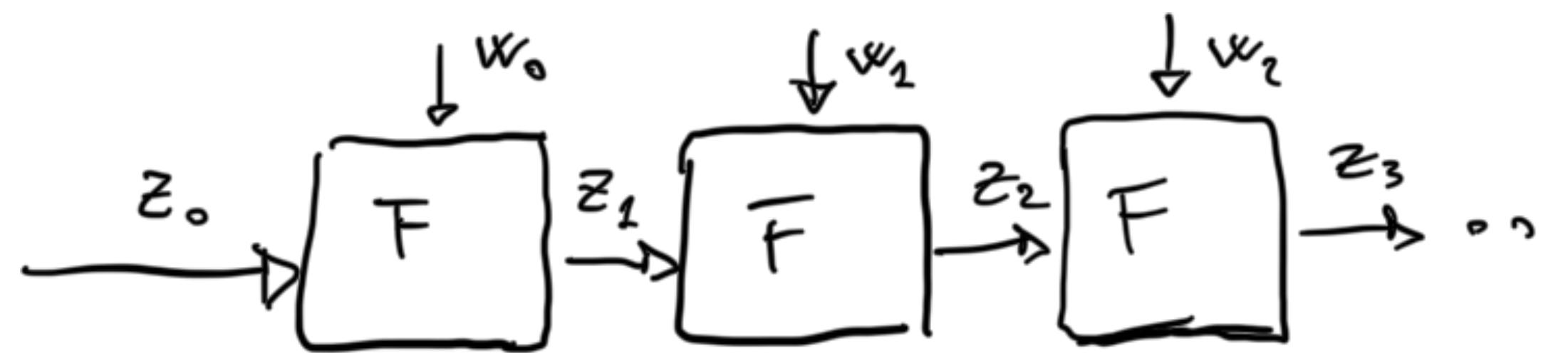


**Advantages**



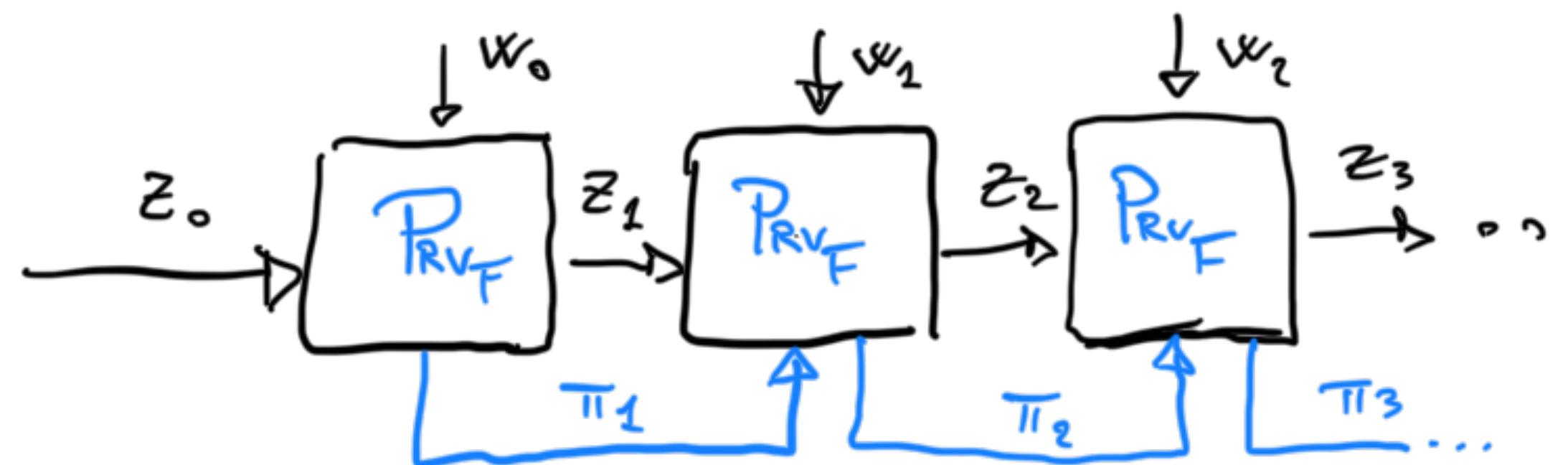
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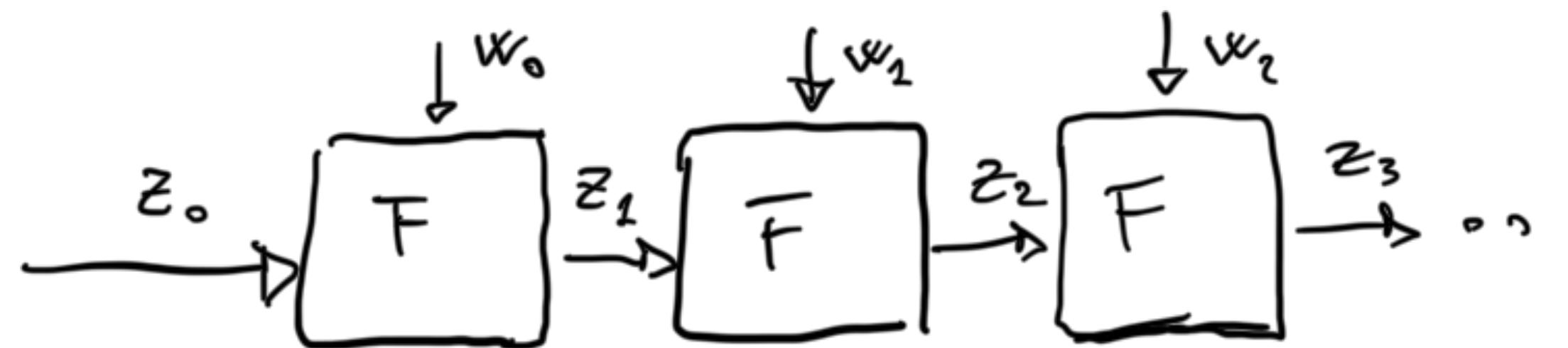
## Advantages

- Low memory footprint



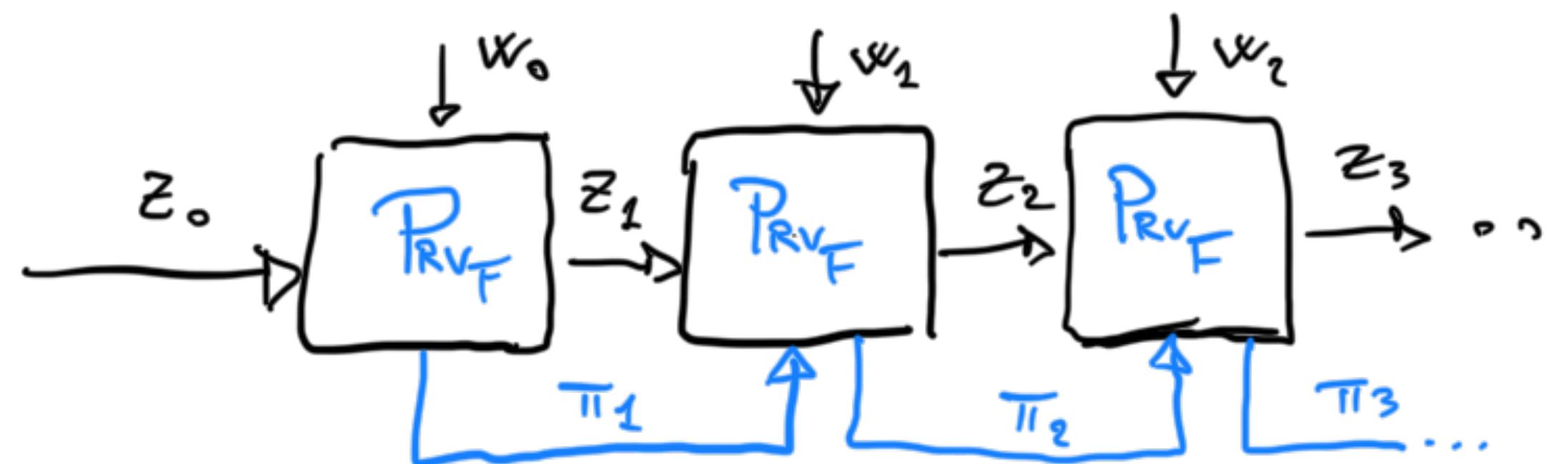
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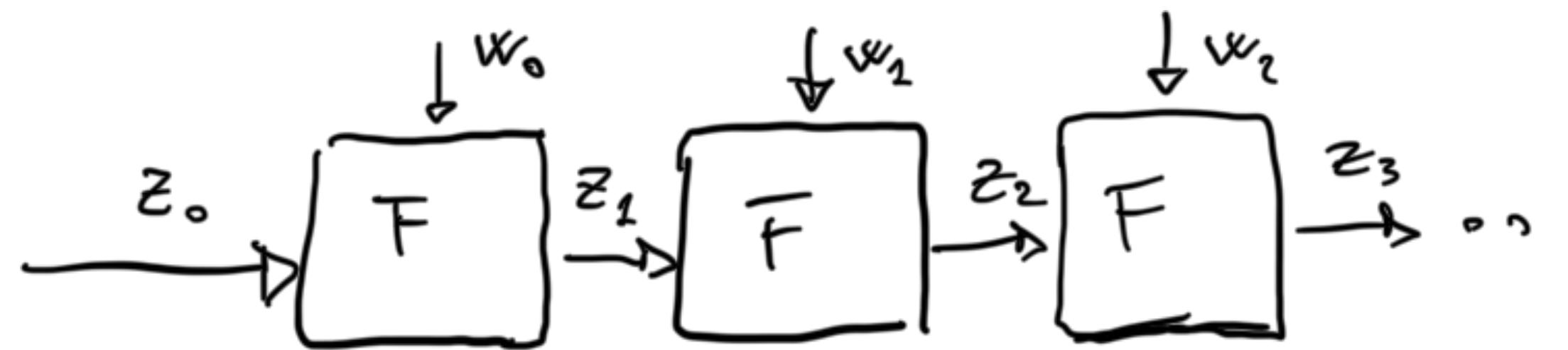
## Advantages

- Low memory footprint
- Pipelining opportunities



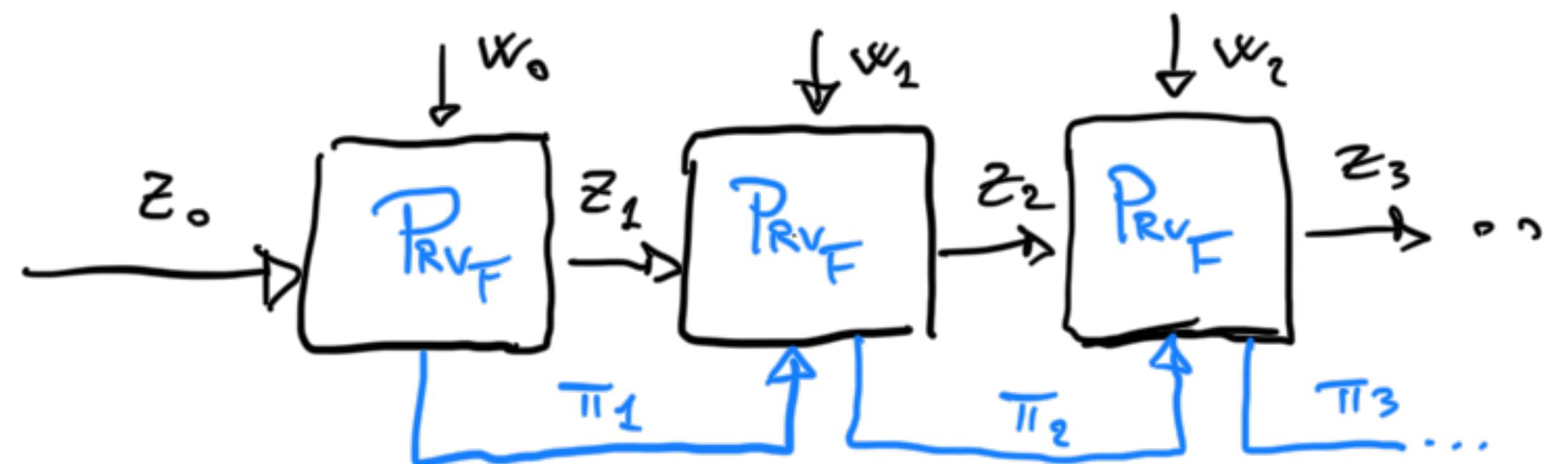
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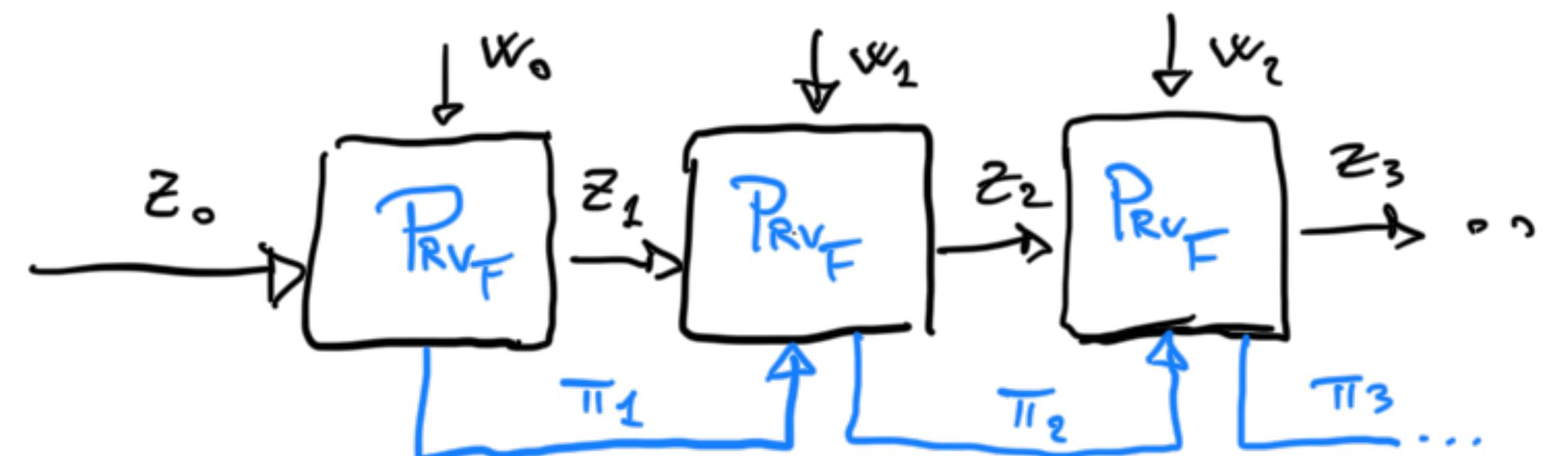
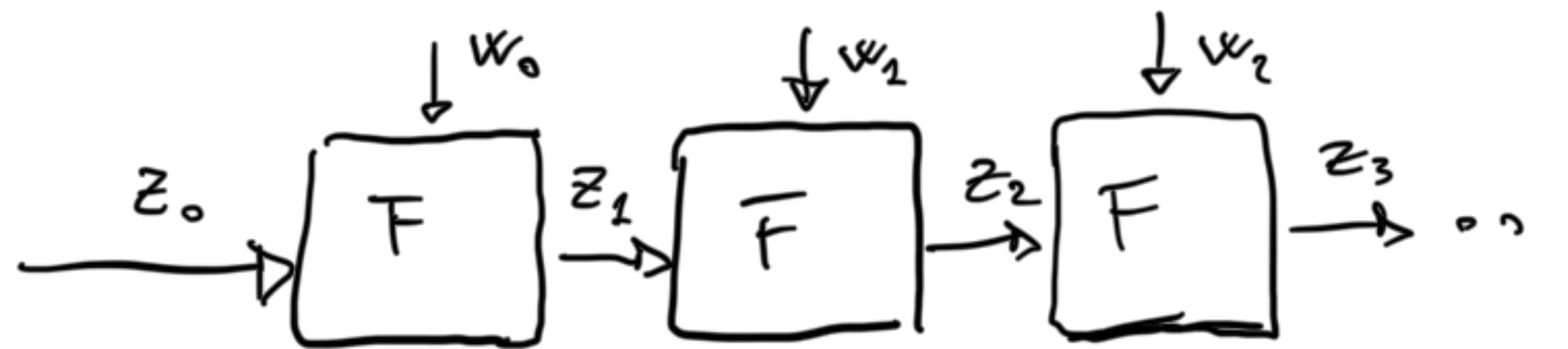
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- Low memory footprint
- Pipelining opportunities
- Natural model for incremental computations



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# Incrementally Verifiable Computations (IVC)



## Advantages

- Low memory footprint
- Pipelining opportunities
- Natural model for incremental computations
- Proofs can be distributed (e.g., in settings with zero-knowledge)

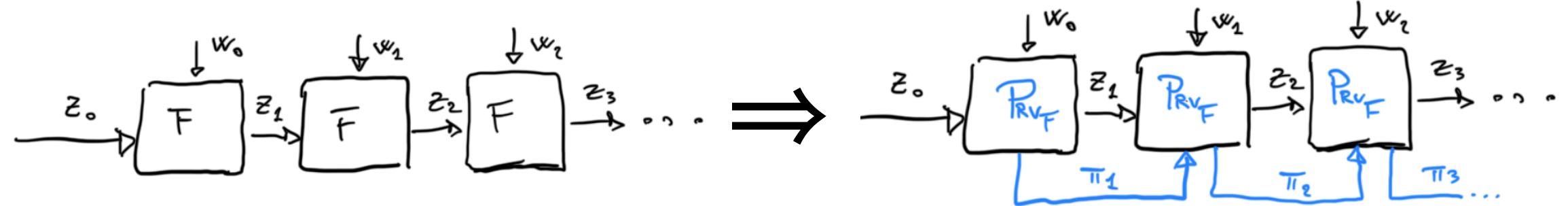
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# **Constructions of IVC**

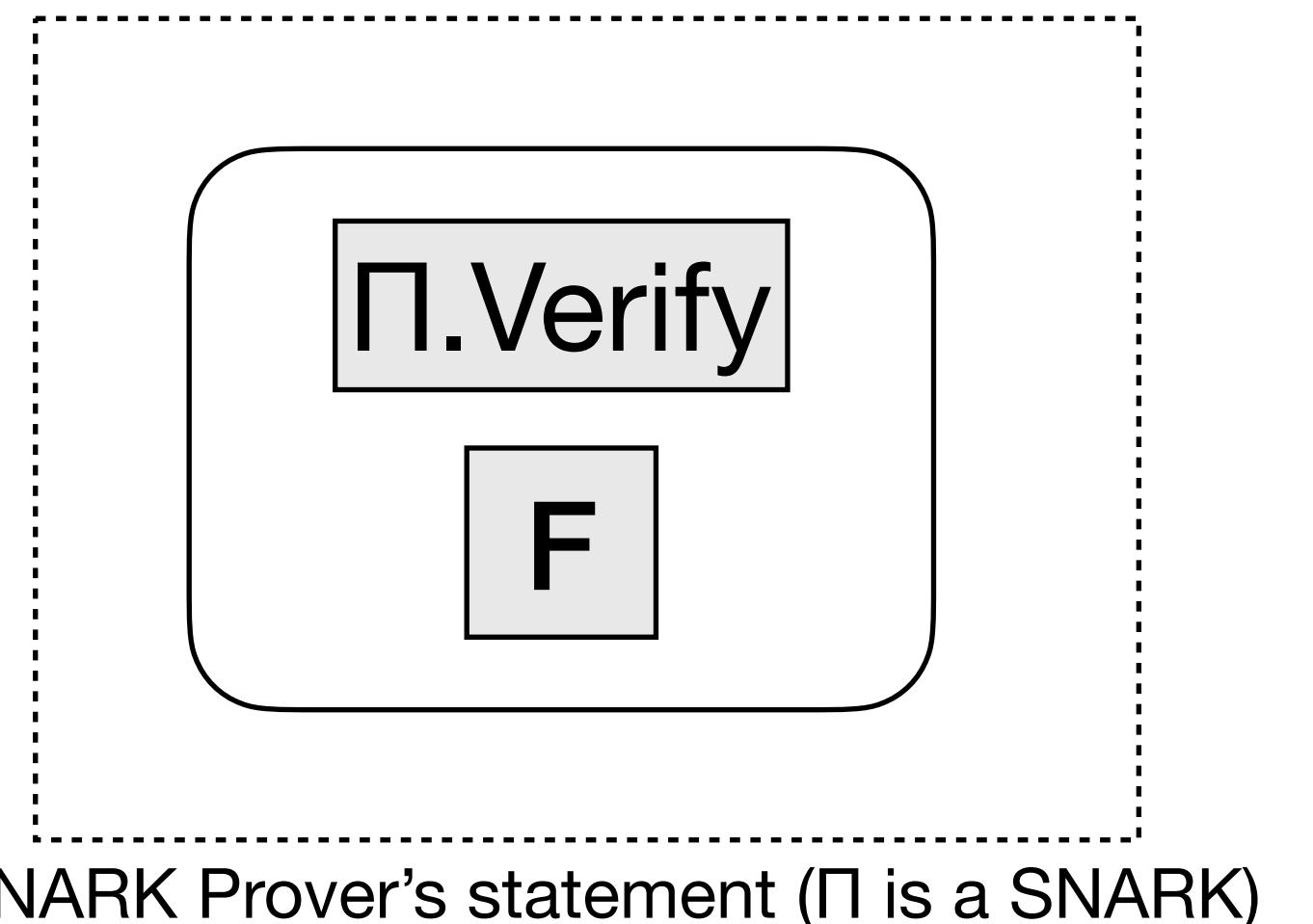
## **(Practical or nearly-practical)**

# Constructions of IVC

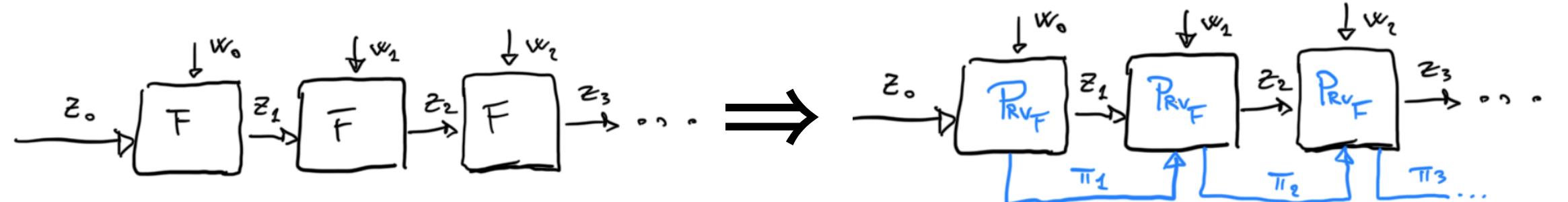
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# Constructions of IVC (Practical or nearly-practical)



**Canonical construction  
(SNARK recursion)**

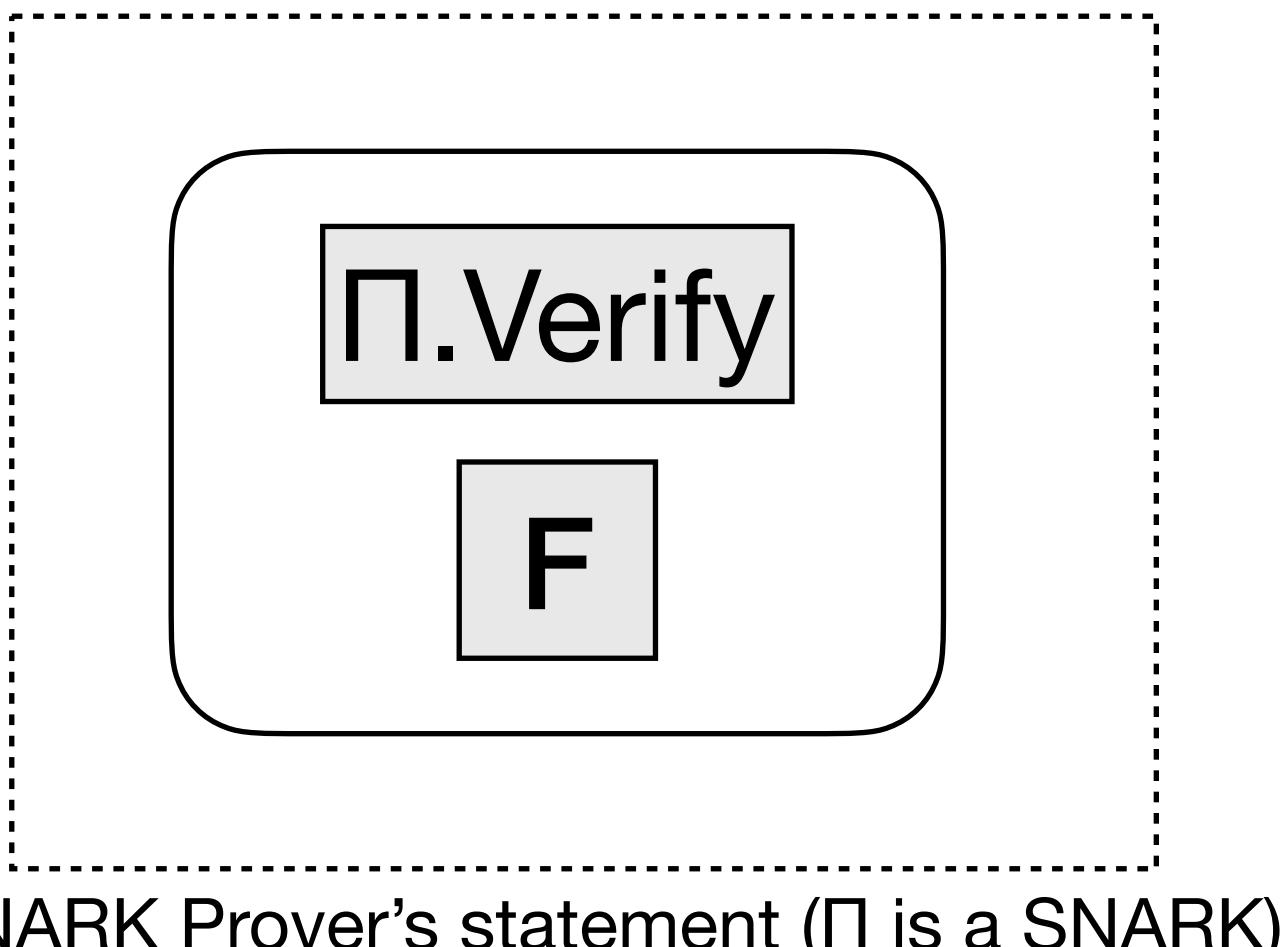


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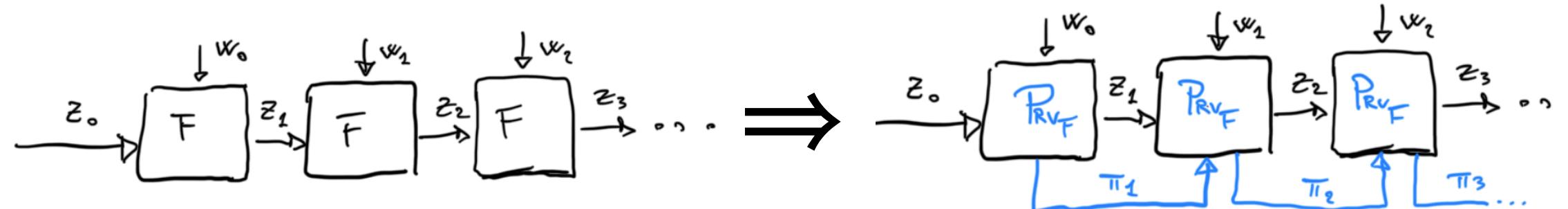
Nova: Recursive Zero-Knowledge Arguments  
from Folding Schemes

Abhiram Kothapalli<sup>†</sup> Srinath Setty<sup>\*</sup> Ioanna Tzialla<sup>‡</sup>

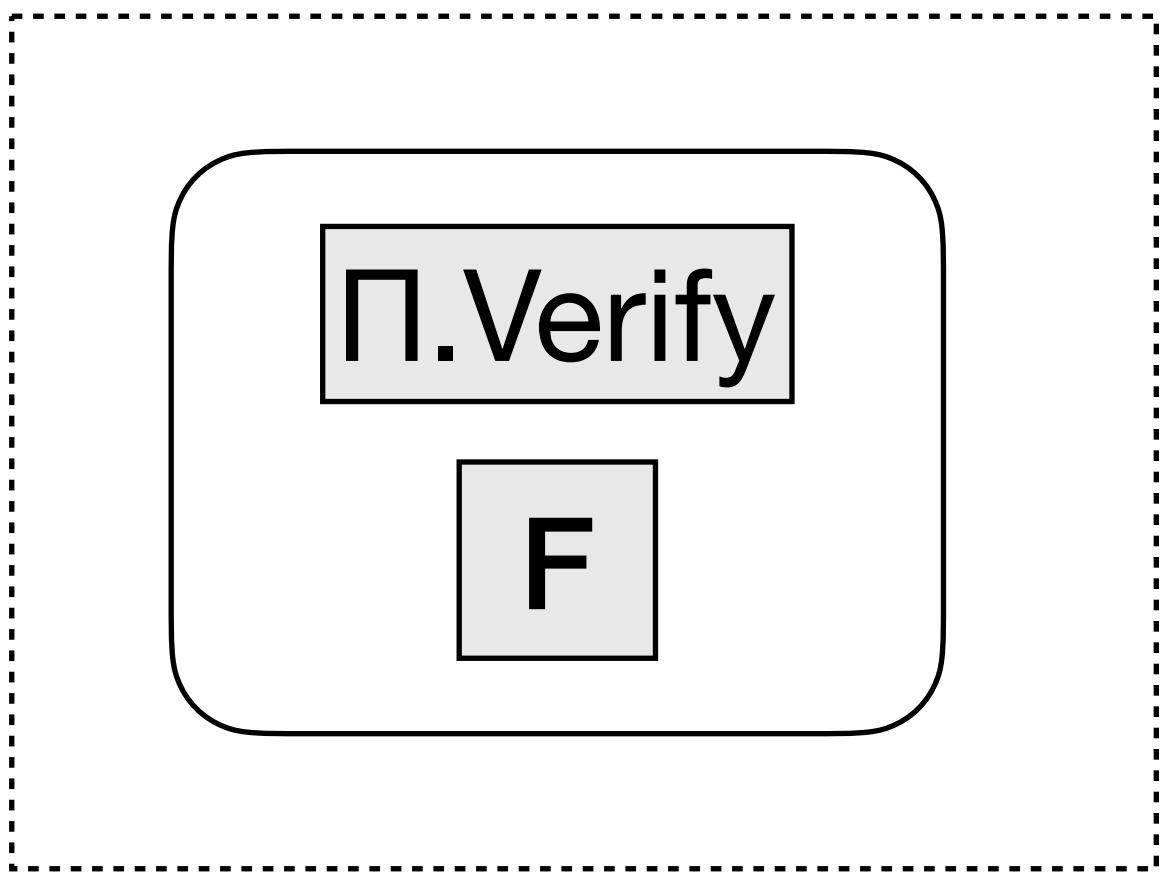
<sup>†</sup>Carnegie Mellon University <sup>\*</sup>Microsoft Research <sup>‡</sup>New York University



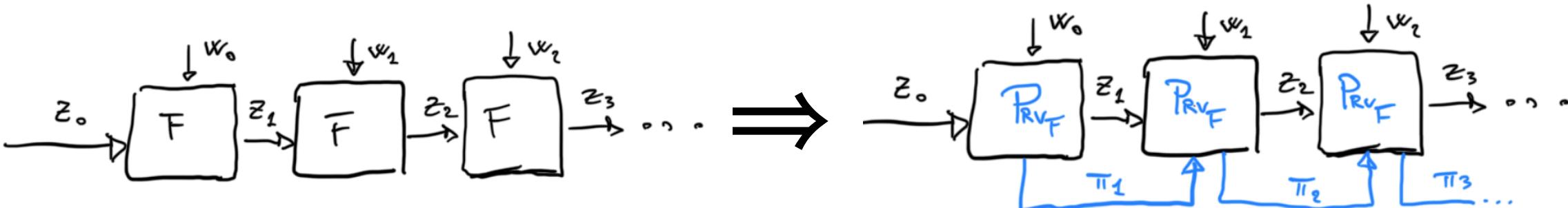
**Canonical construction  
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# Constructions of IVC (Practical or nearly-practical)



## Canonical construction (SNARK recursion)



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Proof-Carrying Data without Succinct Arguments

Benedikt Bünz  
benedikt@cs.stanford.edu  
Stanford University

Alessandro Chiesa  
alexch@berkeley.edu  
UC Berkeley

William Lin  
will.lin@berkeley.edu  
UC Berkeley

Pratyush Mishra  
pratyush@berkeley.edu  
UC Berkeley

Nicholas Spooner  
nspoone@bu.edu  
Boston University

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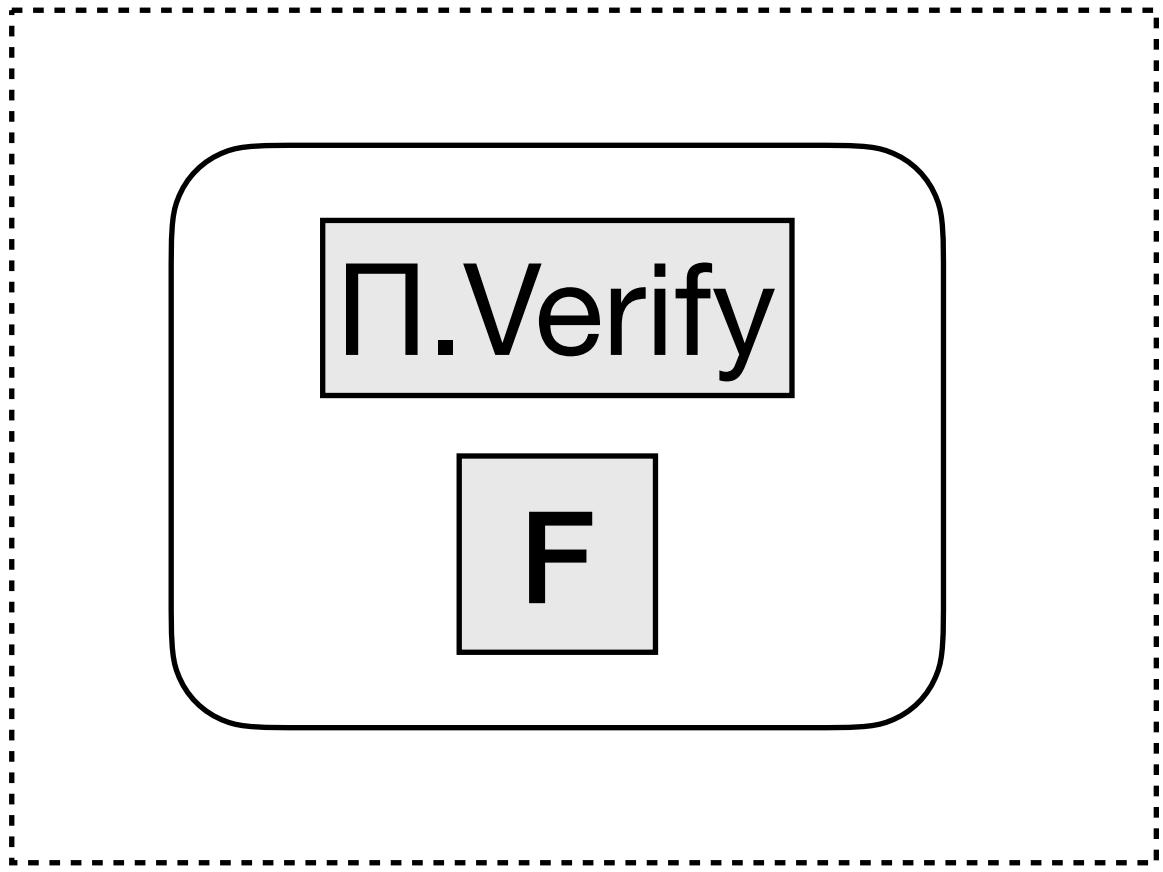
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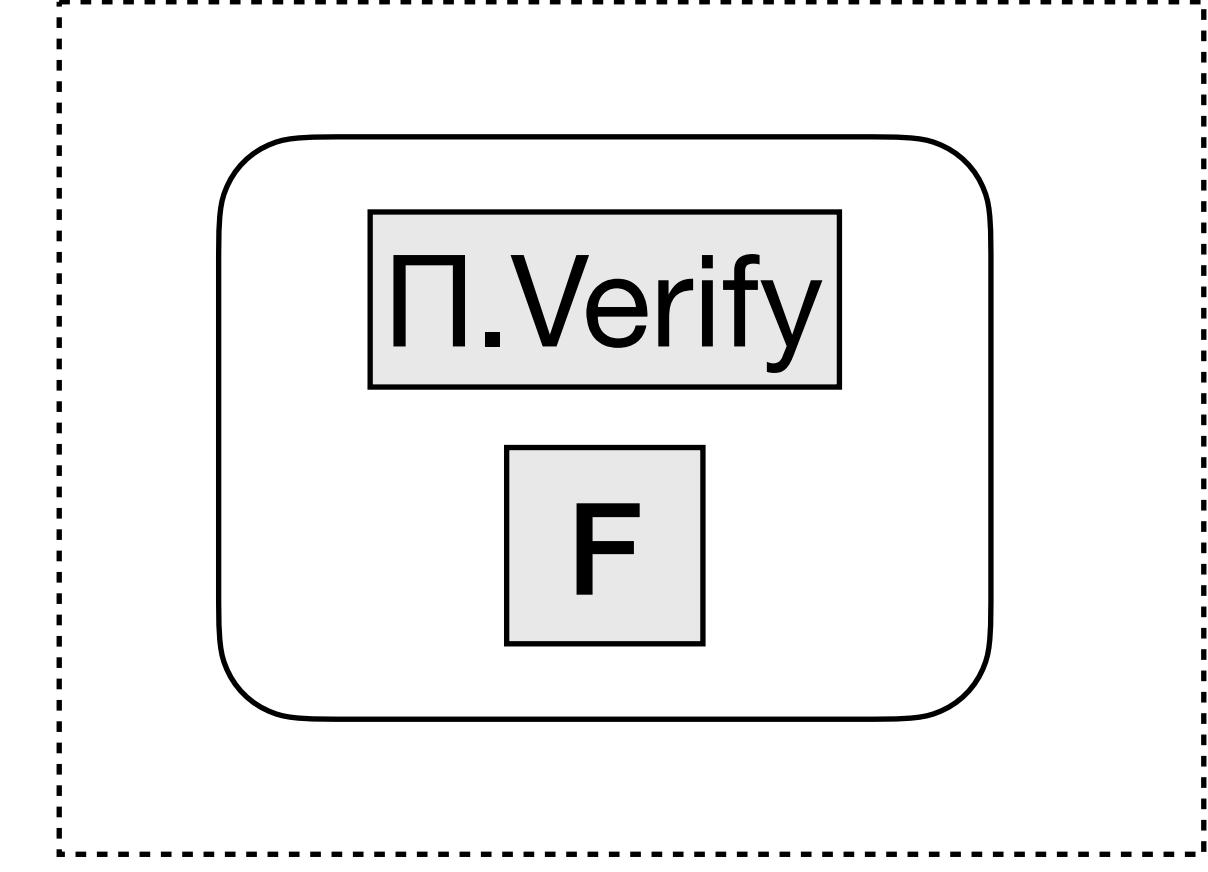
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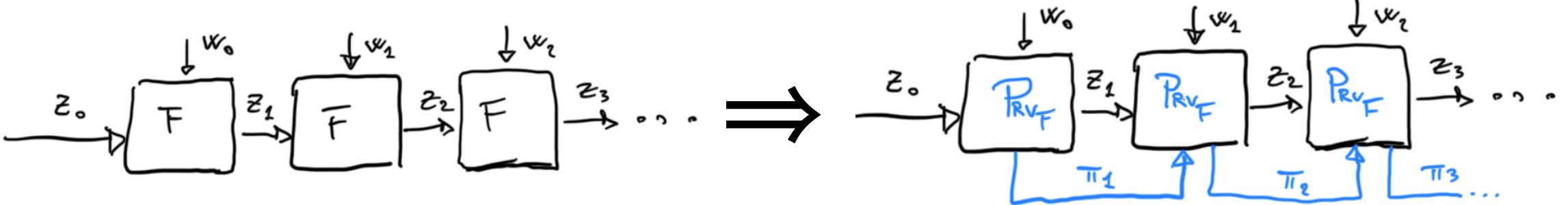
SNARK Prover's statement ( $\Pi$  is a SNARK)

**Canonical construction  
(SNARK recursion)**



Folding/acc. Prover's statement ( $\Pi$  is a folding/acc. scheme)

**Lightweight version  
(folding/accumulation recursion)**



\* very approximate rendition (there are more details)

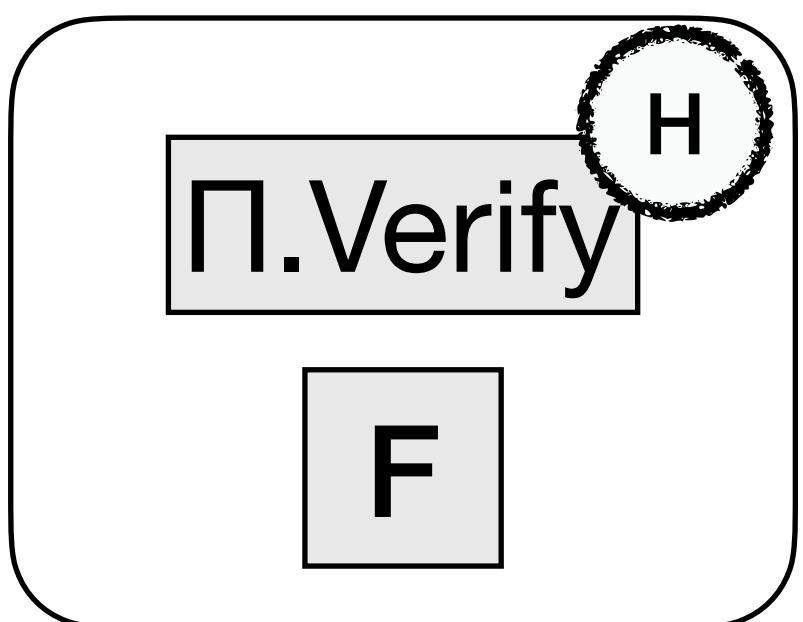
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- **First challenge:** idealized models and “theoretical hygiene”

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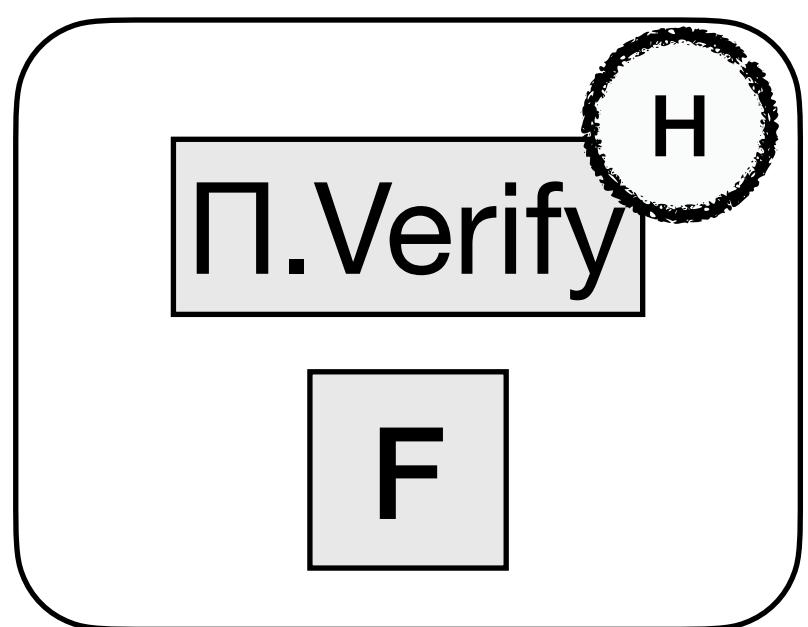
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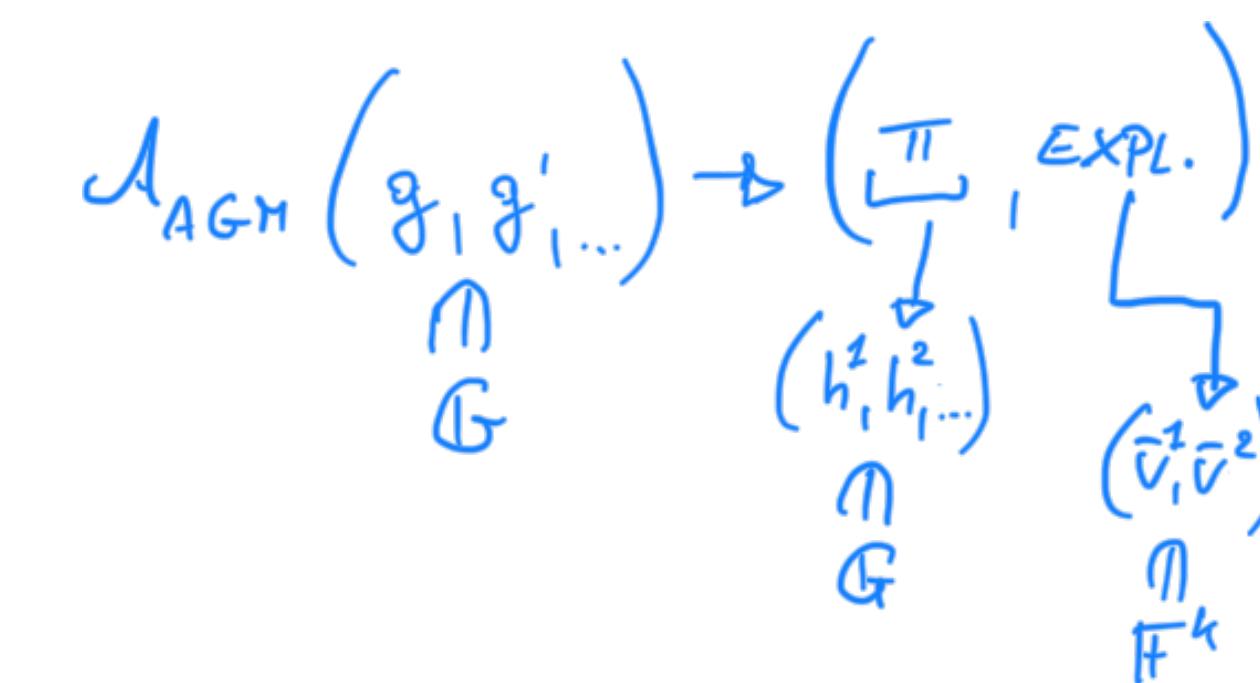
Random Oracle

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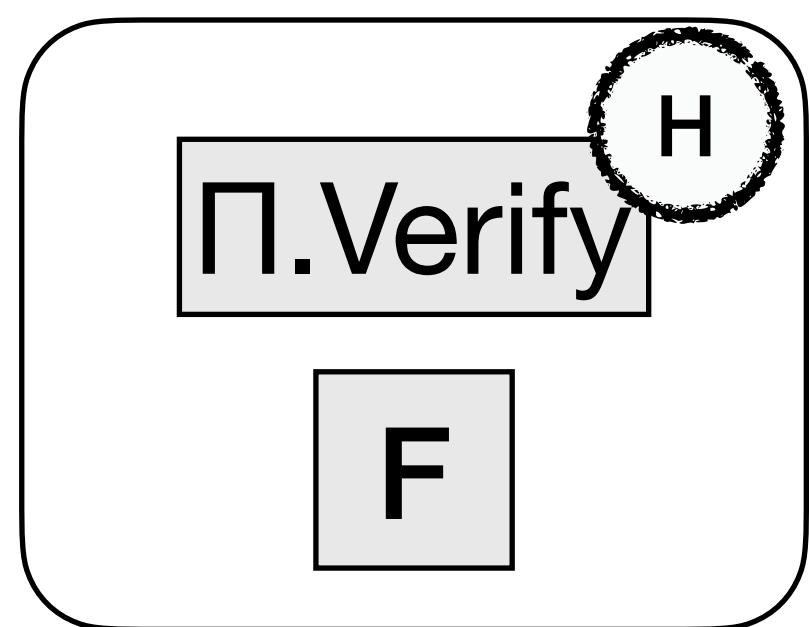
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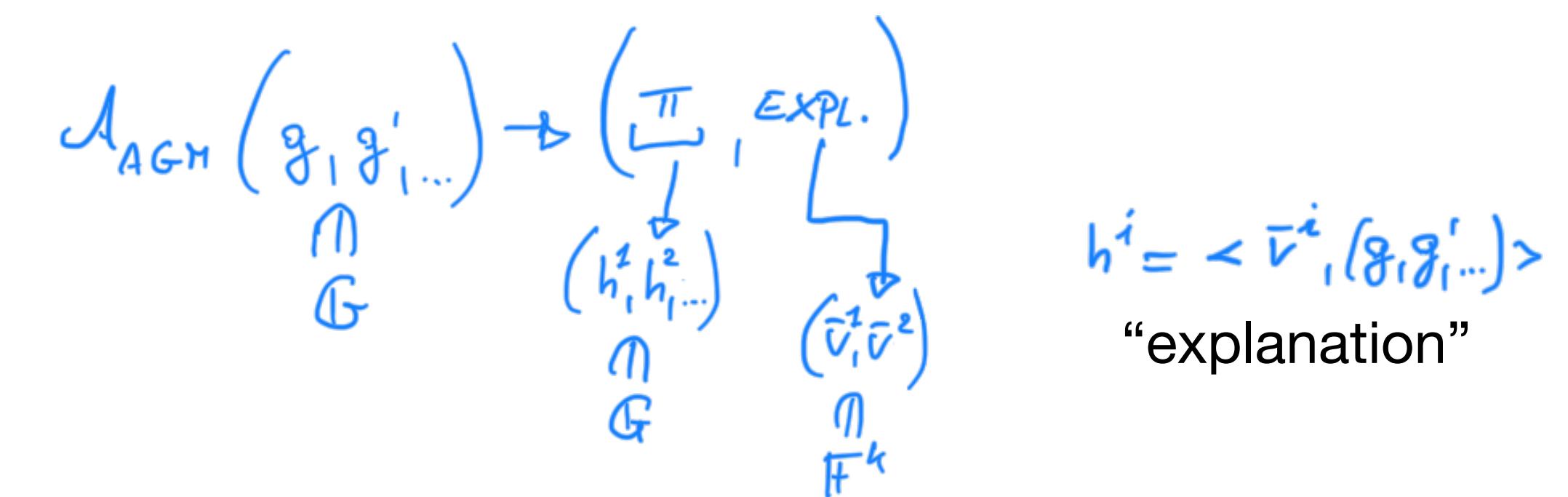
Algebraic Group Model (AGM)

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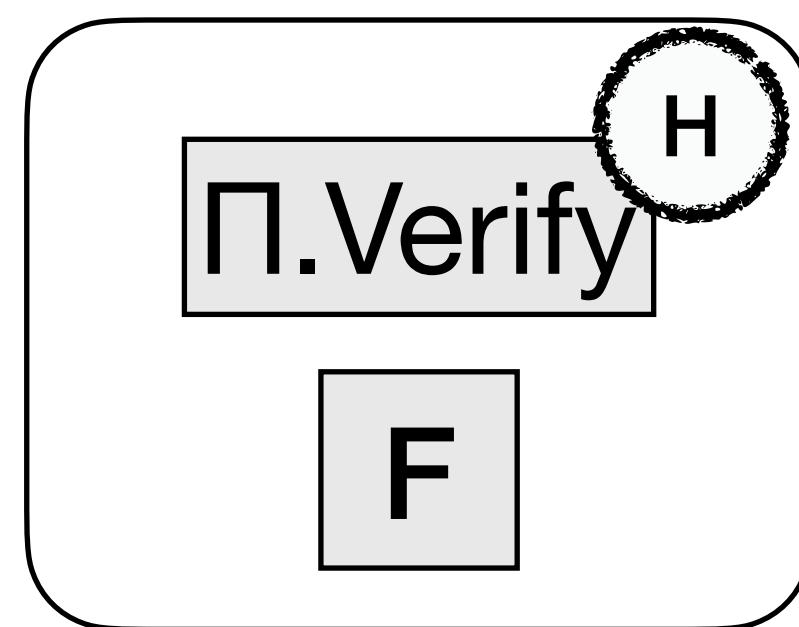
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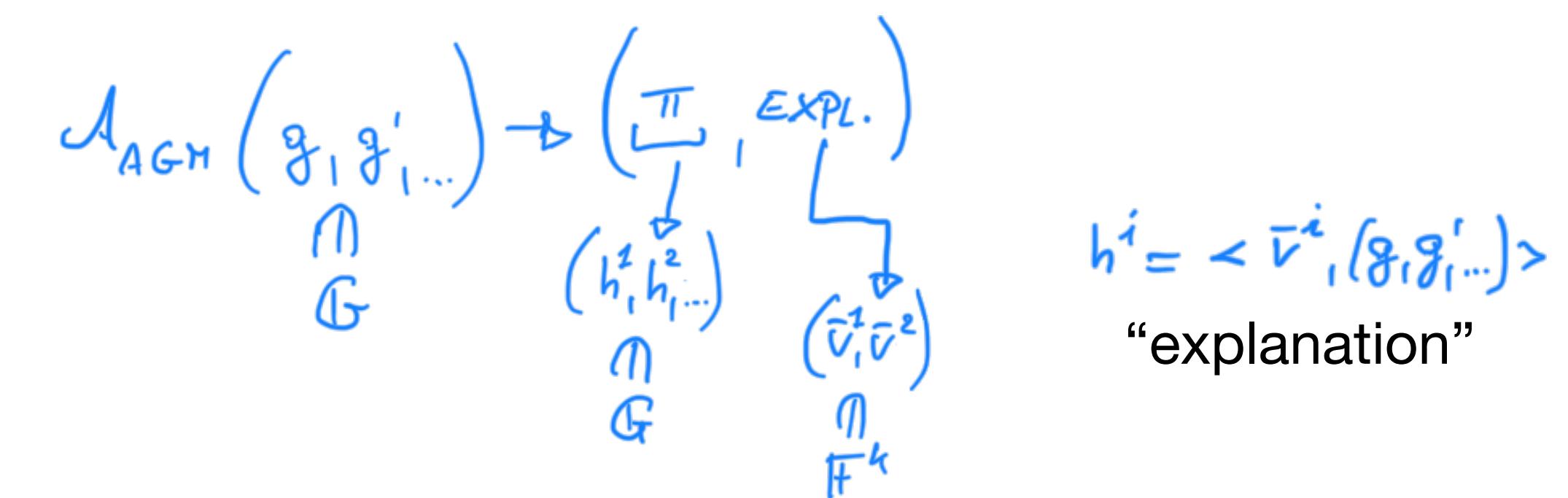
Algebraic Group Model (AGM)

# Challenges in Proving the Security of IVC

- **First challenge:** idealized models and “theoretical hygiene”



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Algebraic Group Model (AGM)

- **Second challenge (our focus):** depth of the computation

# How Do We Usually Prove Security in IVC?

A glimpse of what can go wrong and what depth has to do with it

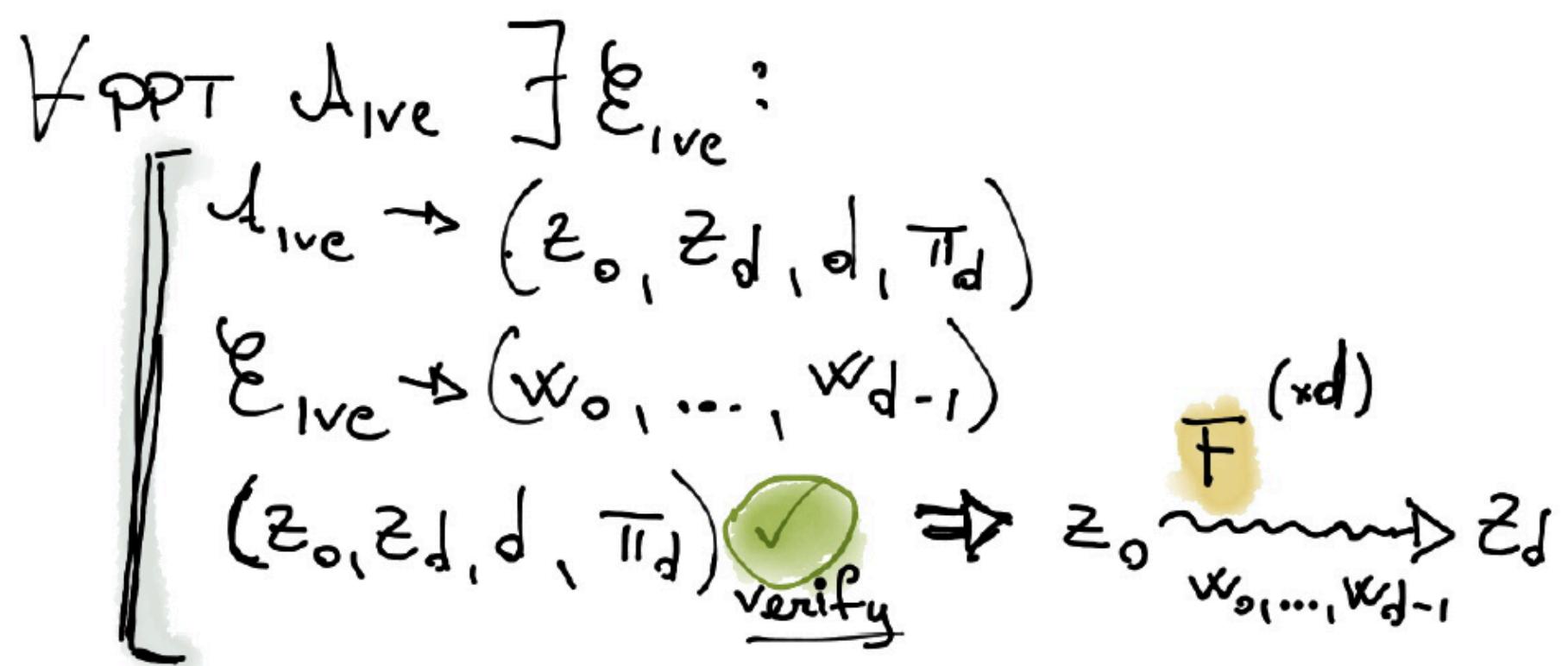
$$A_{\text{live}} \rightarrow (z_0, z_d, d, \pi_d)$$

$$z_0 \xrightarrow[w_0]{F} z_1 \xrightarrow[w_1]{F} \dots \xrightarrow[w_{d-2}]{F} z_{d-1} \xrightarrow[w_{d-1}]{F} z_d$$

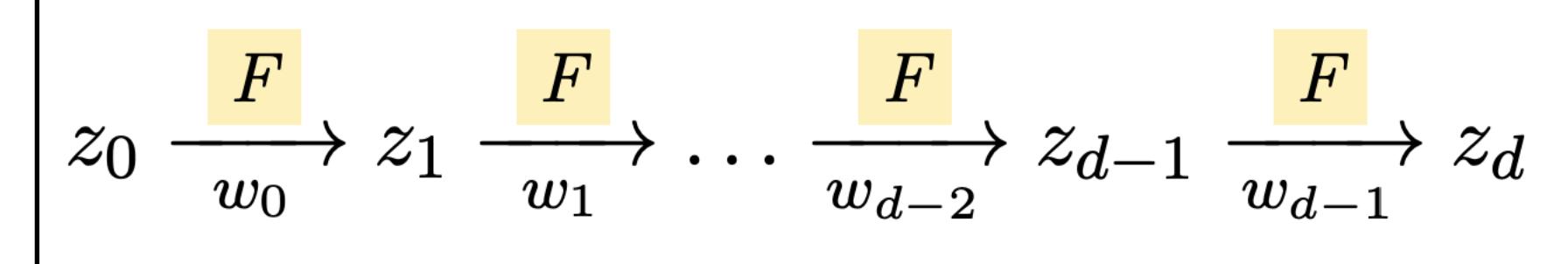
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## IVC extractability



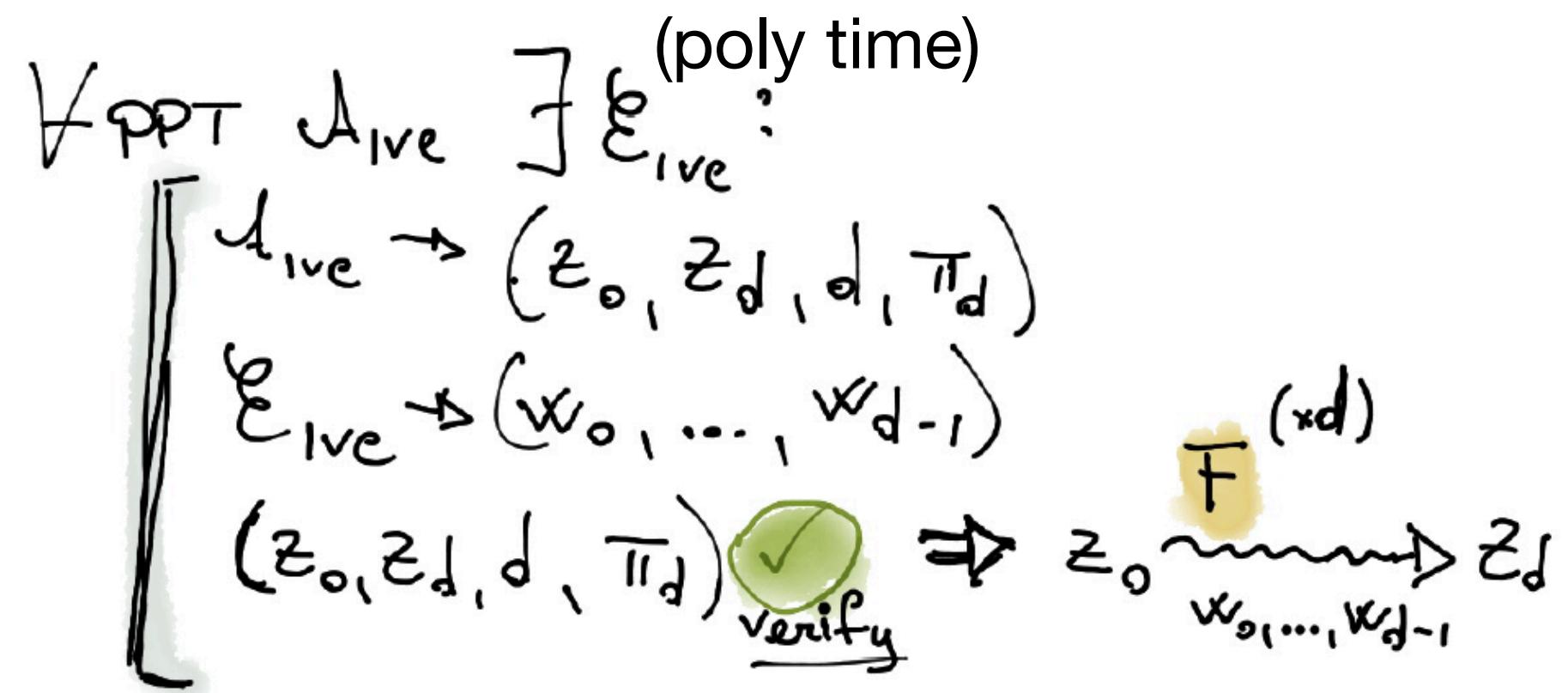
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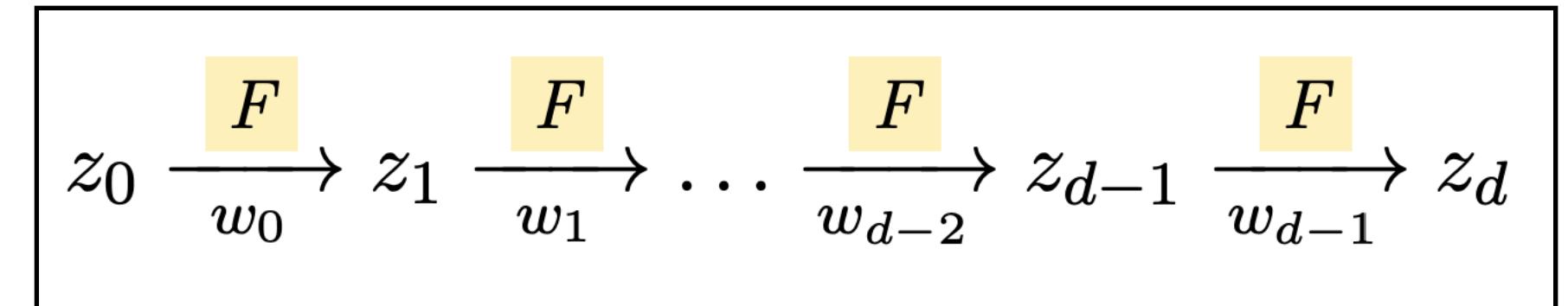
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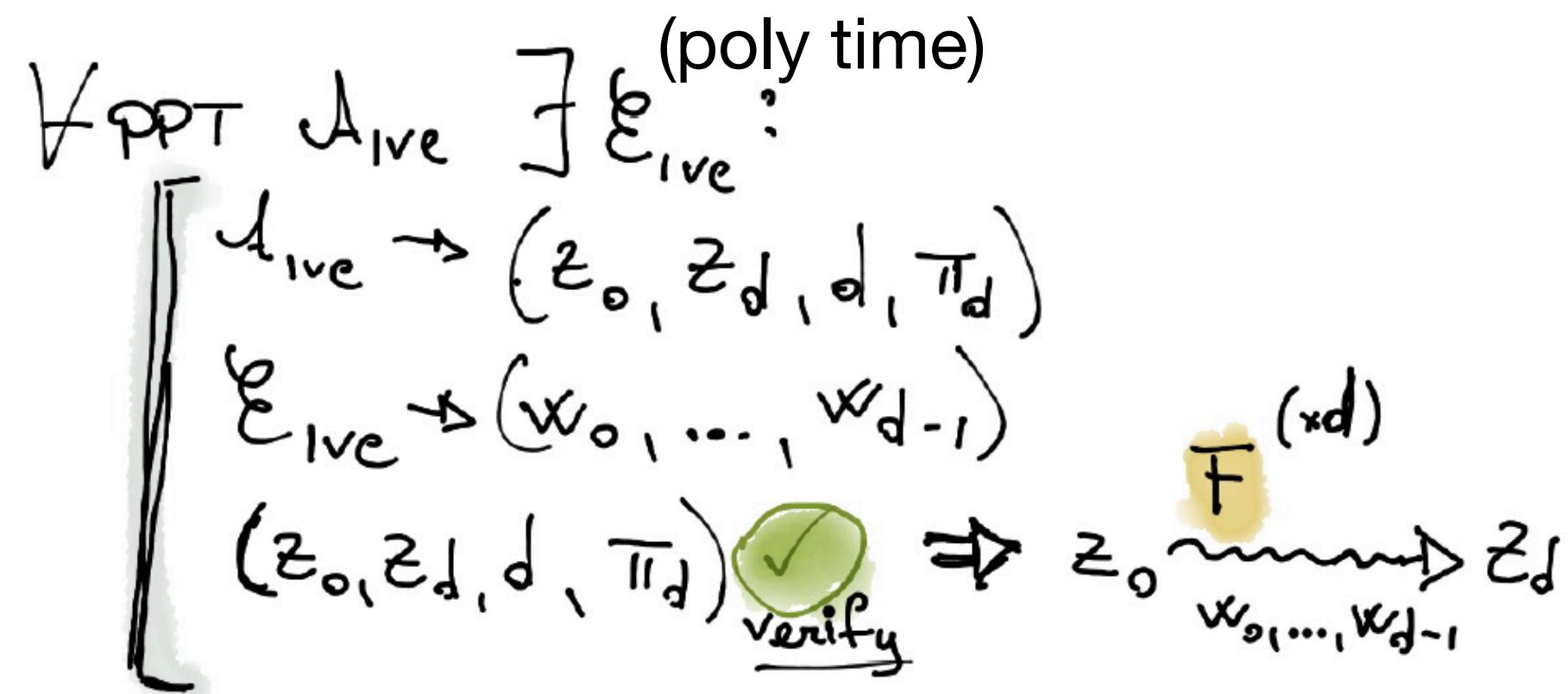
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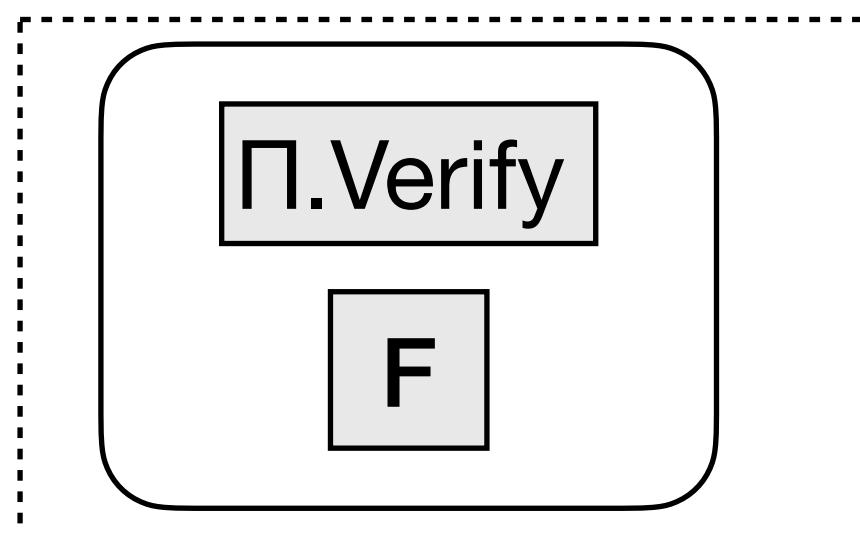
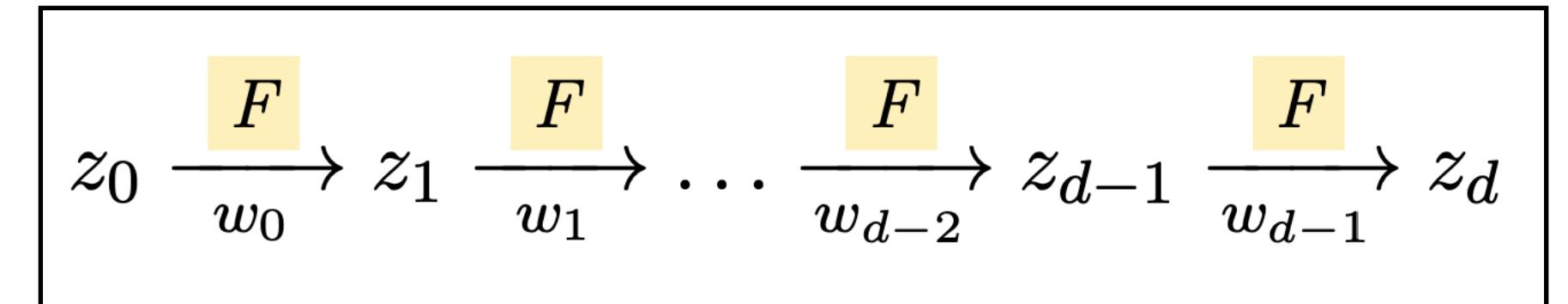
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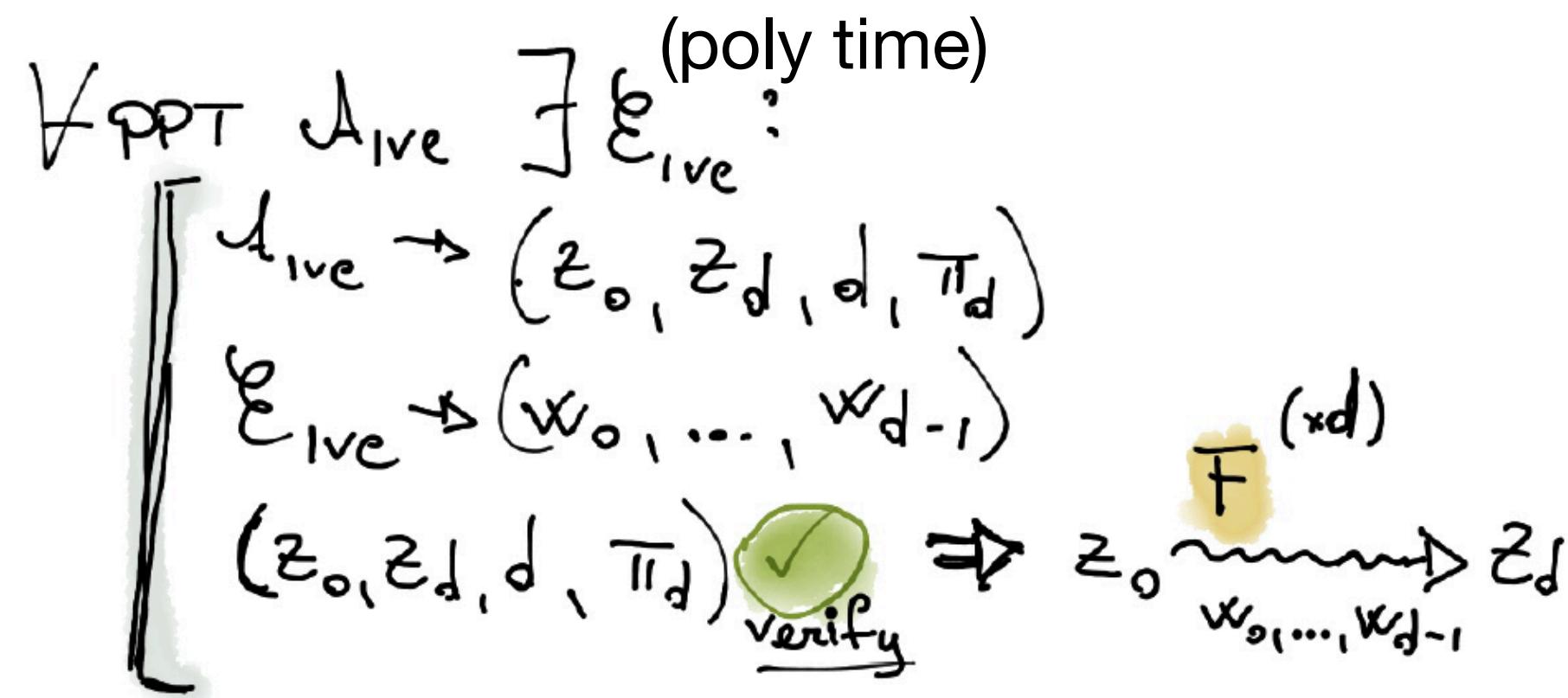


SNARK Prover's statement ( $\Pi$  is a SNARK)

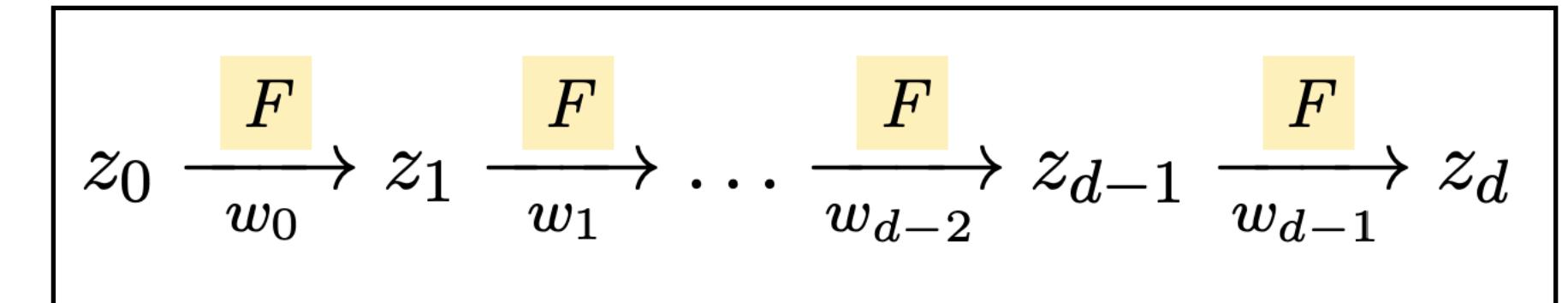
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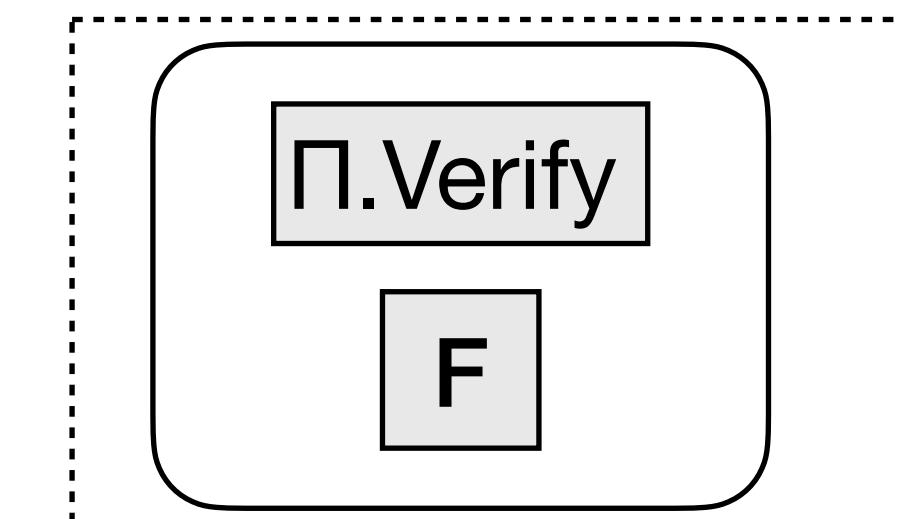
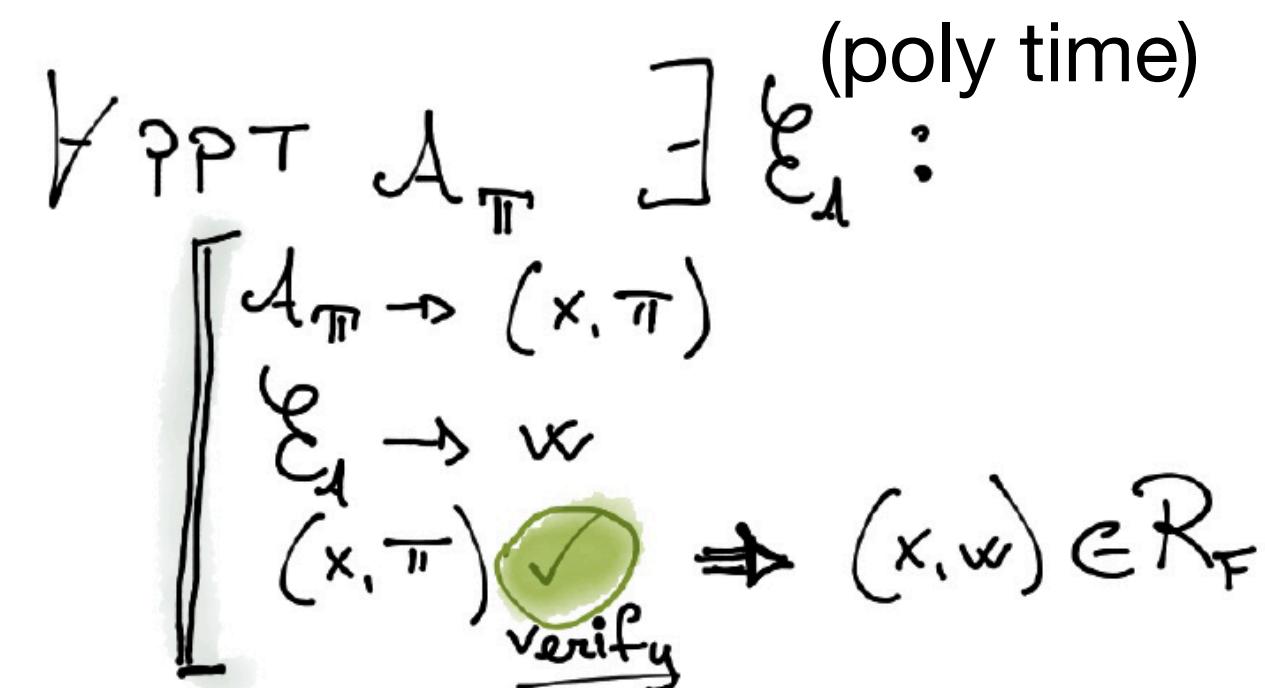
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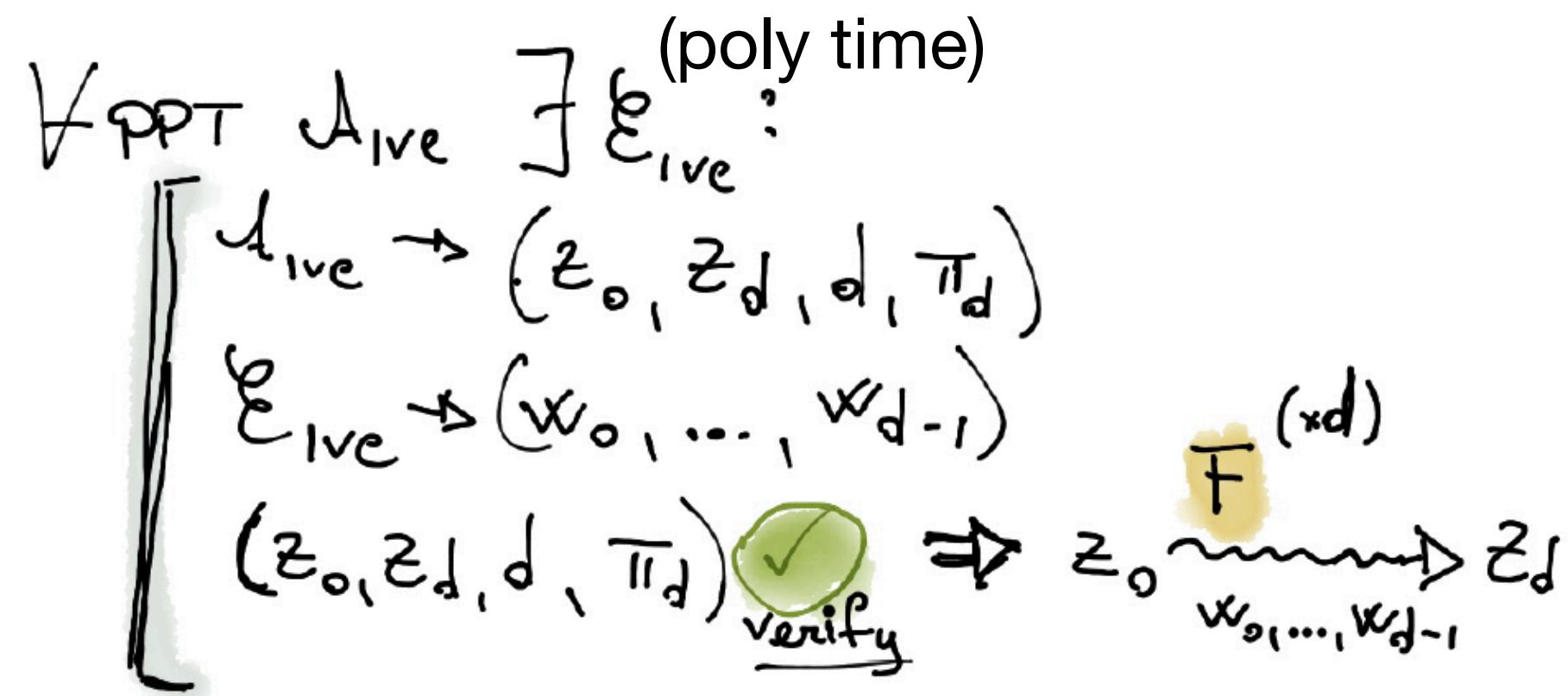


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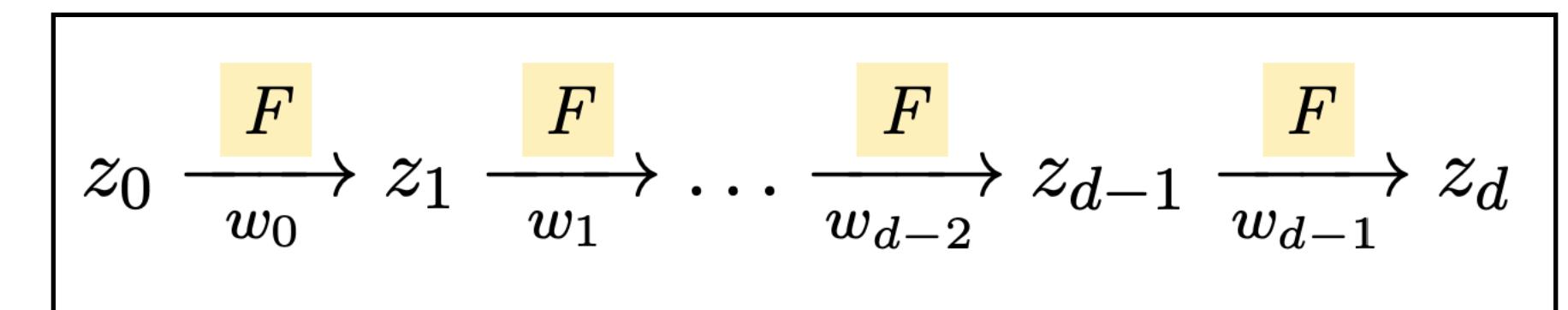
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A glimpse of what can go wrong and what depth has to do with it

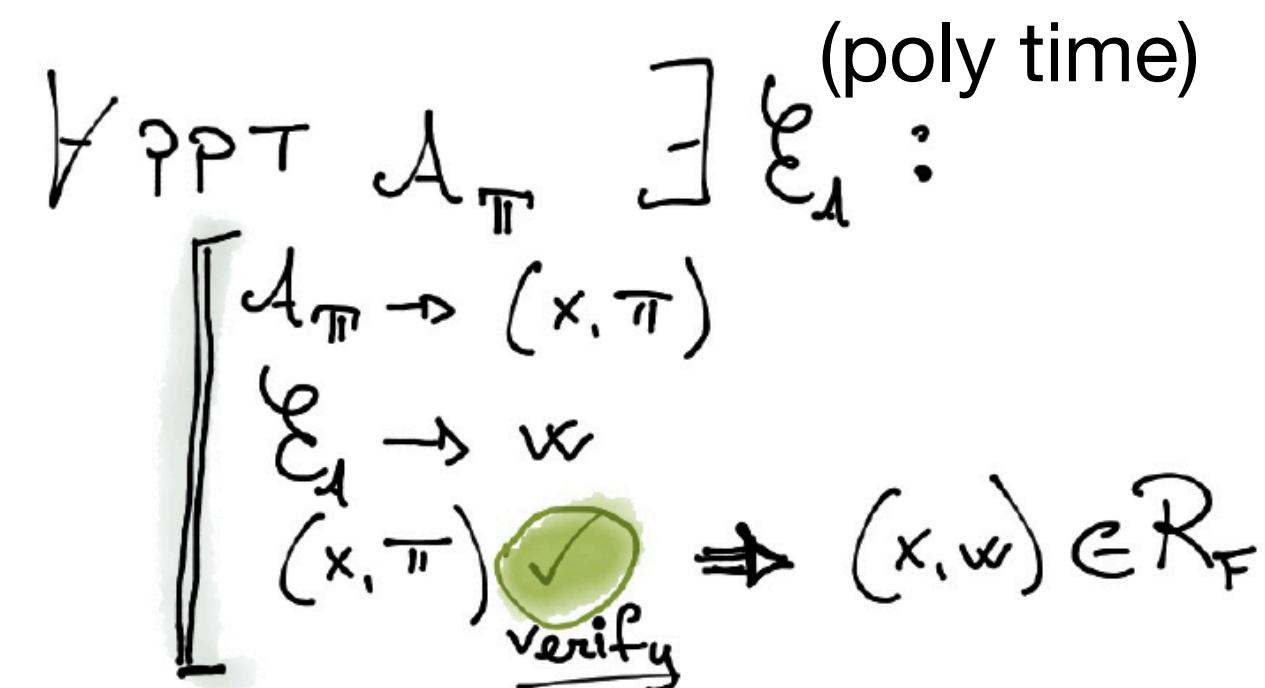
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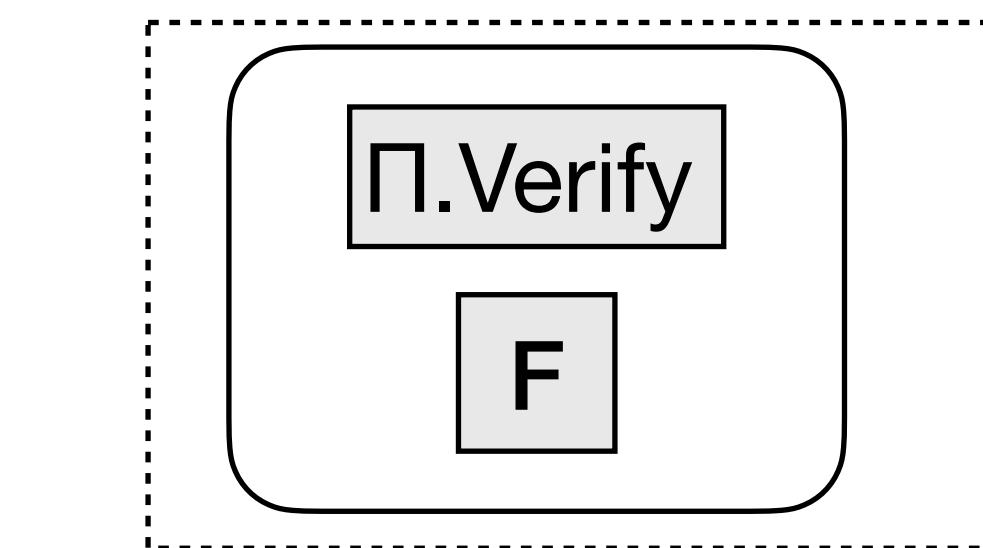


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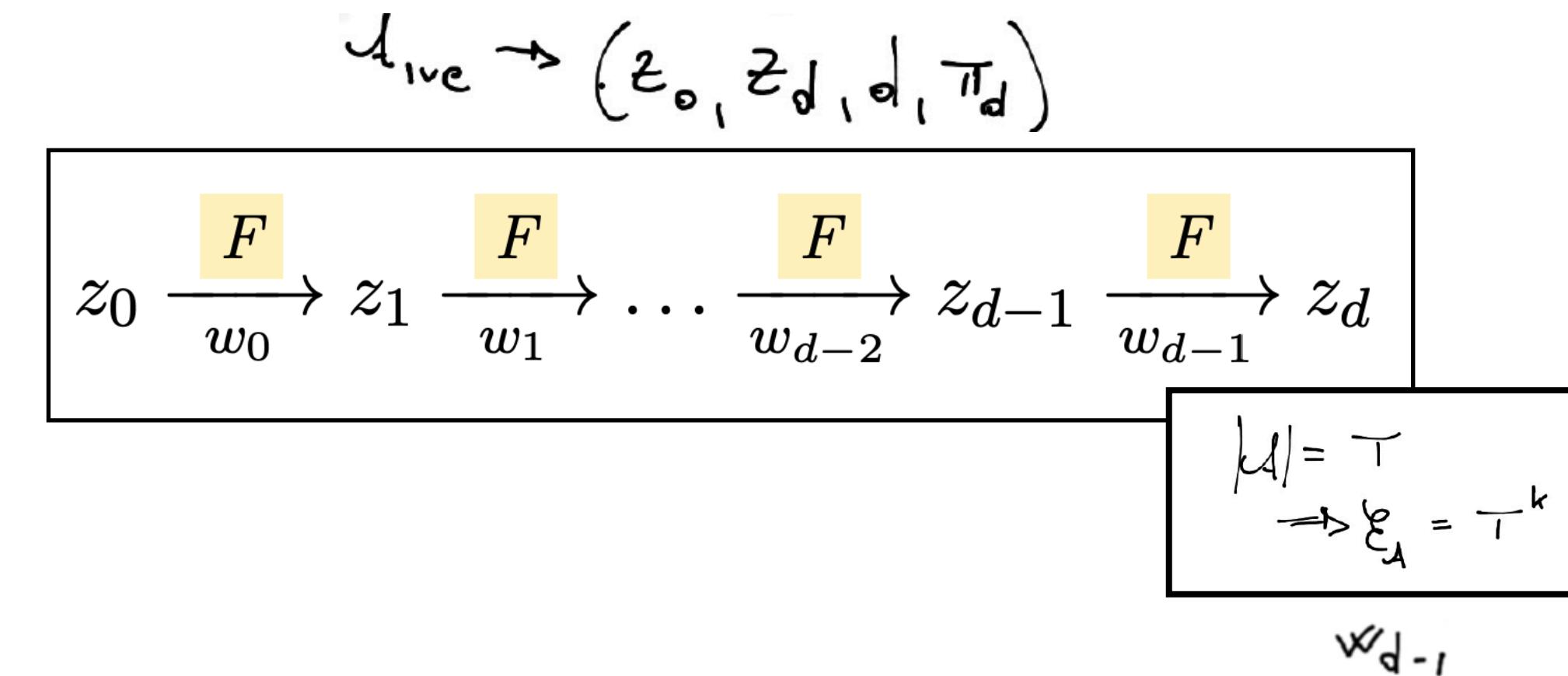
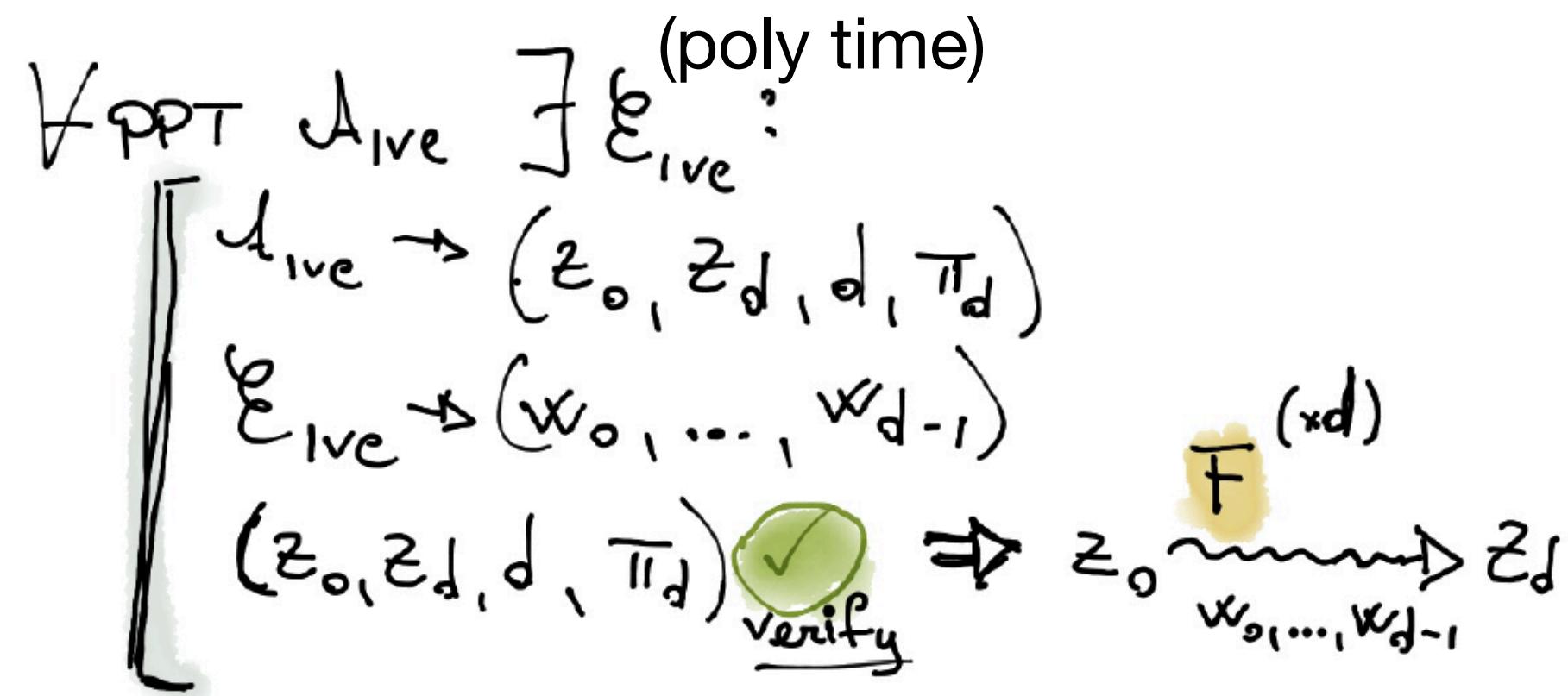


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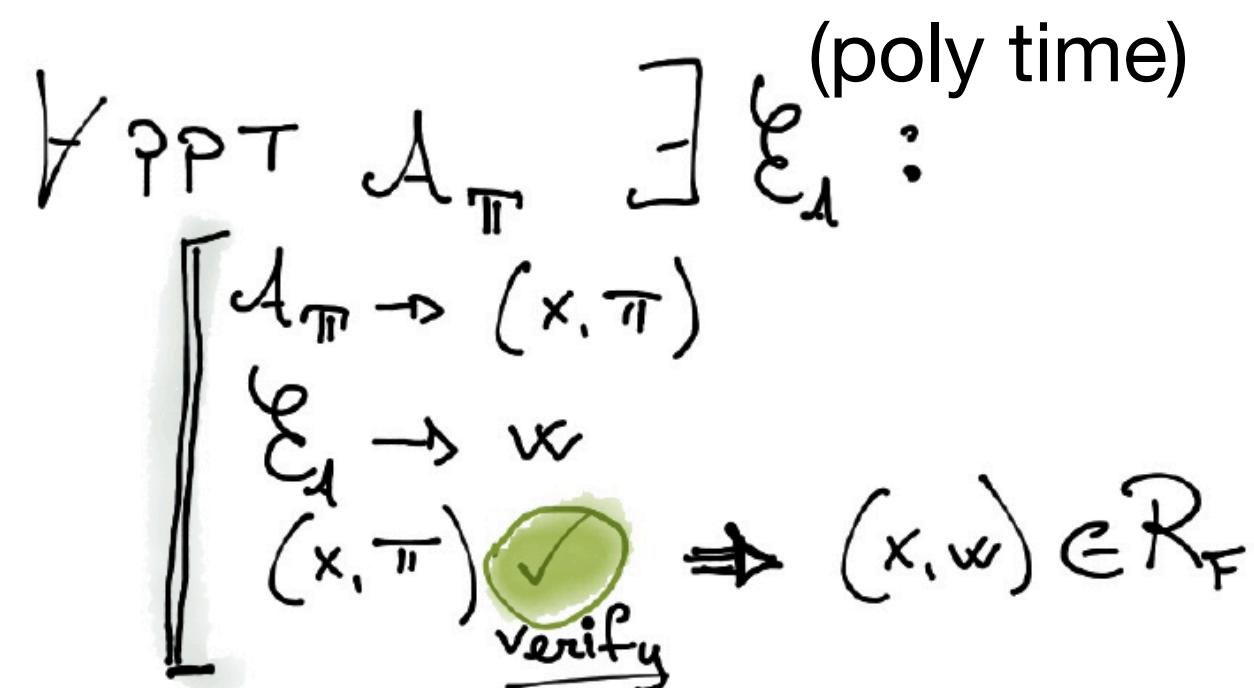
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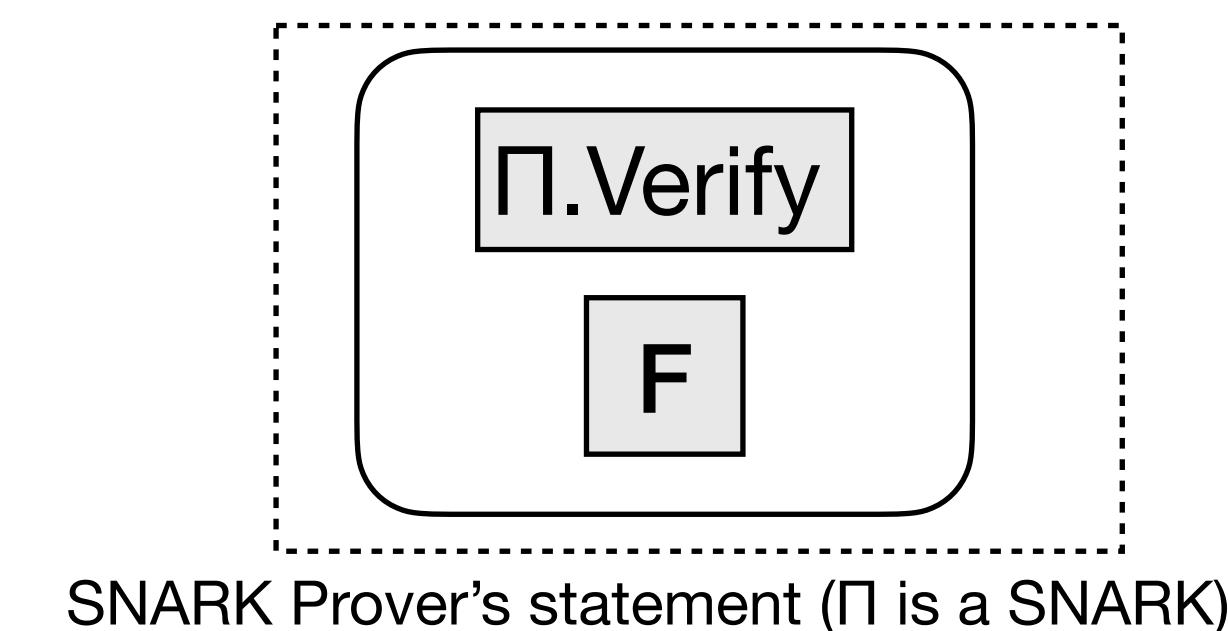


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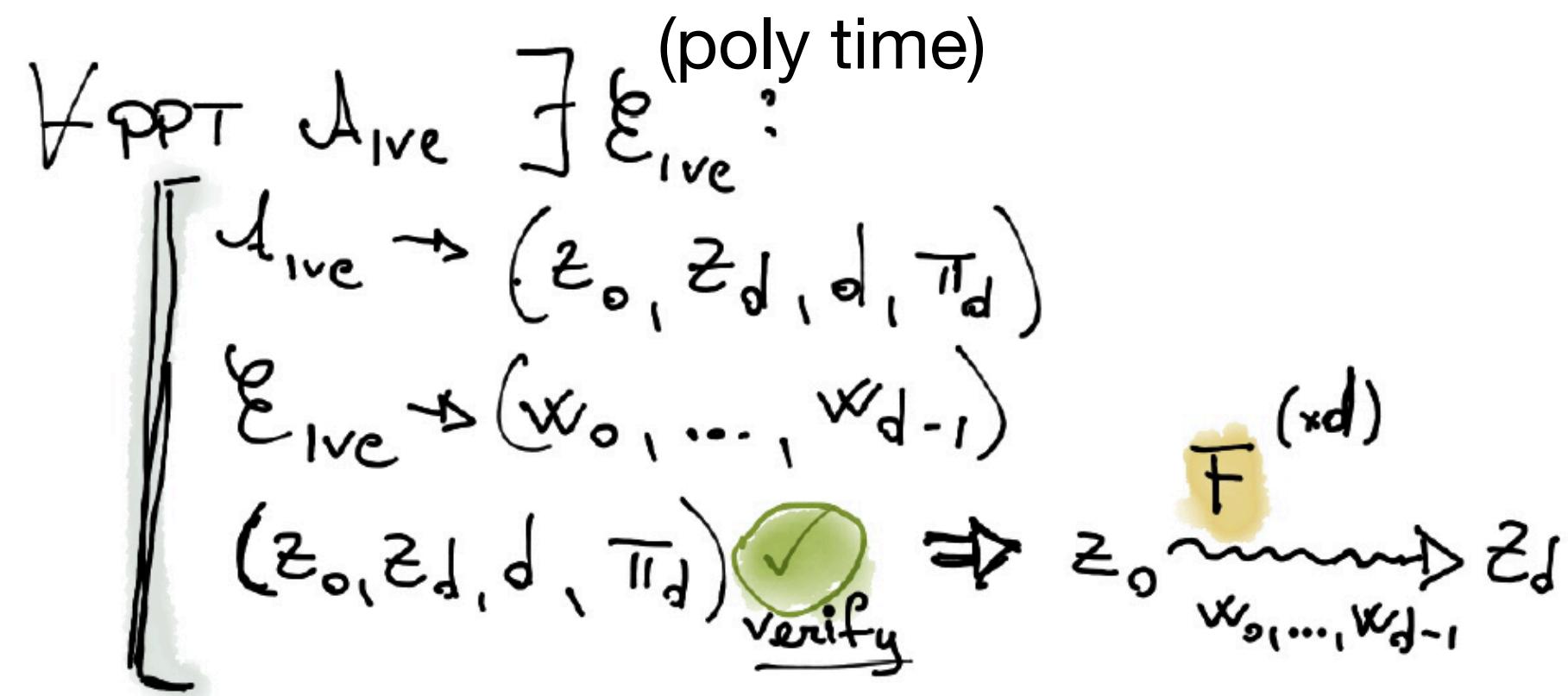
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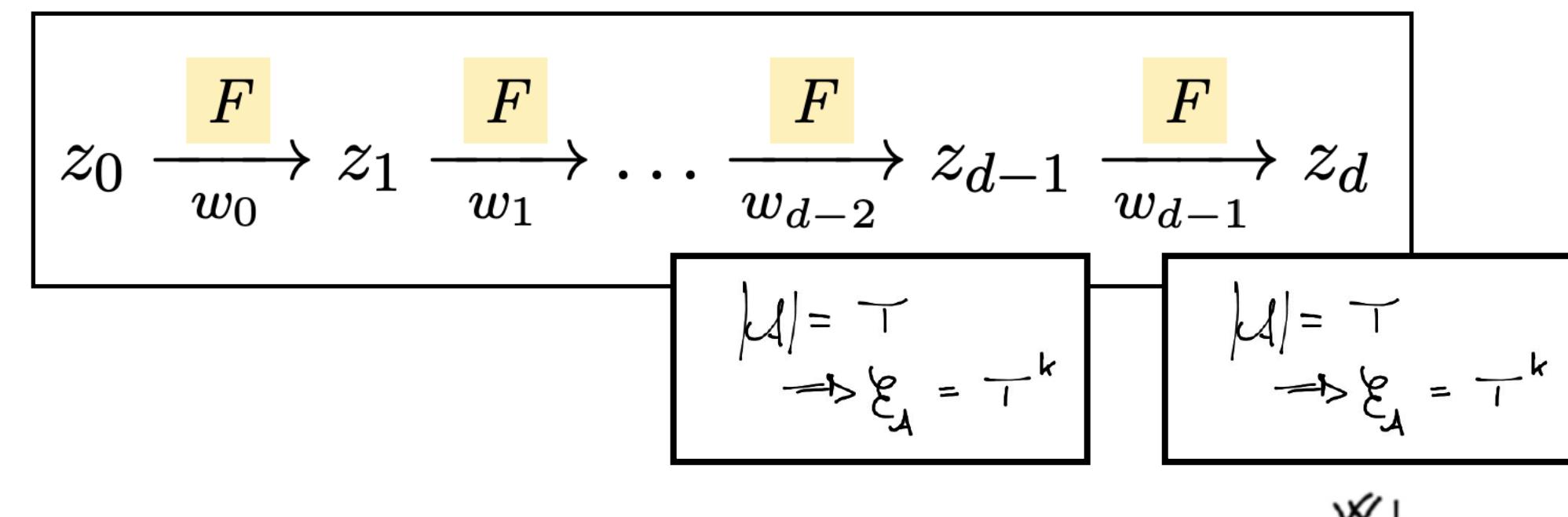
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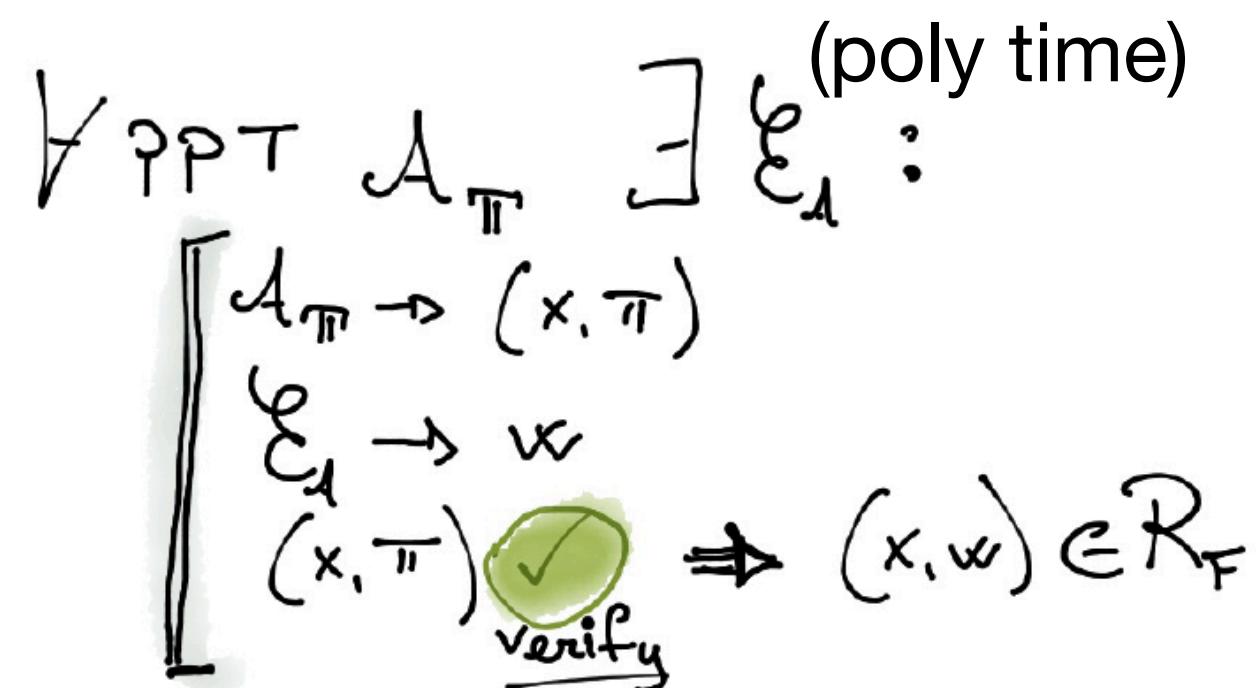
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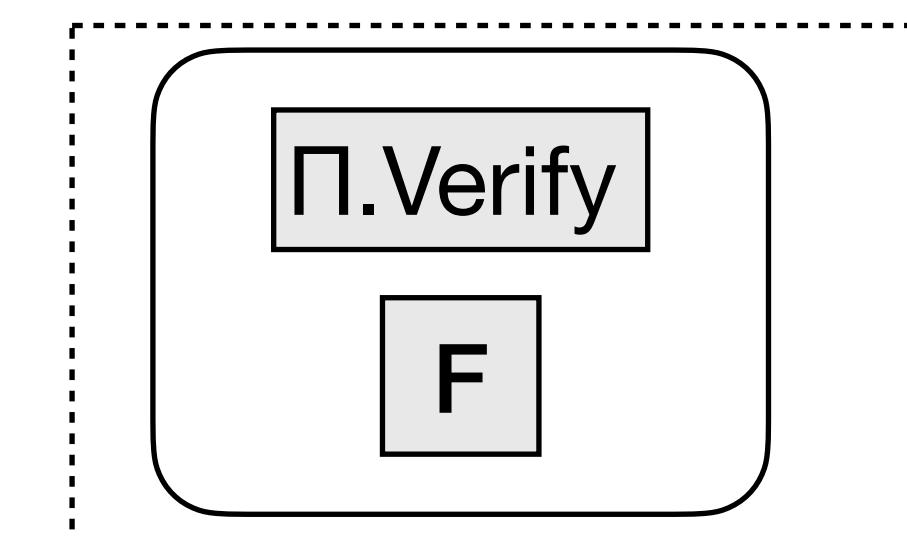


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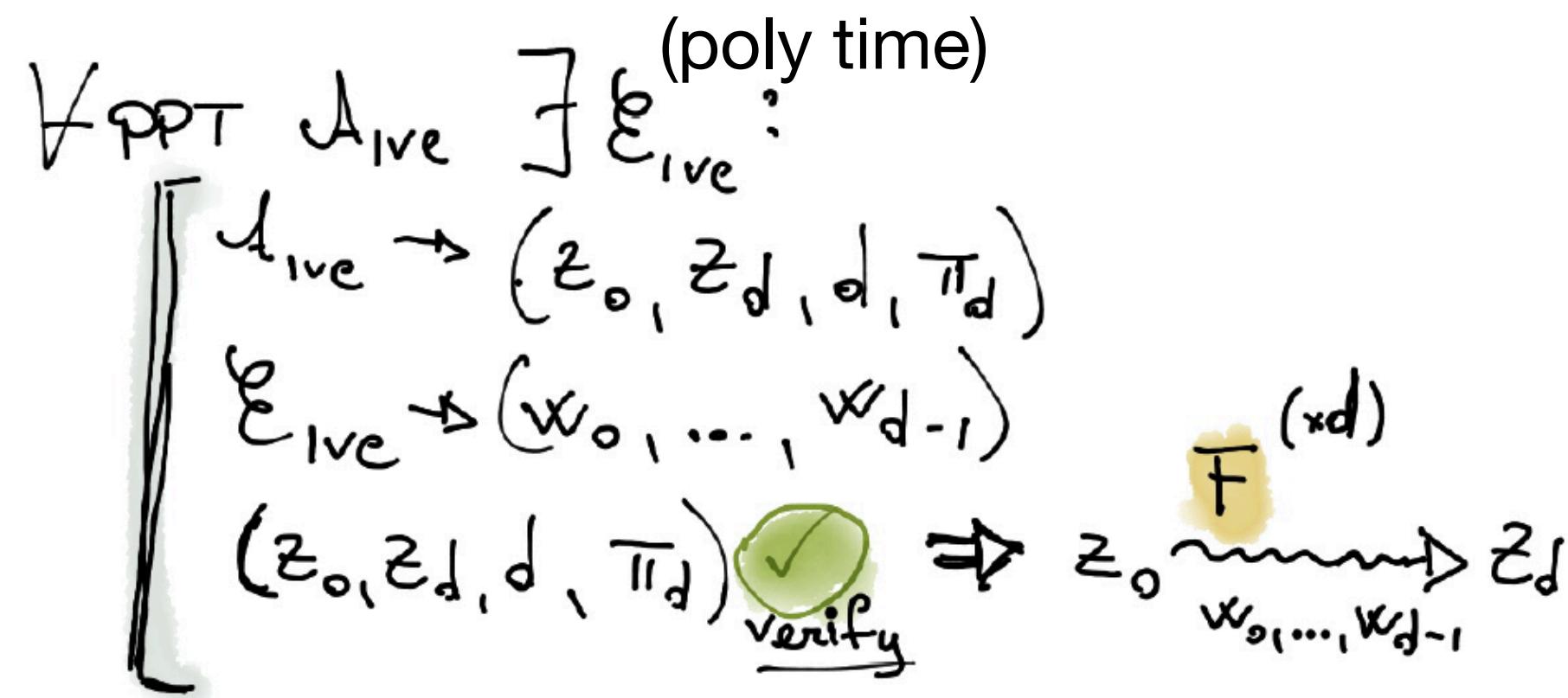


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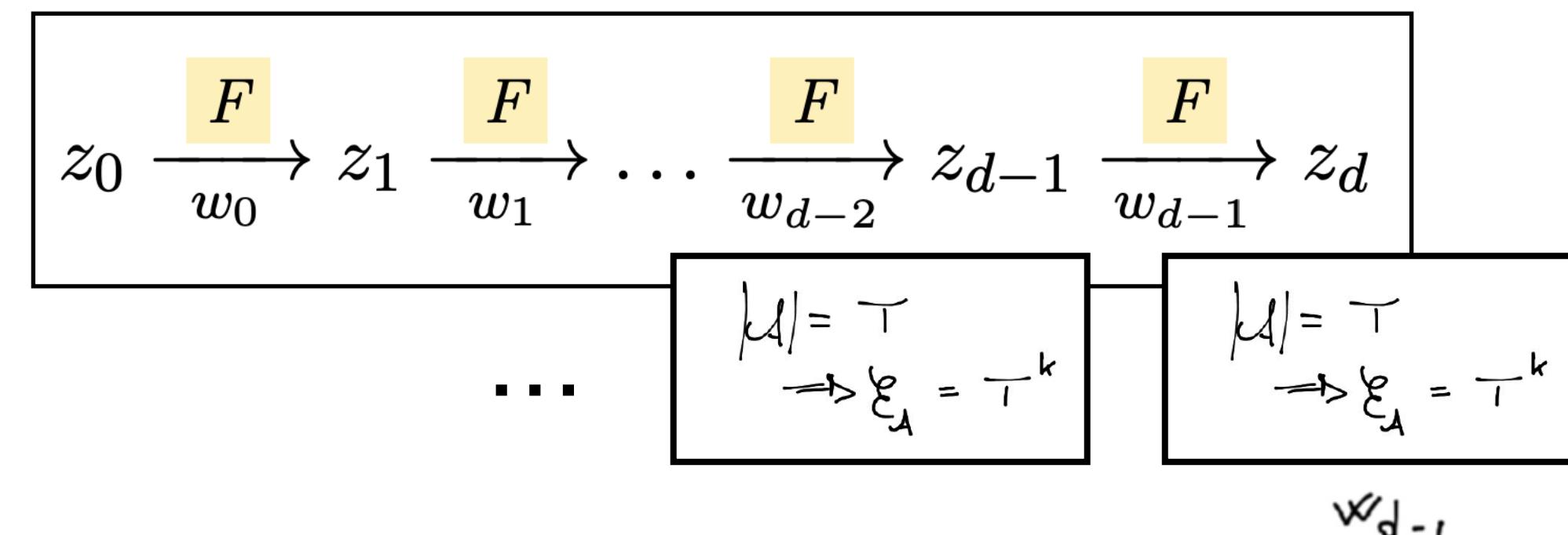
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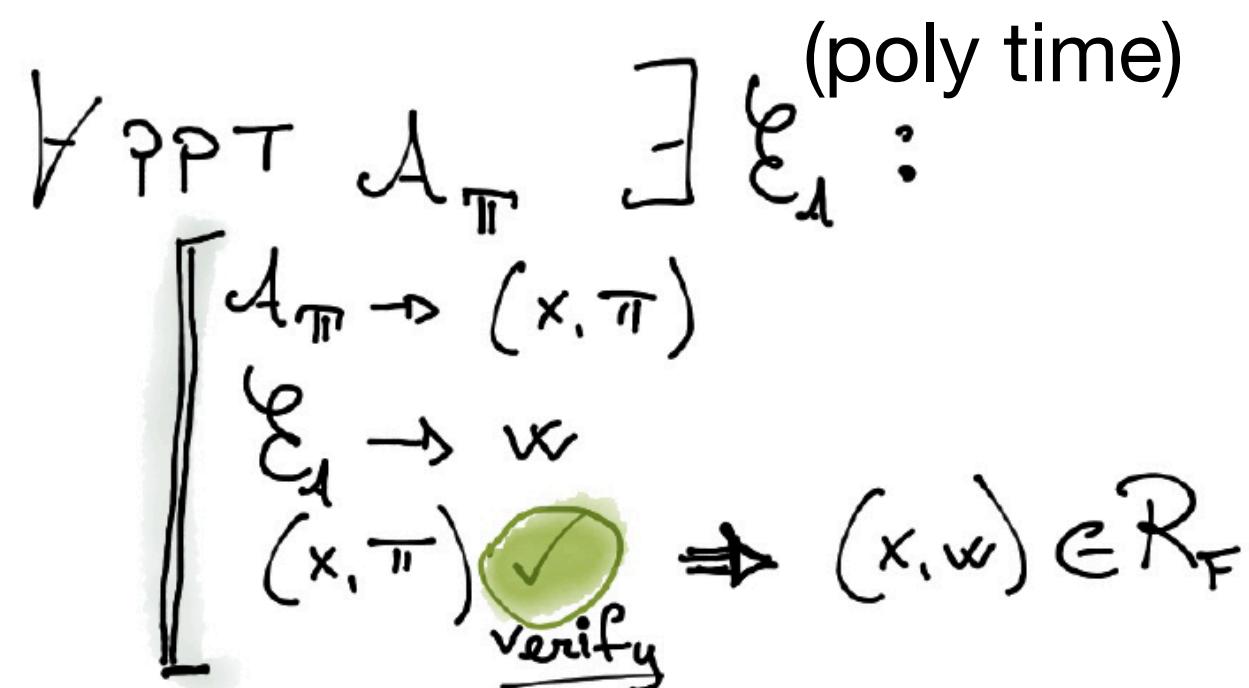
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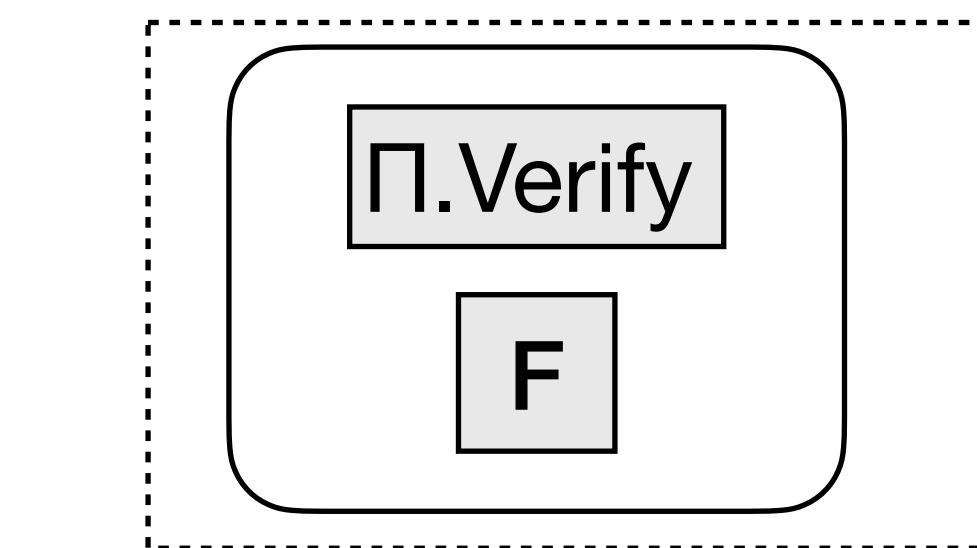


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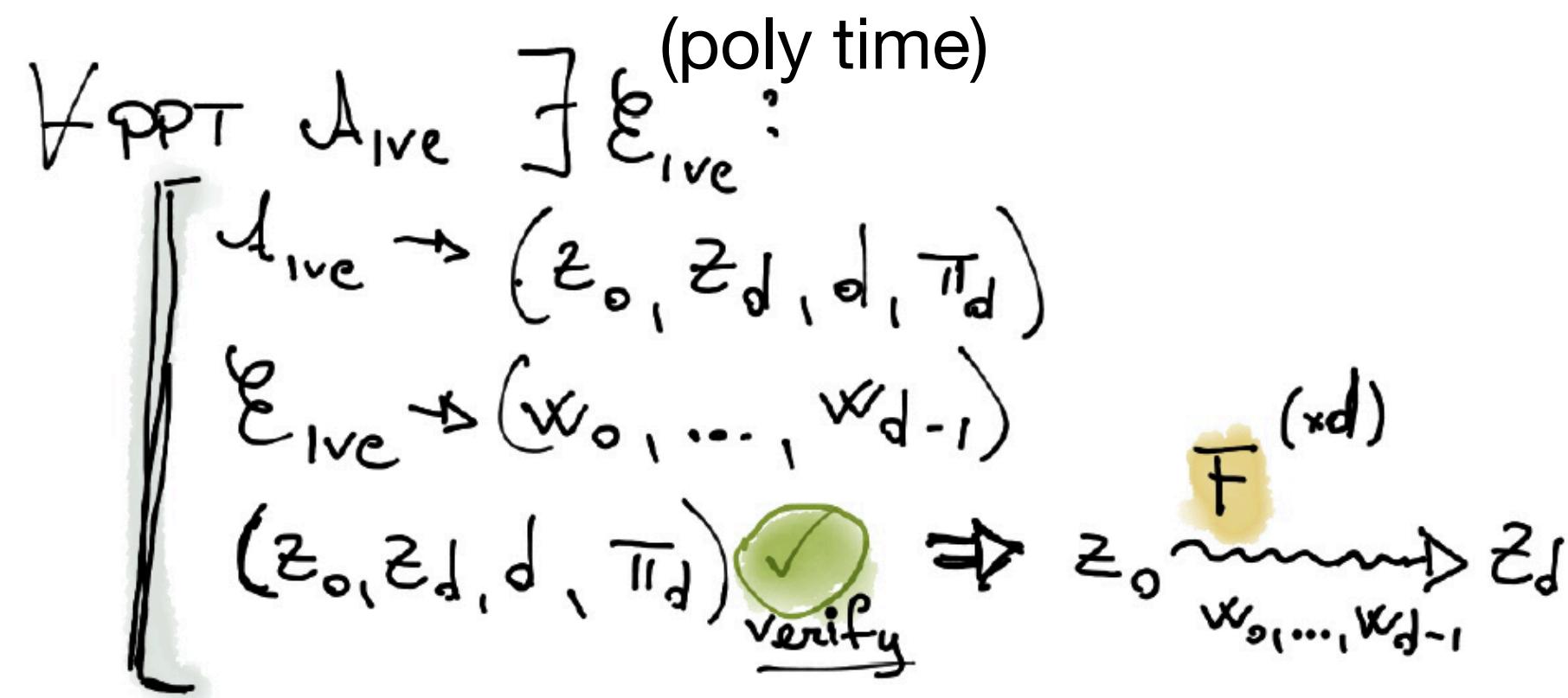


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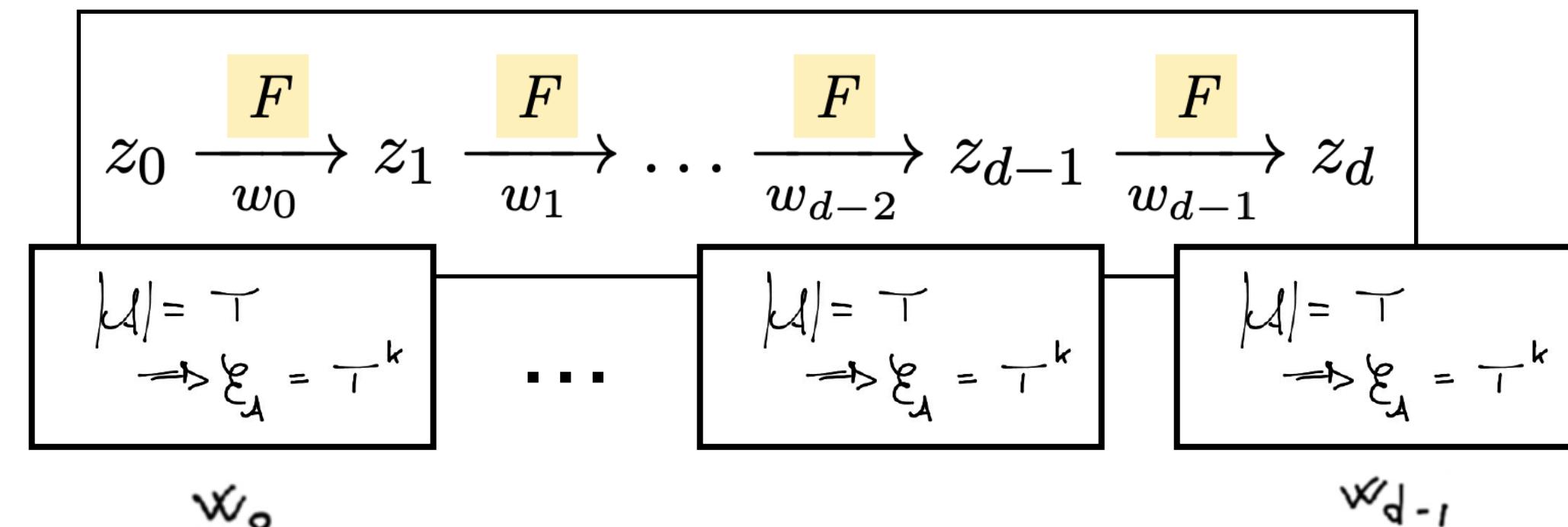
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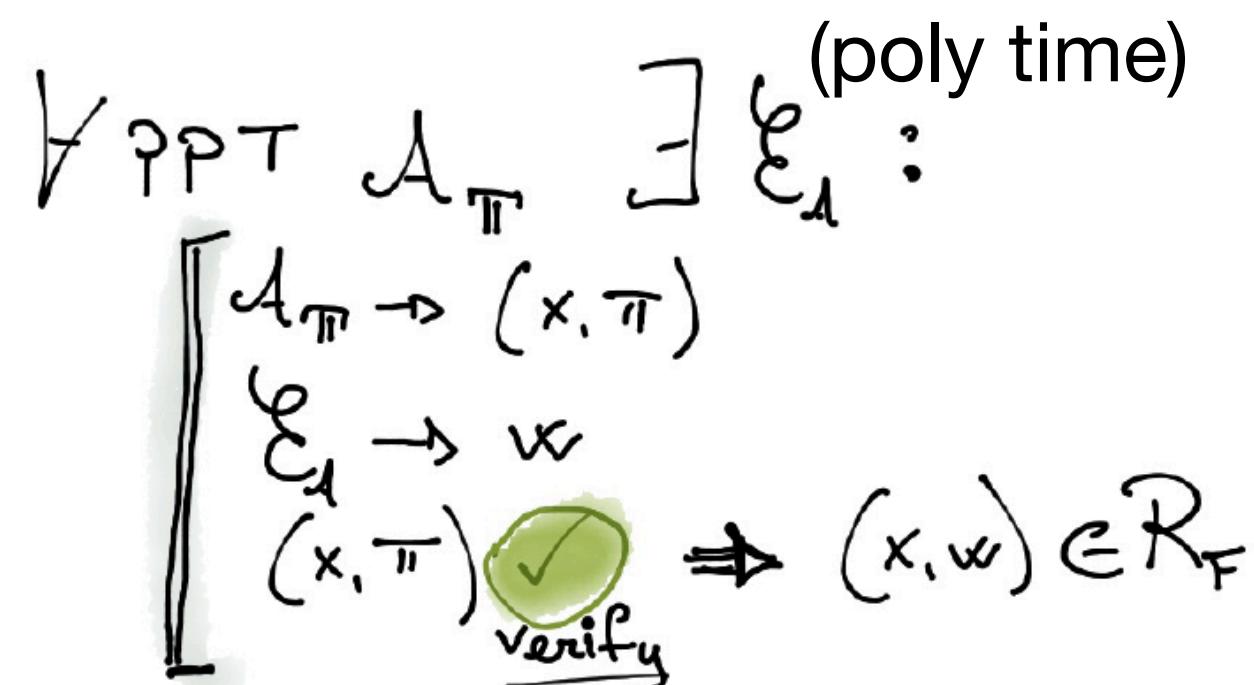
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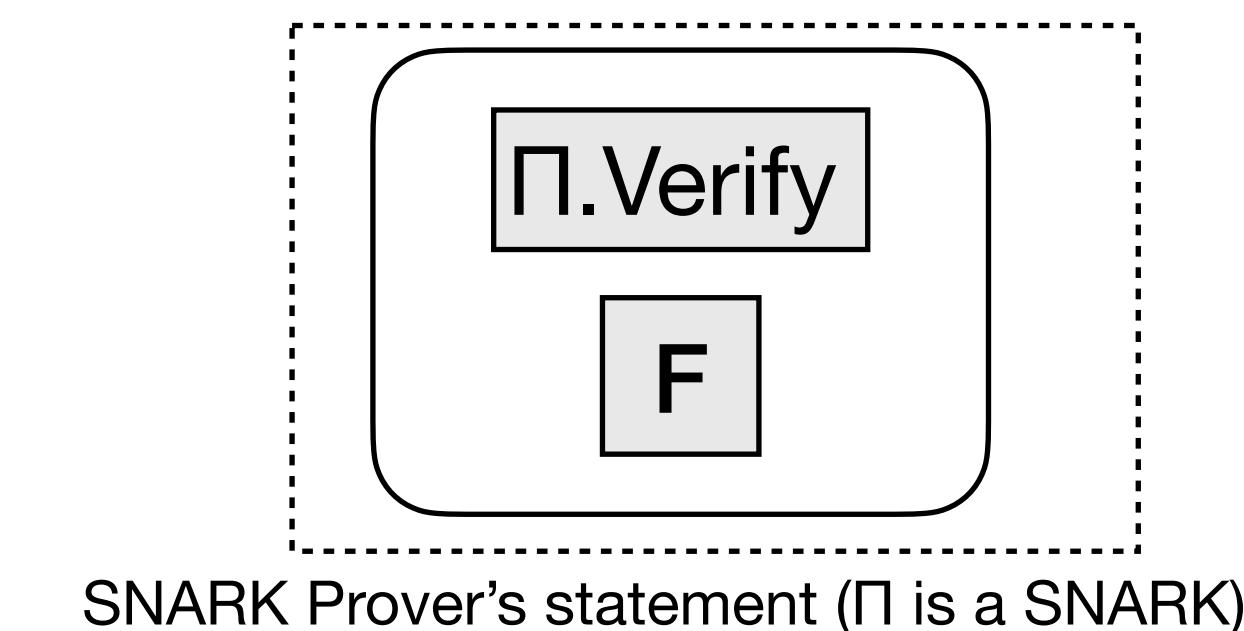


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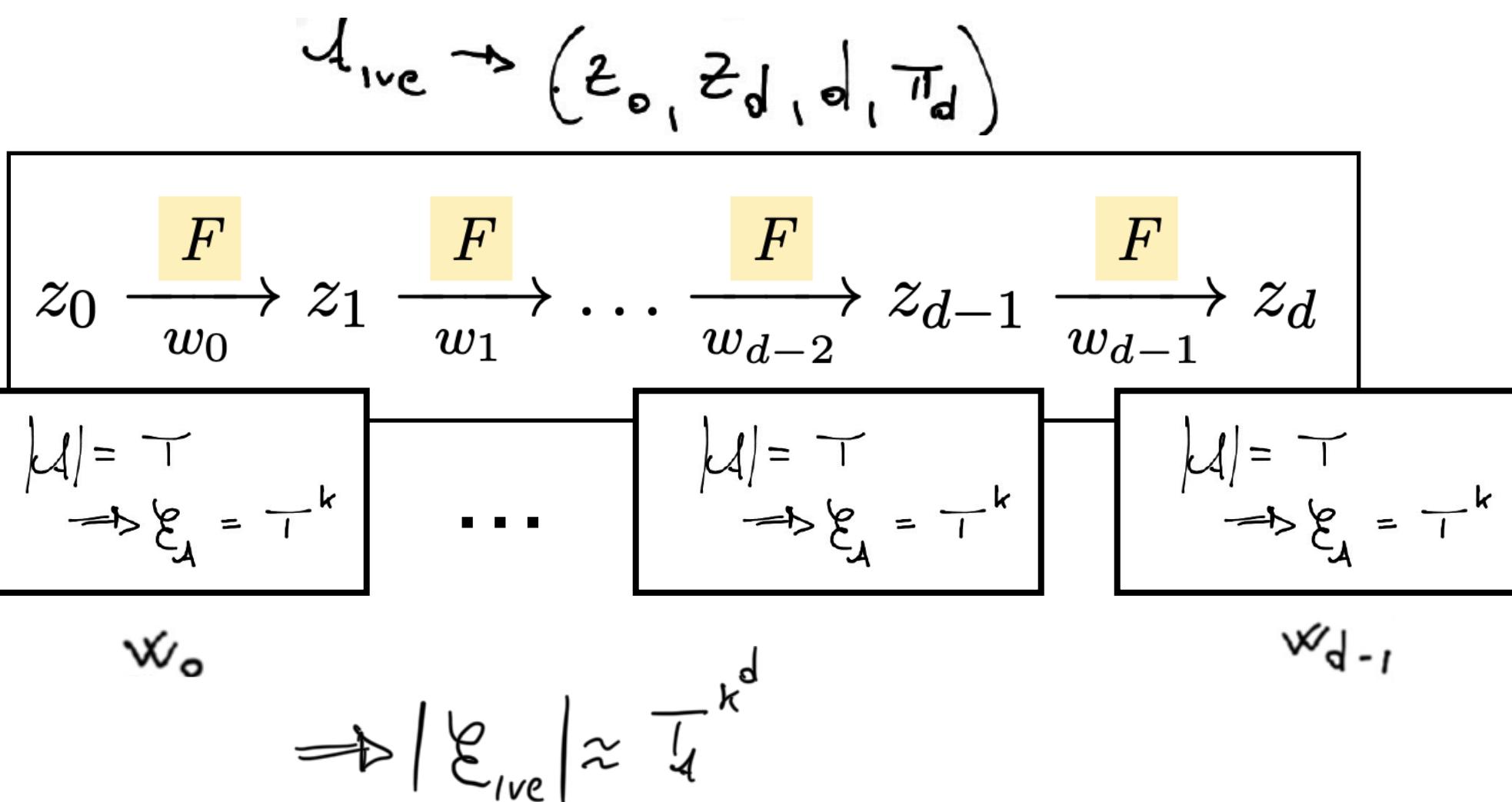
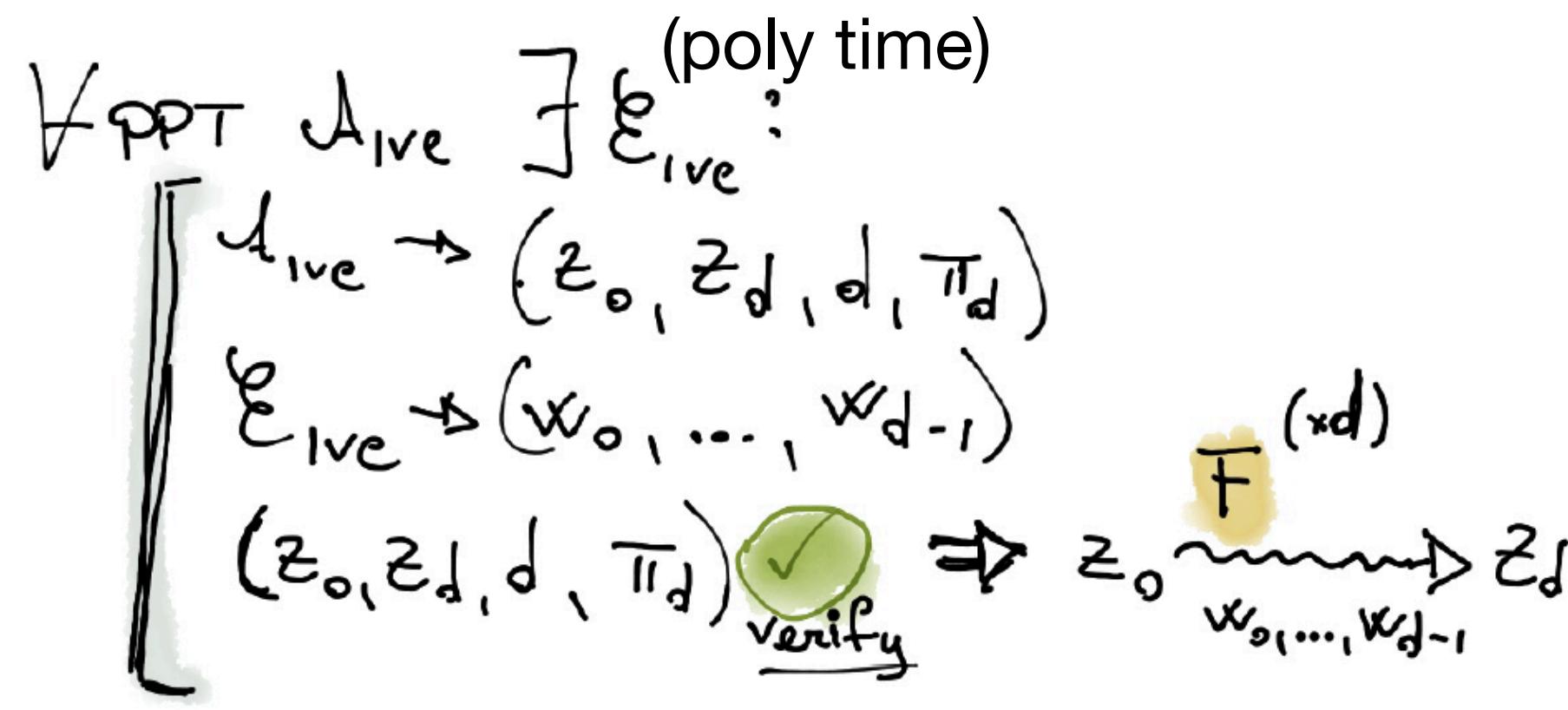
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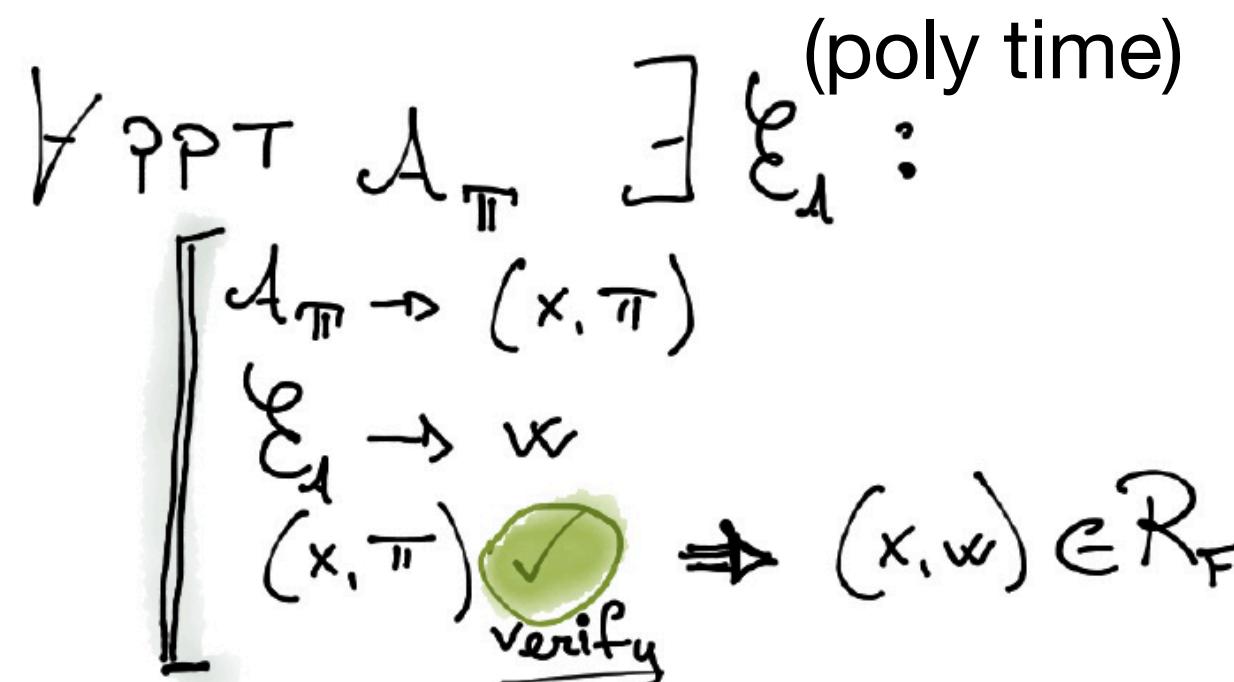
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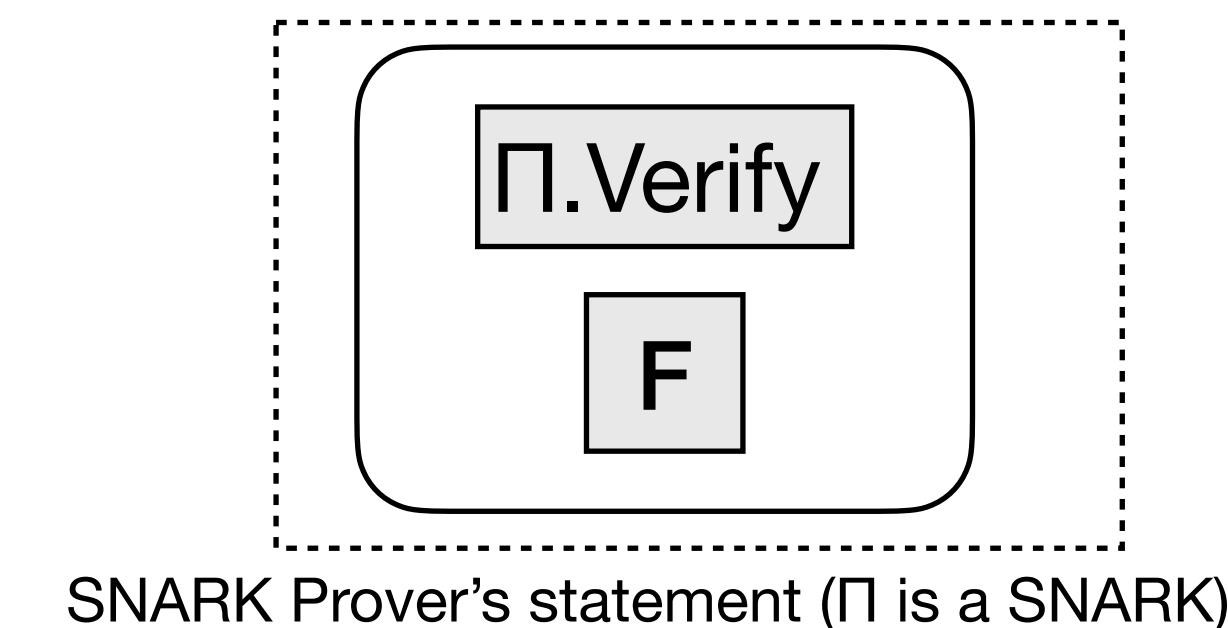


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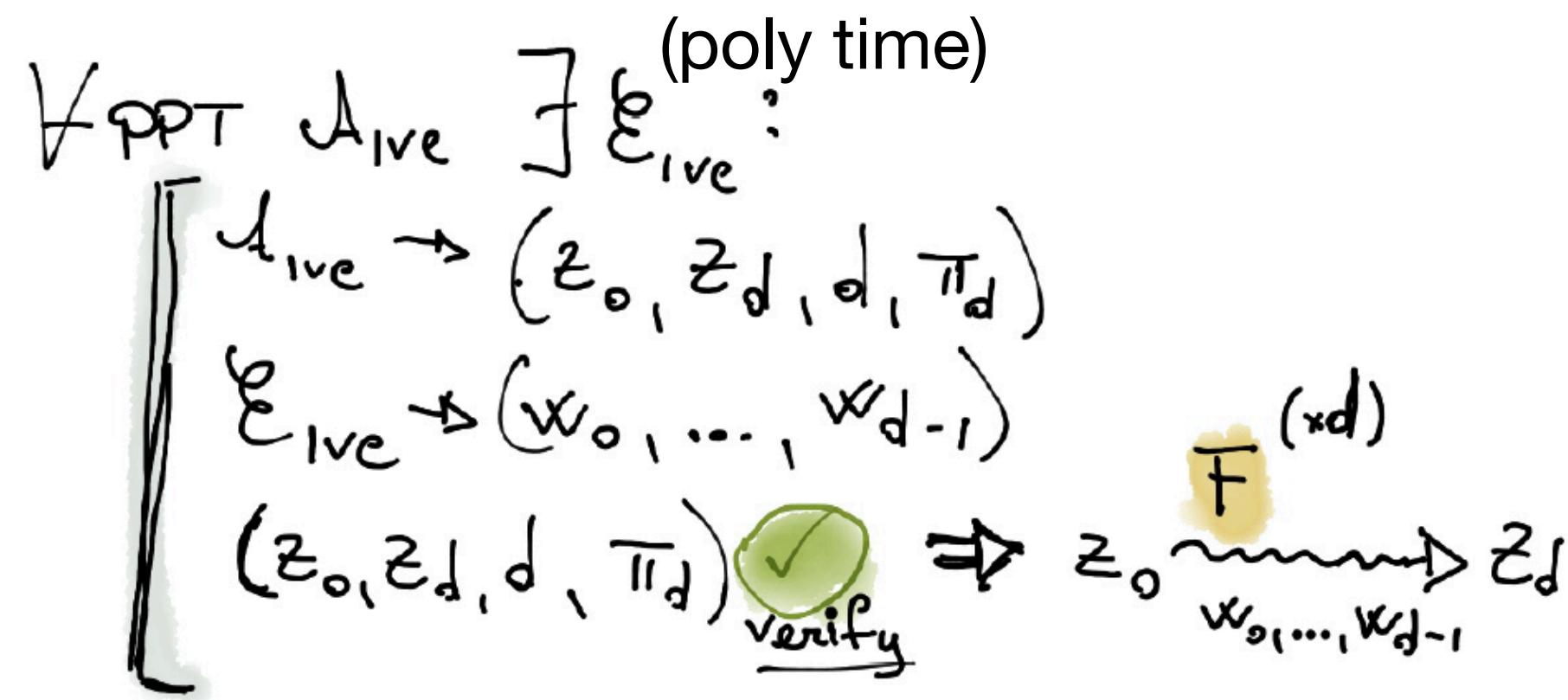
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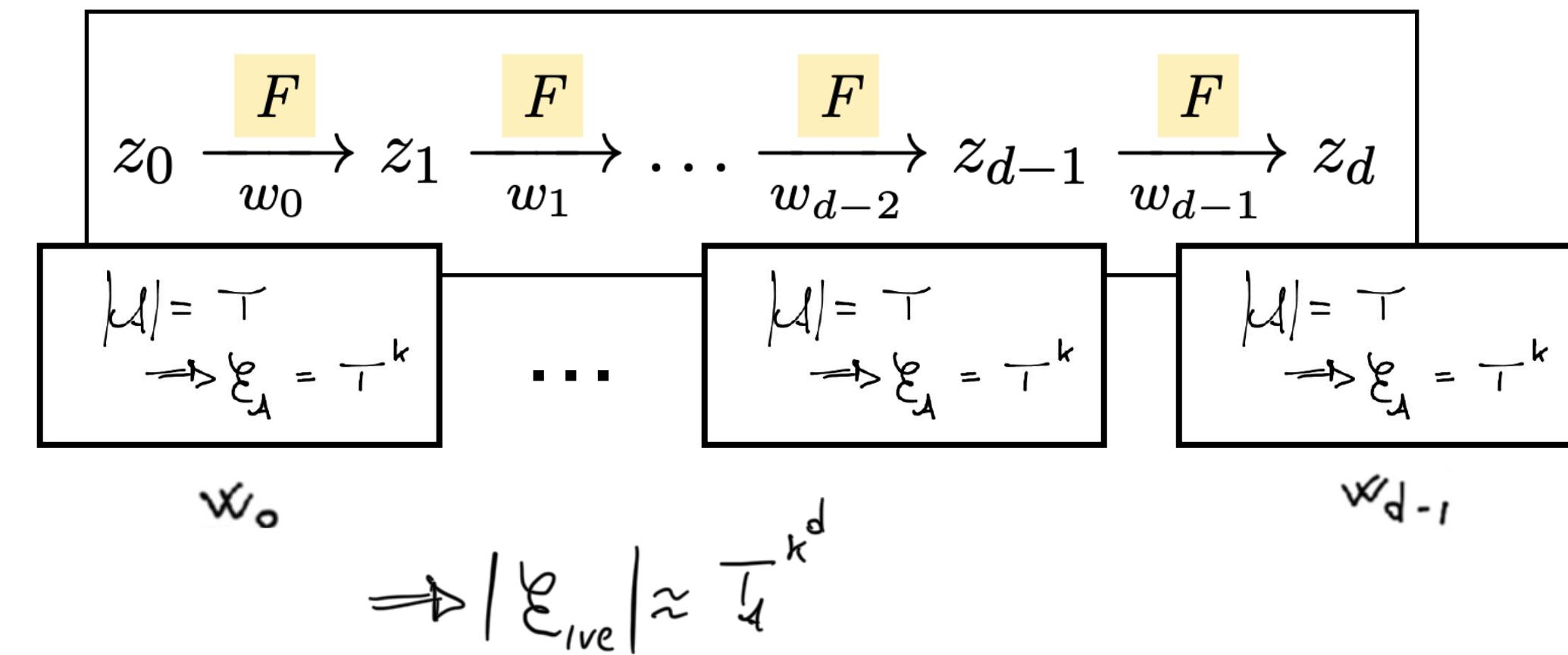
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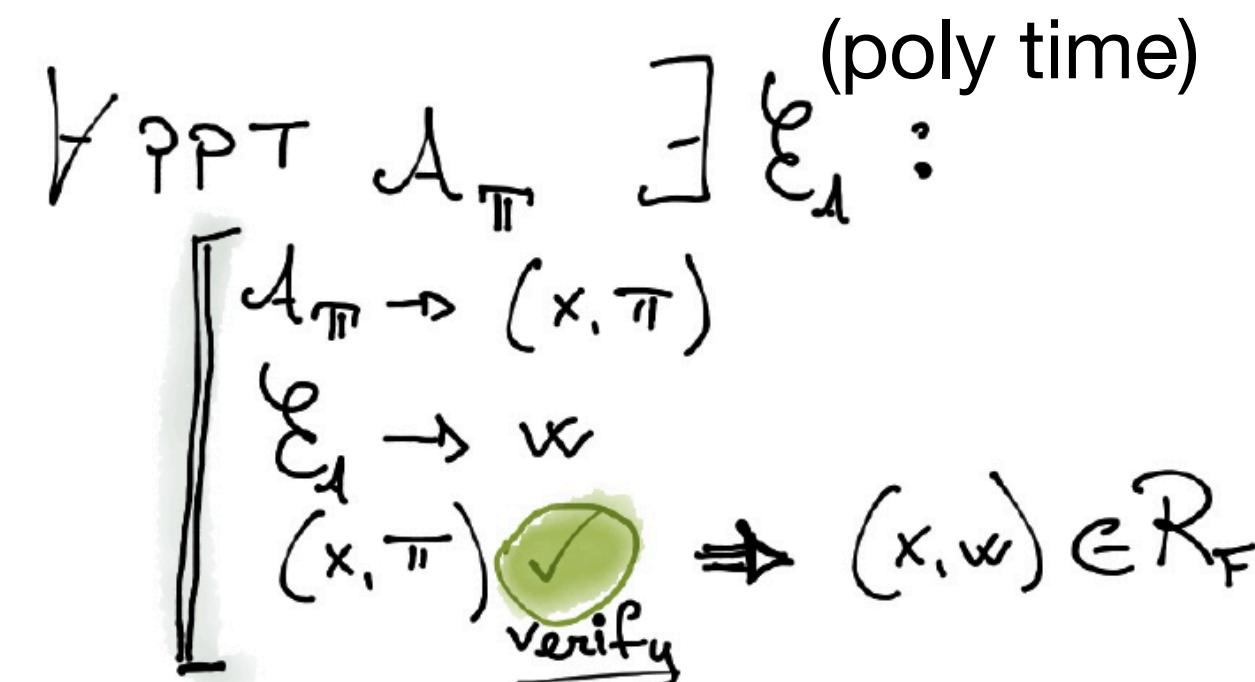
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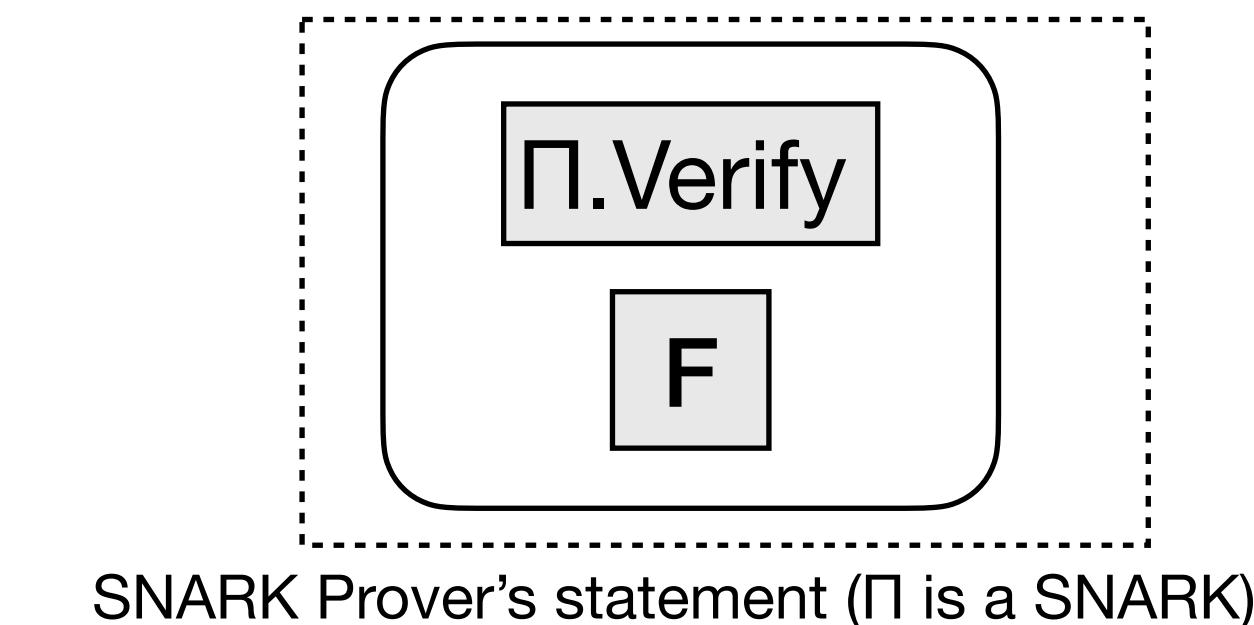


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# **The “Tree Approach”**

**A canonical way to go around the problem we just saw (via extractability)**

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## Recursive Composition and Bootstrapping for SNARKs and Proof-Carrying Data

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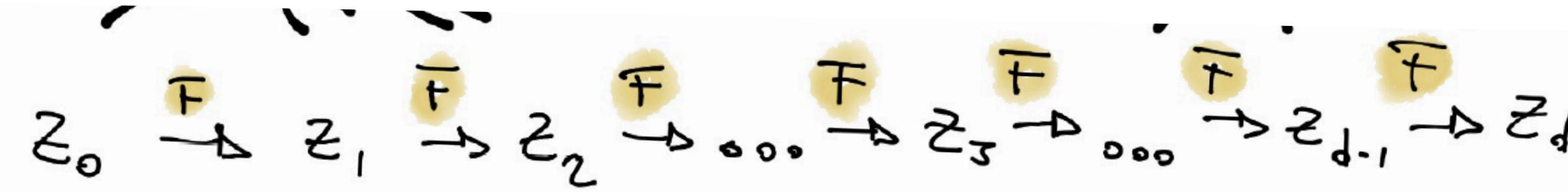
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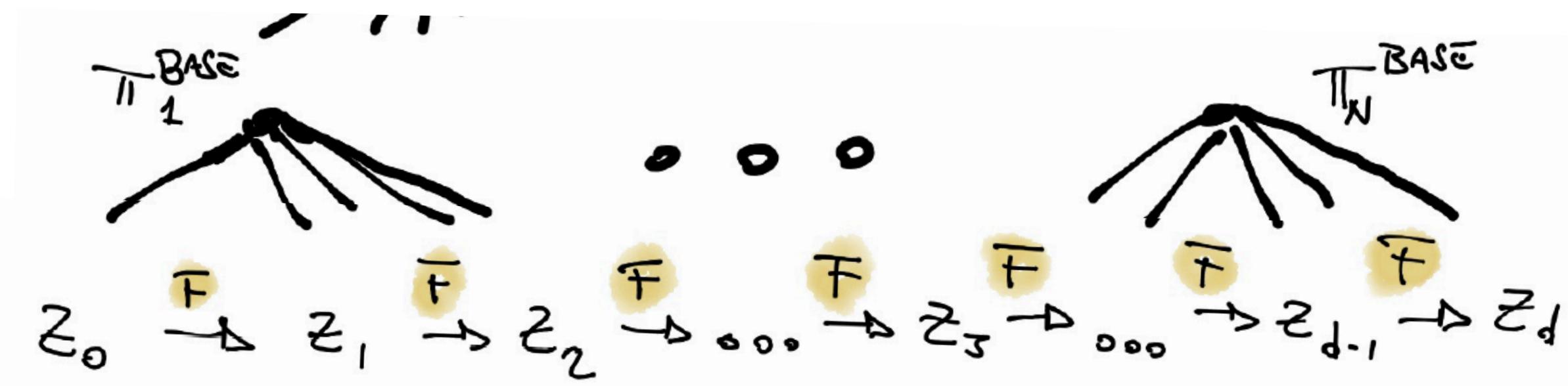
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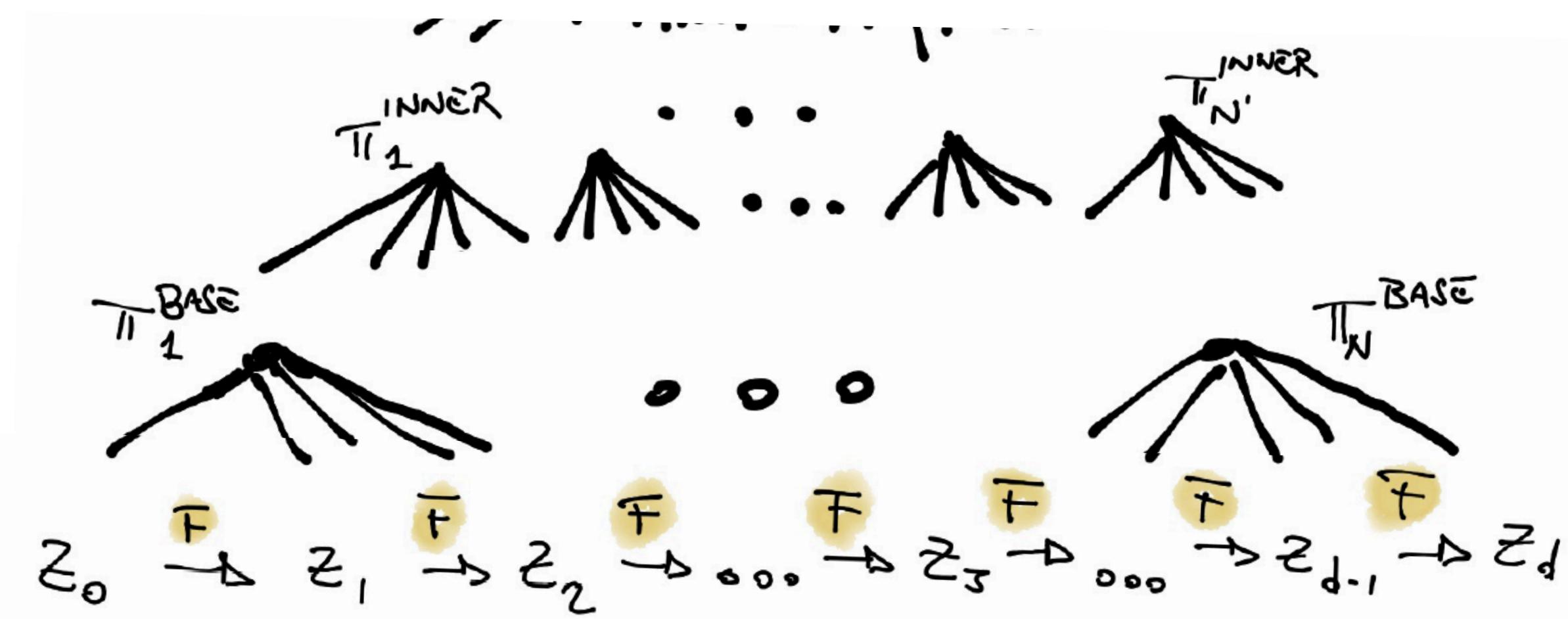
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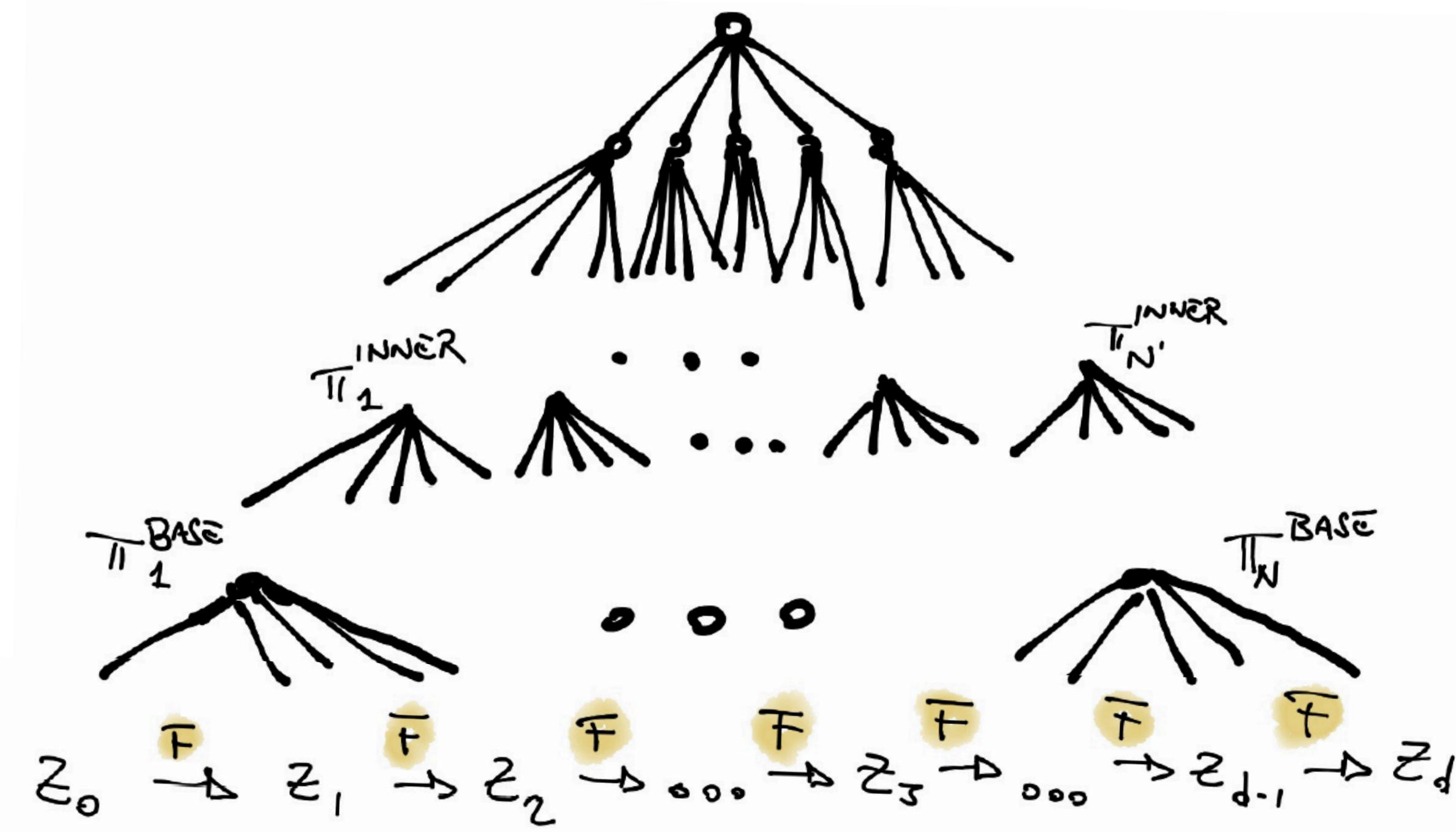
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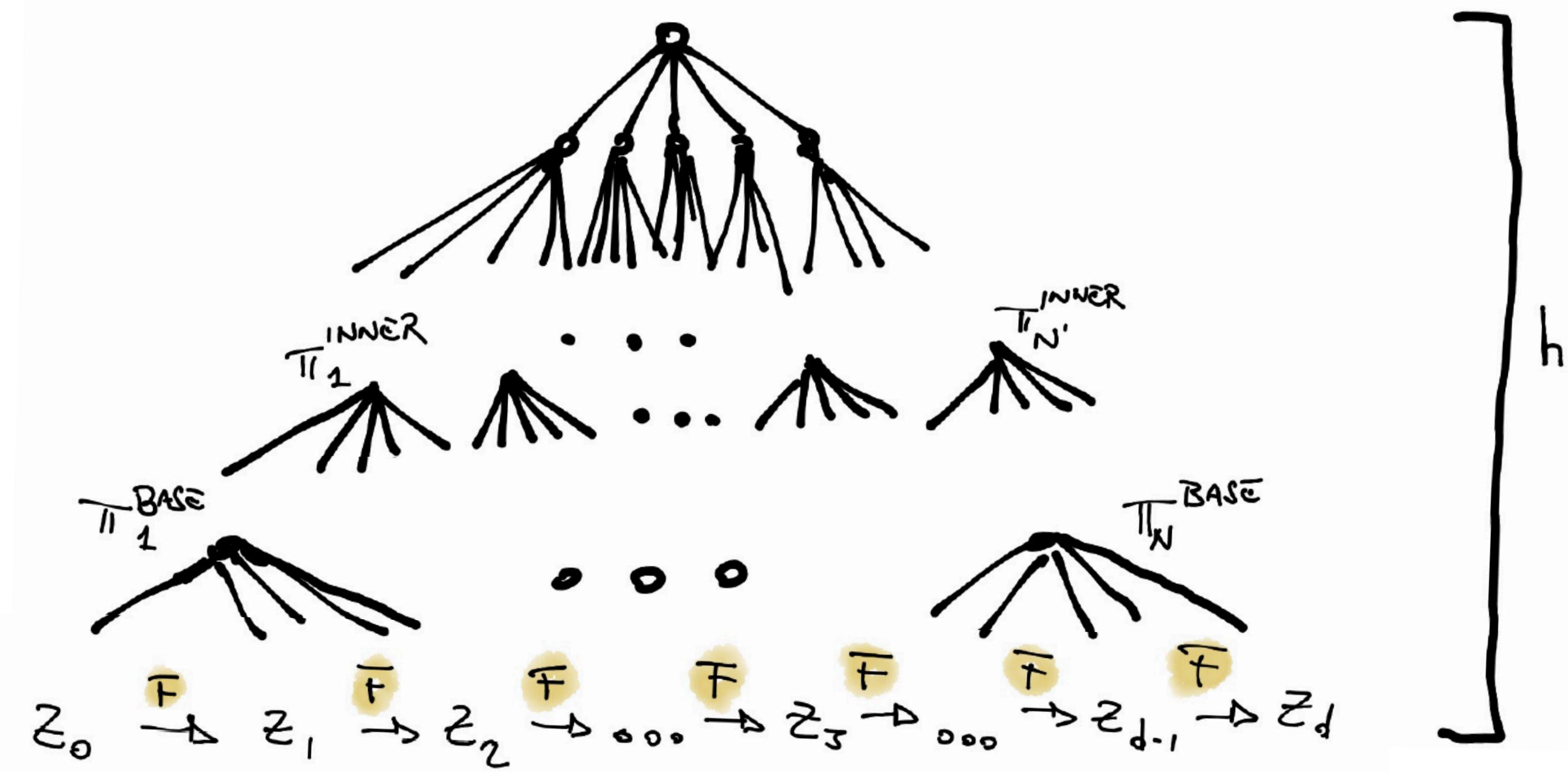
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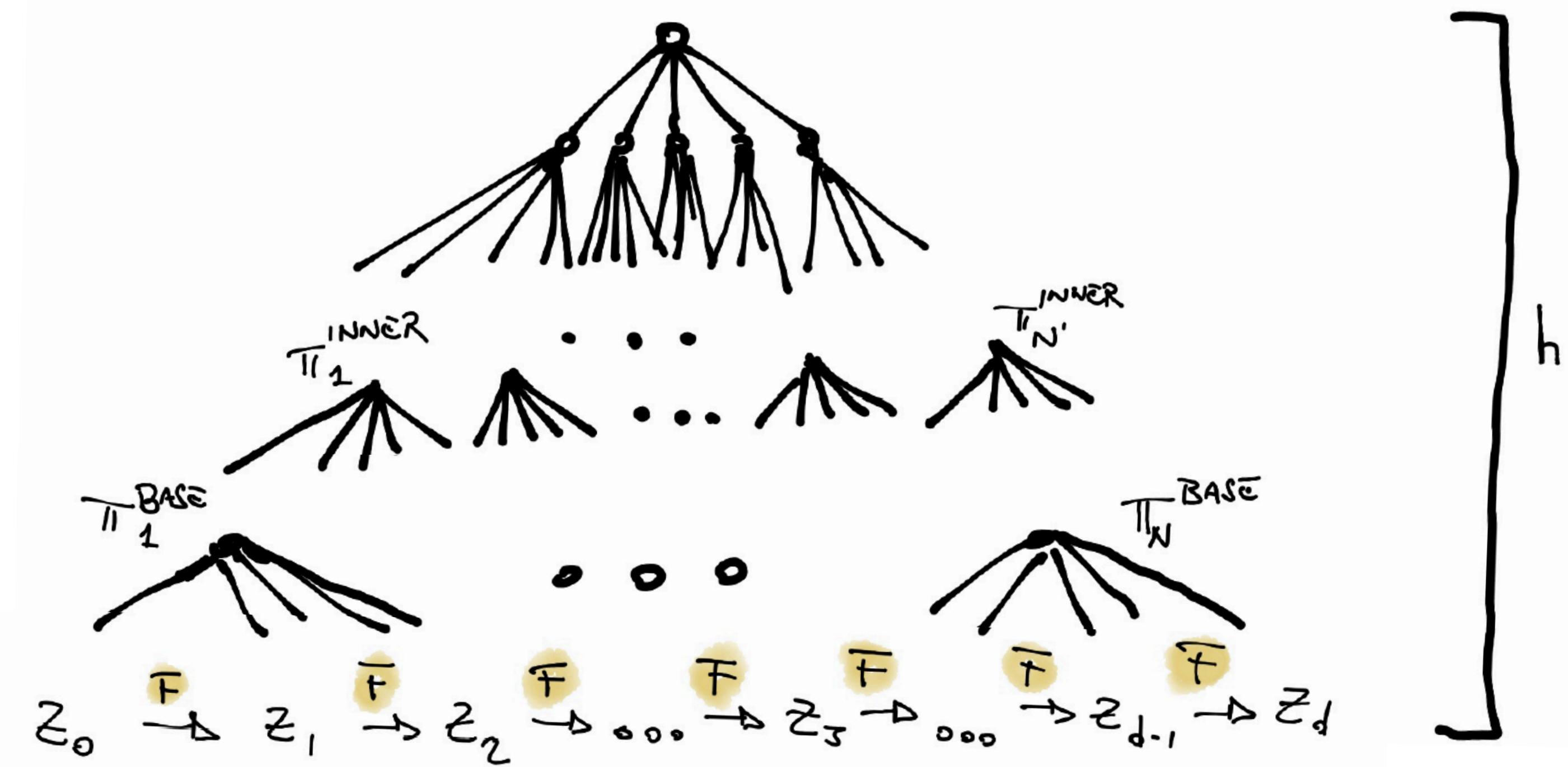
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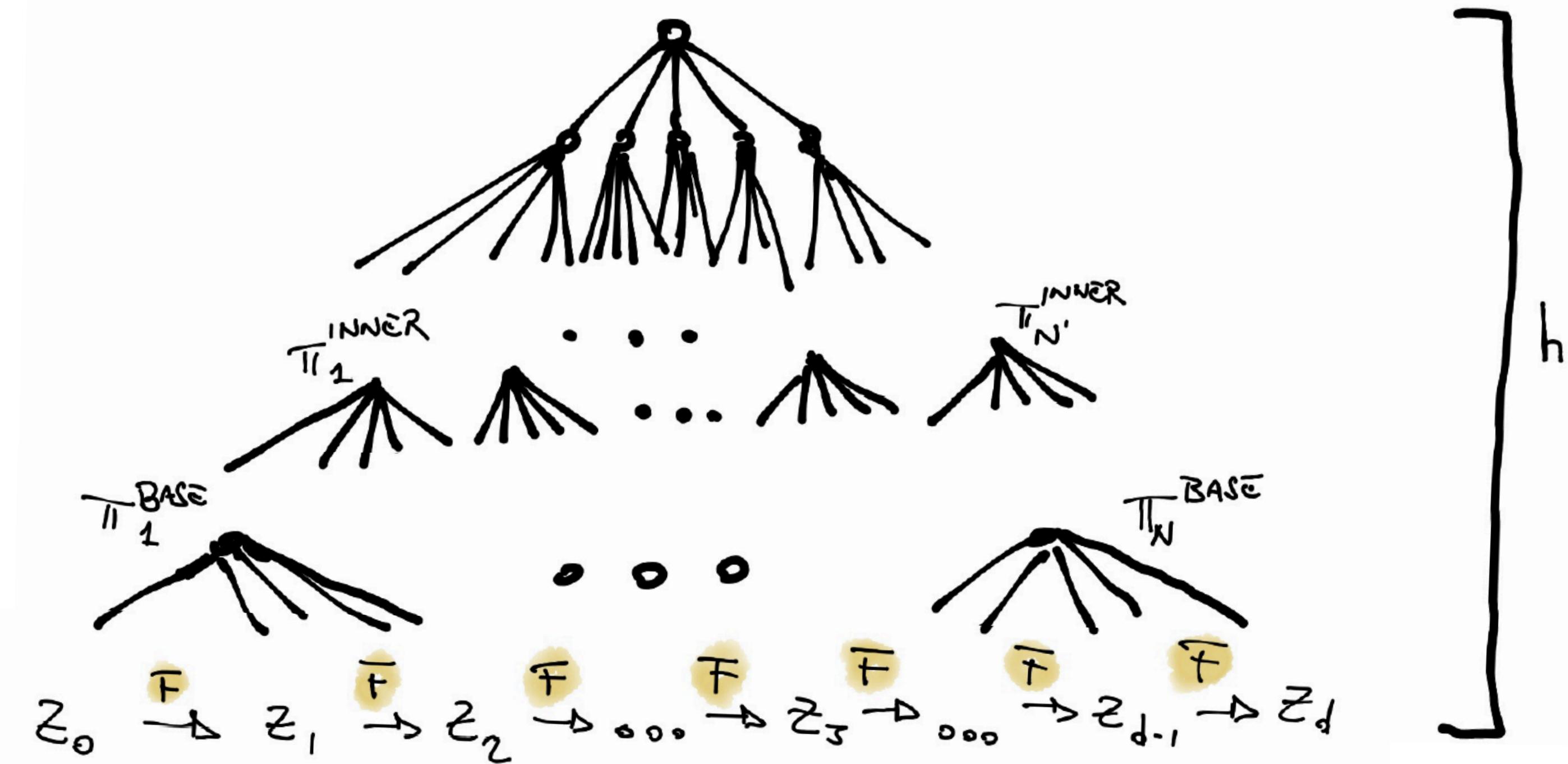
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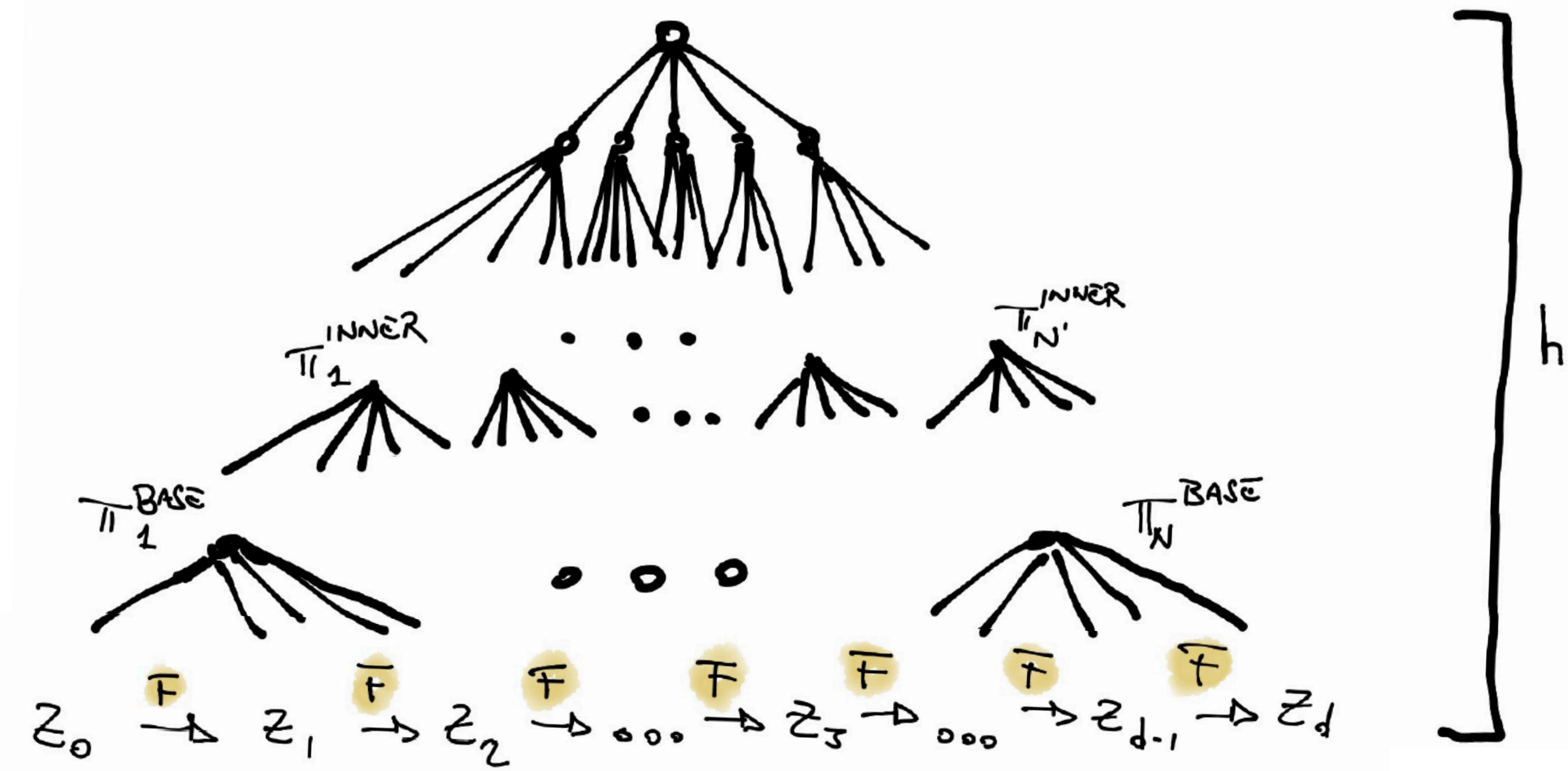
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This construction works but it is not the  
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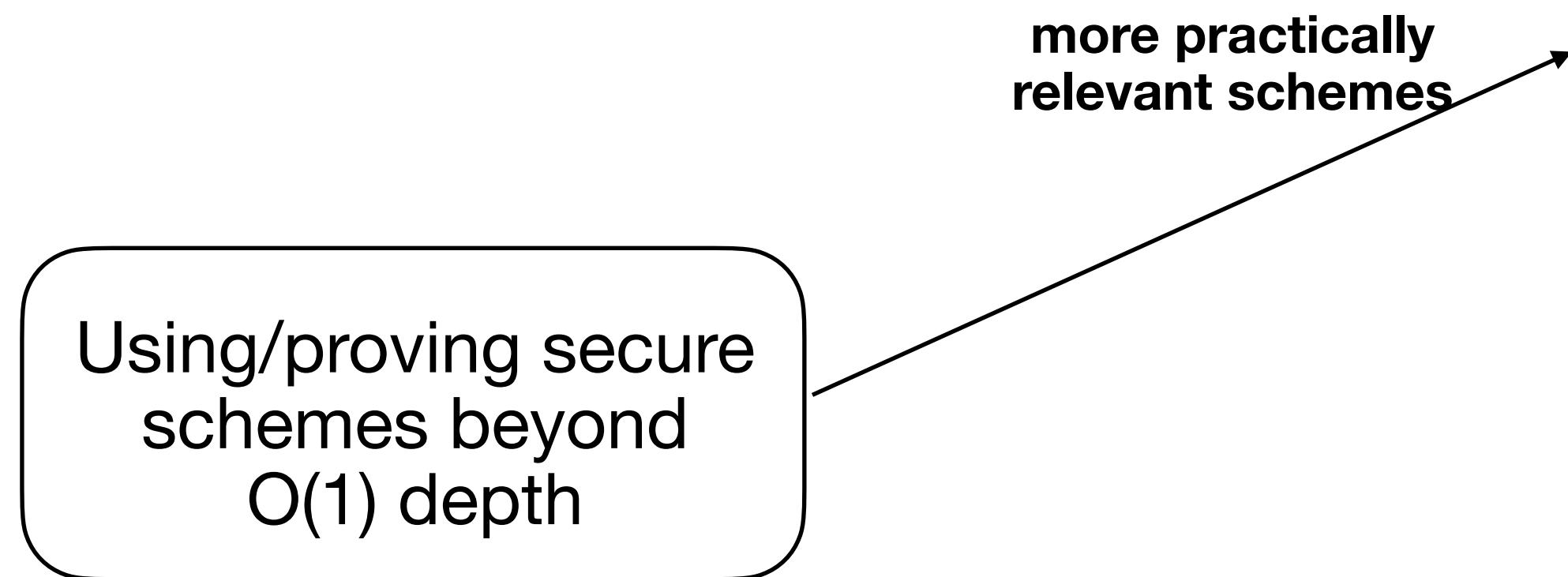
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  - General improved understanding of *where* we can use *which* constructions

# How the Community Has Addressed This—A Landscape

Using/proving secure  
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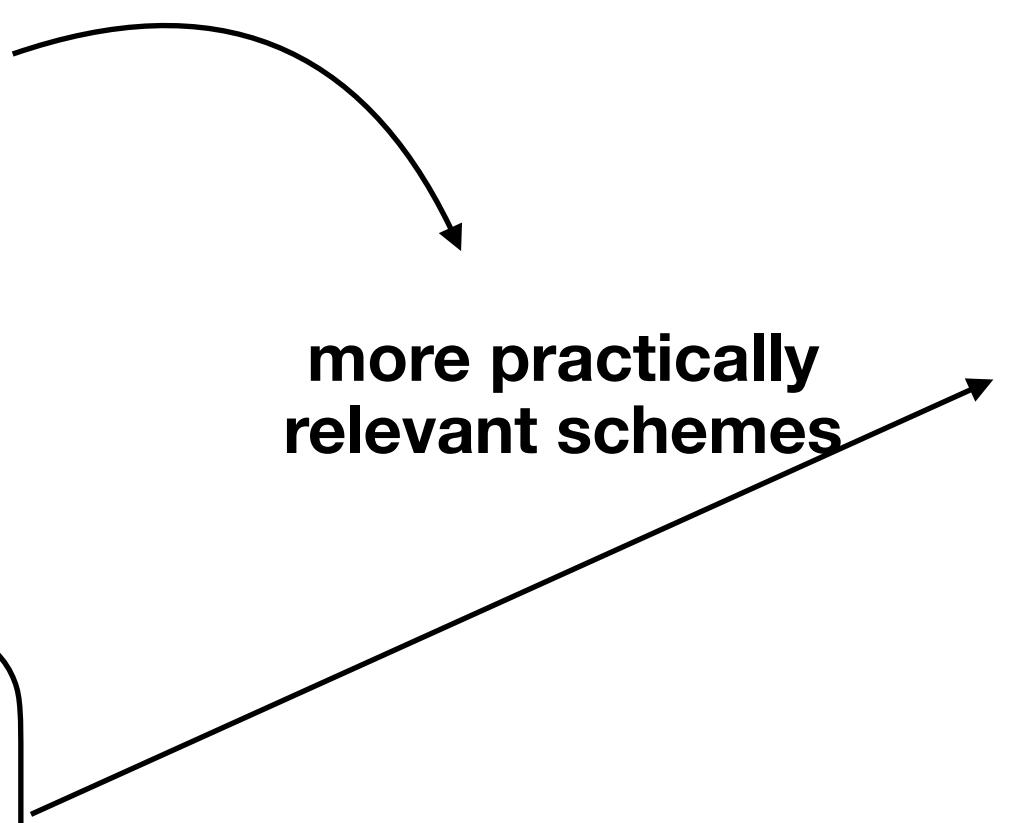


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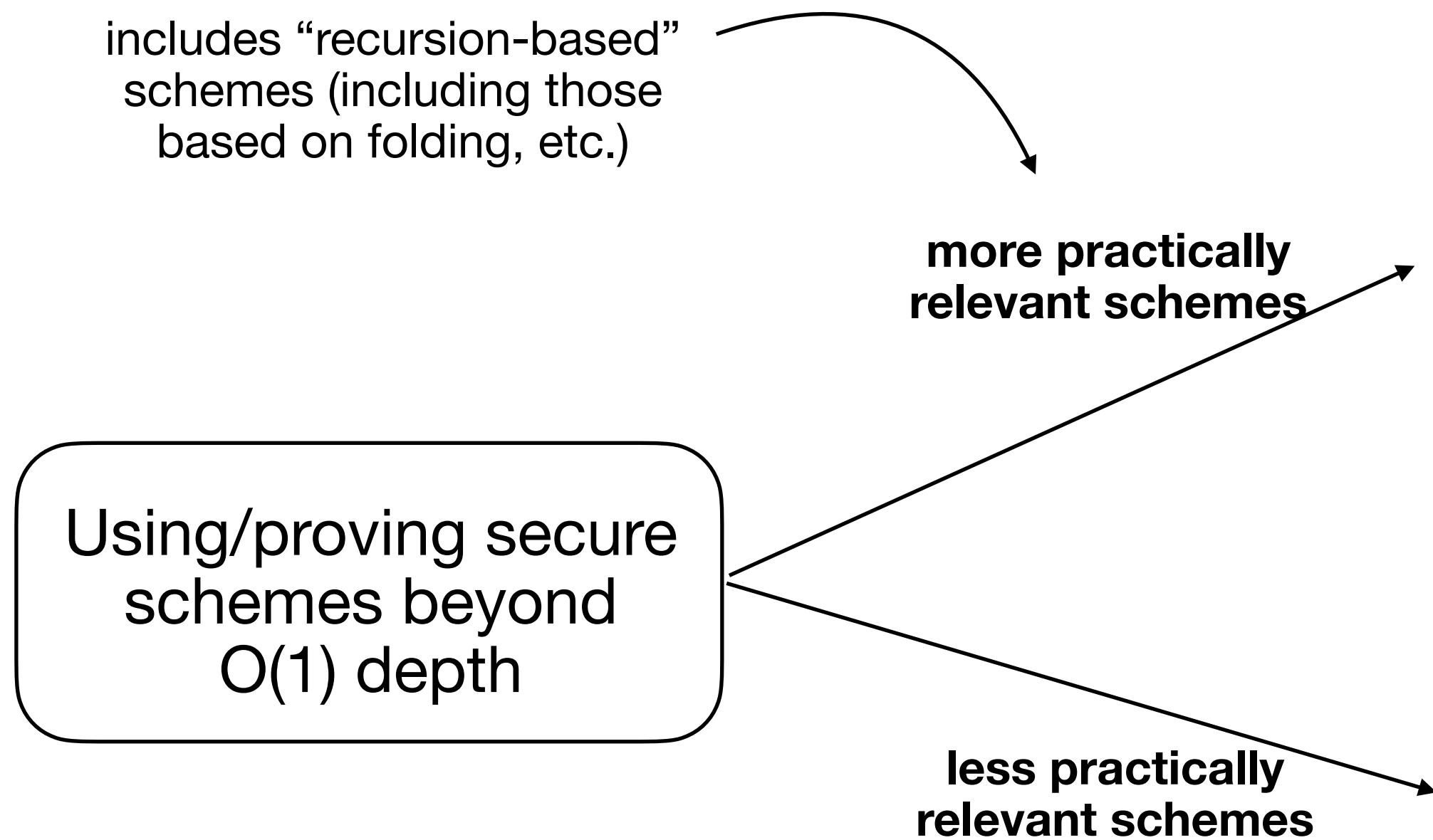
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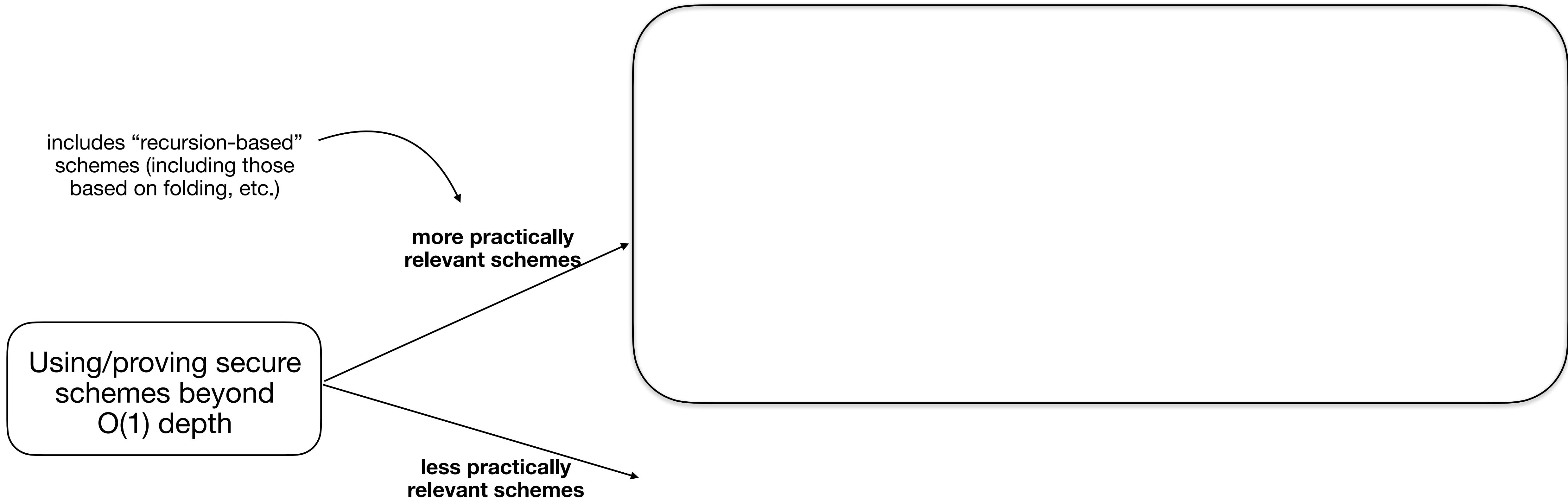
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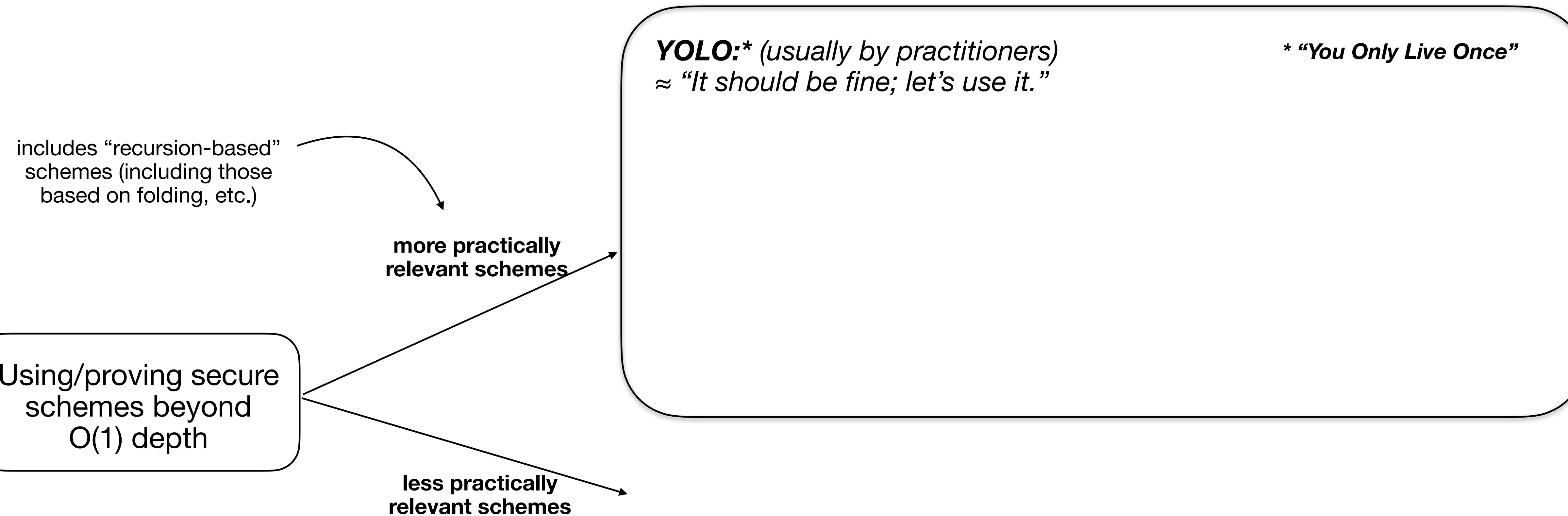
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## Malleable SNARKs and Their Applications

Suvradip Chakraborty<sup>1</sup>, Dennis Hofheinz<sup>2</sup>, Roman Langrehr<sup>1</sup>, Jesper Buus Nielsen<sup>3</sup>, Christoph Striecks<sup>1</sup>, and Daniele Venturi<sup>1</sup>

extractors run in polynomial time. If we want to allow any bound  $D$  polynomial in the security parameter, we have to assume fast extraction (meaning that the extractor for an adversary running in time  $t$  takes only time  $t + \text{poly}(\lambda)$  for a polynomial  $\text{poly}$  independent of the adversary) to avoid an exponential blow-up

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**Straight-Line Extraction**

# How the Community Has Addressed This—A Landscape

includes “recursion-based” schemes (including those based on folding, etc.)

more practically relevant schemes

Using/proving secure schemes beyond  $O(1)$  depth

less practically relevant schemes

**YOLO:**\* (usually by practitioners)  
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EPFL

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Shahar Samocha  
shahars@starkware.co  
StarkWare

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includes “recursion-based” schemes (including those based on folding, etc.)

Using/proving secure schemes beyond  $O(1)$  depth

more practically relevant schemes

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## Limitations

Often it might not be warranted.

Modifies schemes (or applicable only at times)

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More complex, more inefficient

more practically relevant schemes

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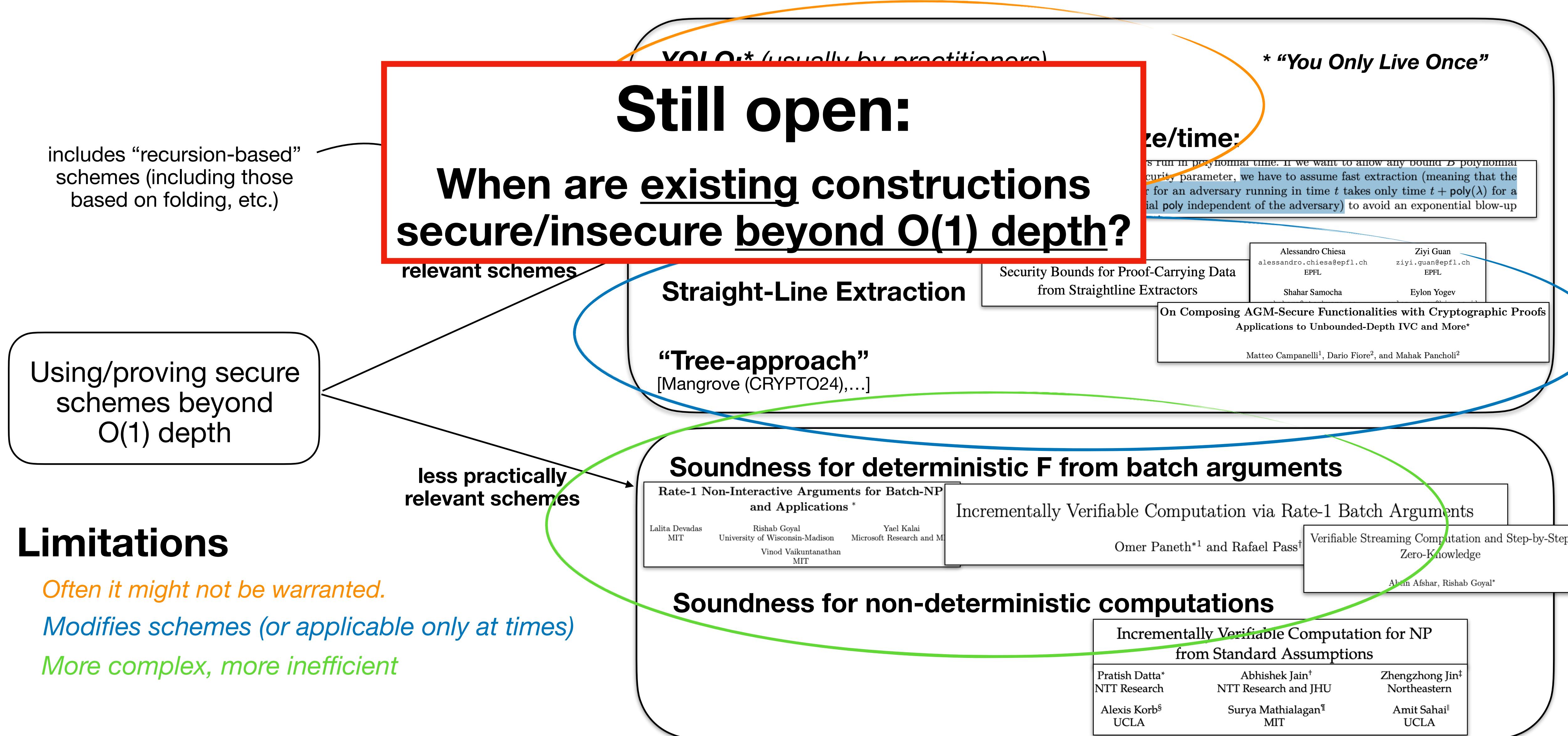
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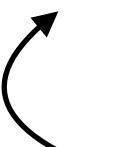
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# This Work's Question

**Still open:**  
When are existing constructions  
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The problem at hand

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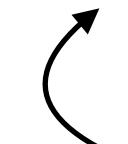
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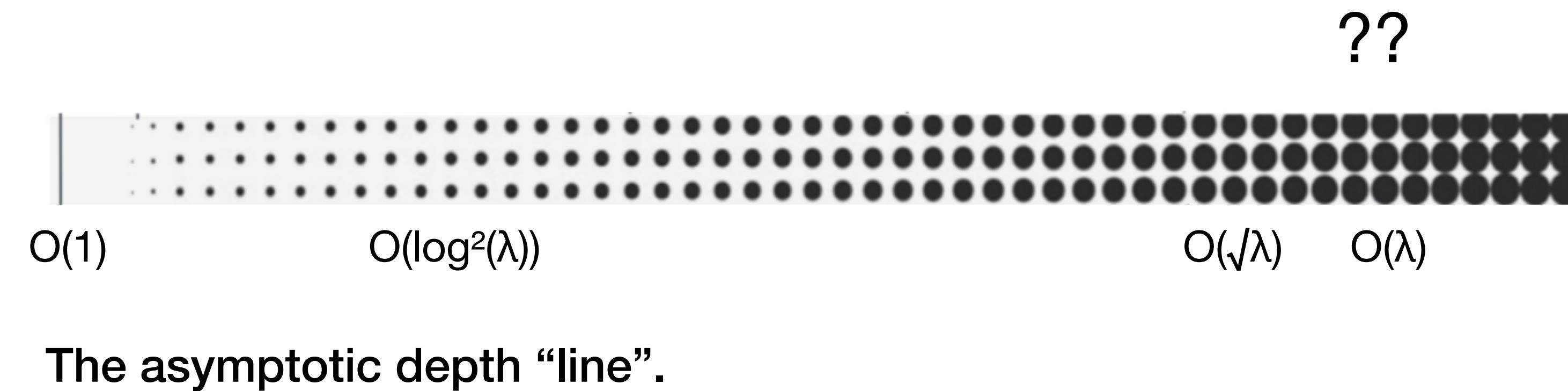
The problem at hand

**When is any construction secure/insecure  
beyond O(1) depth?**

We approach this question through two main conceptual lenses.

# Lens 1: “*Depth*” as a Core Object of Study

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The asymptotic depth “line”.

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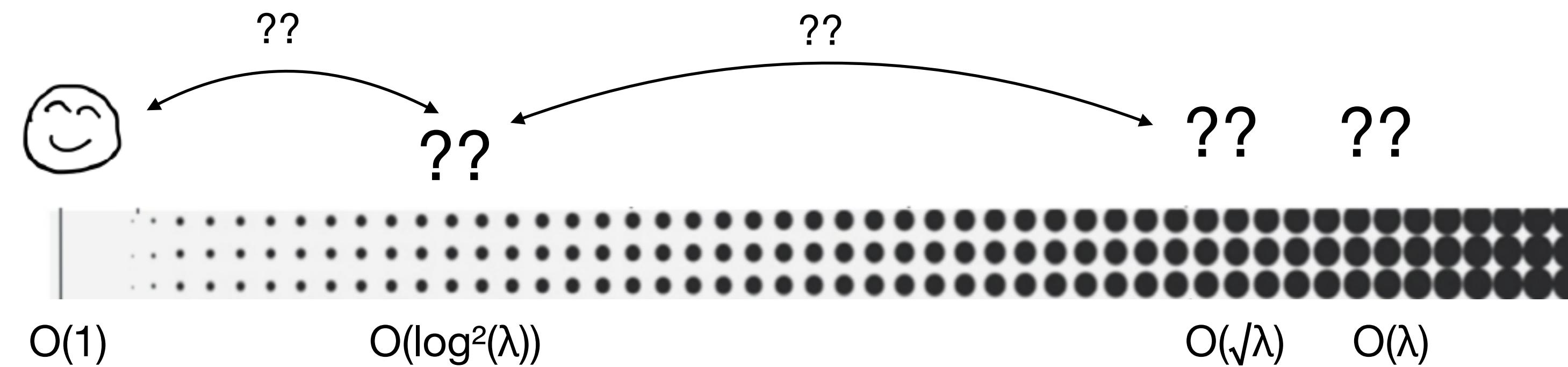
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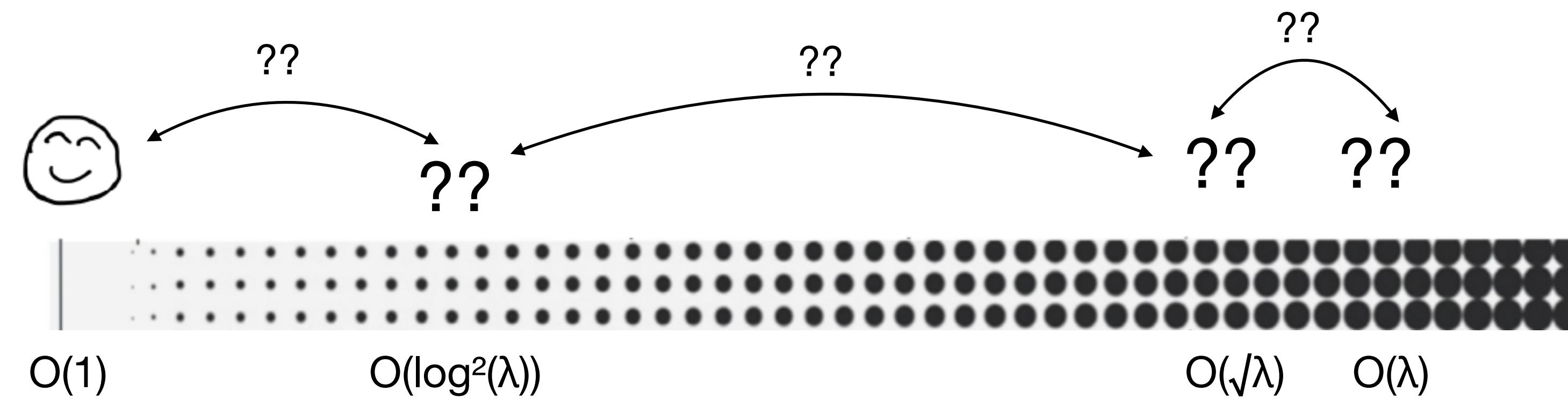
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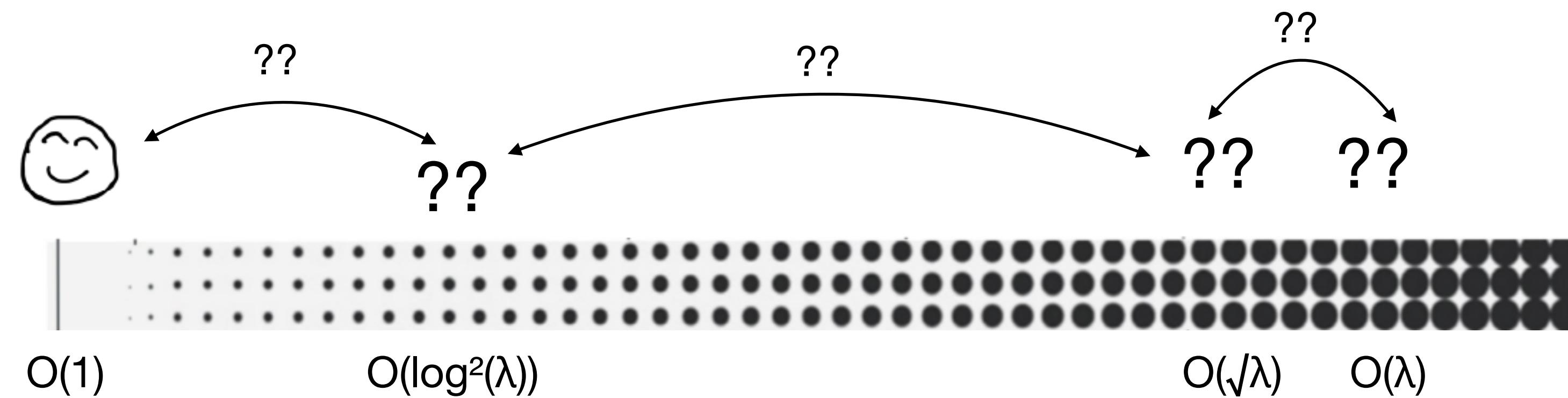
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The asymptotic depth “line”.

**A note on abuse of language:**  
I will say  
“big/bigger” to mean “fast/er growing”;  
“small/smaller” to mean “slow/er growing”

# **Lens 2: Keeping Extractable Security in the Background**

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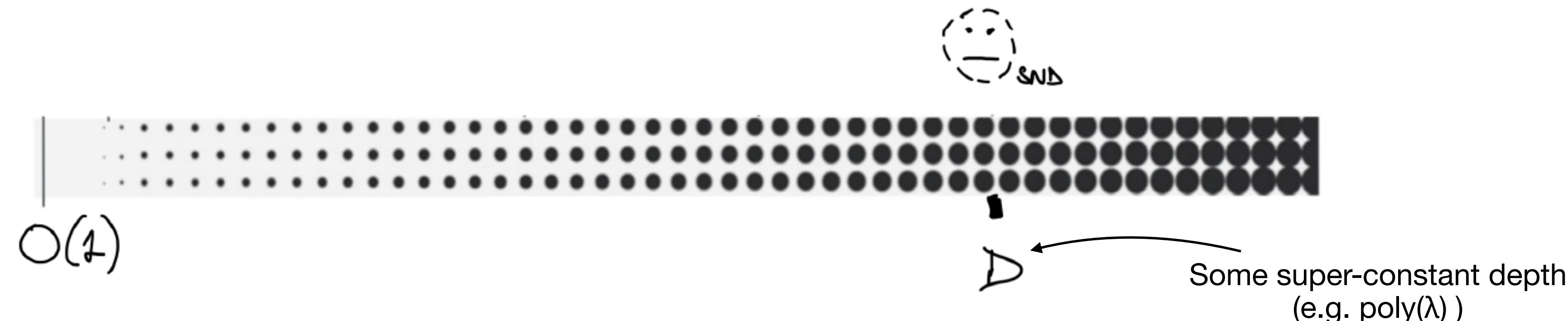
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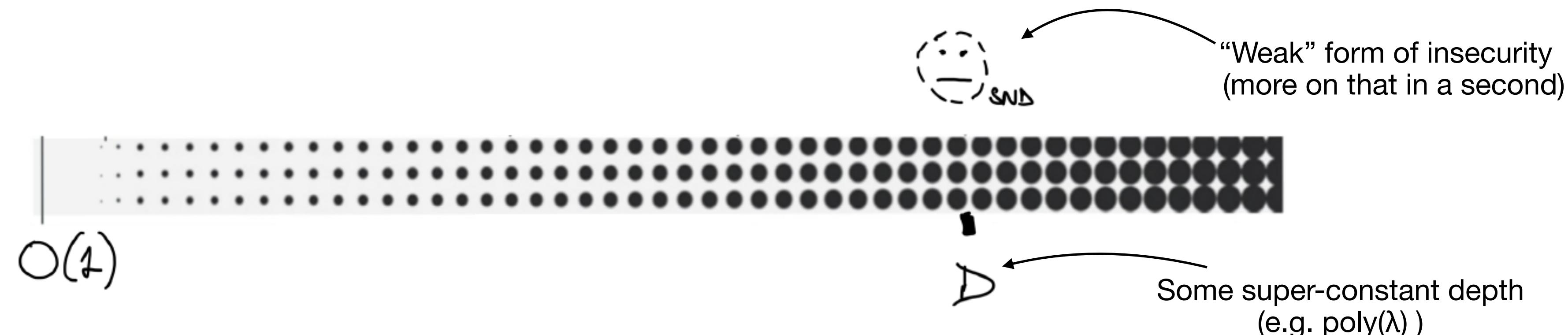
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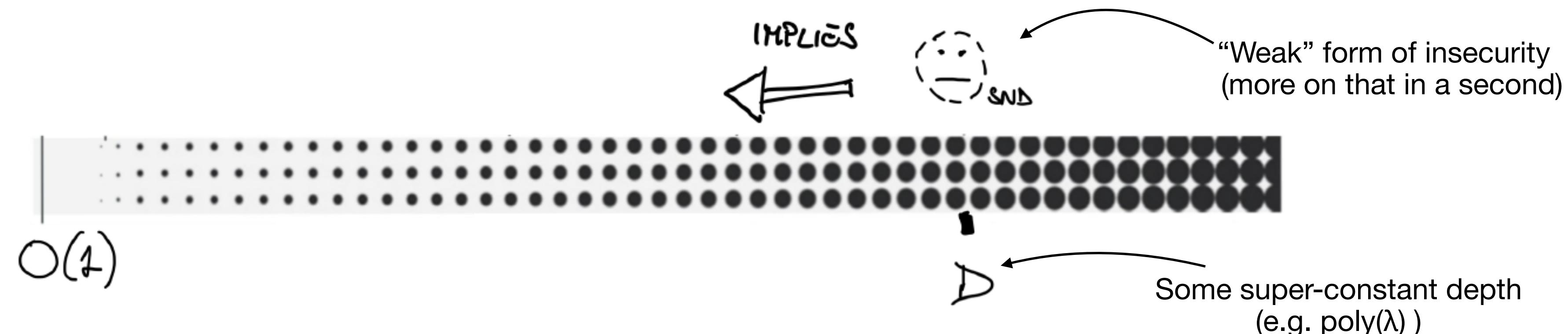
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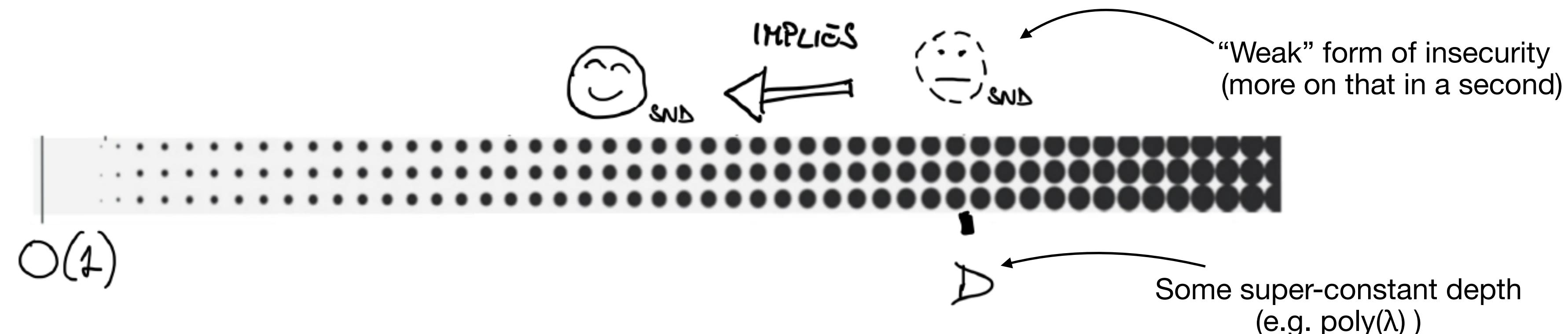
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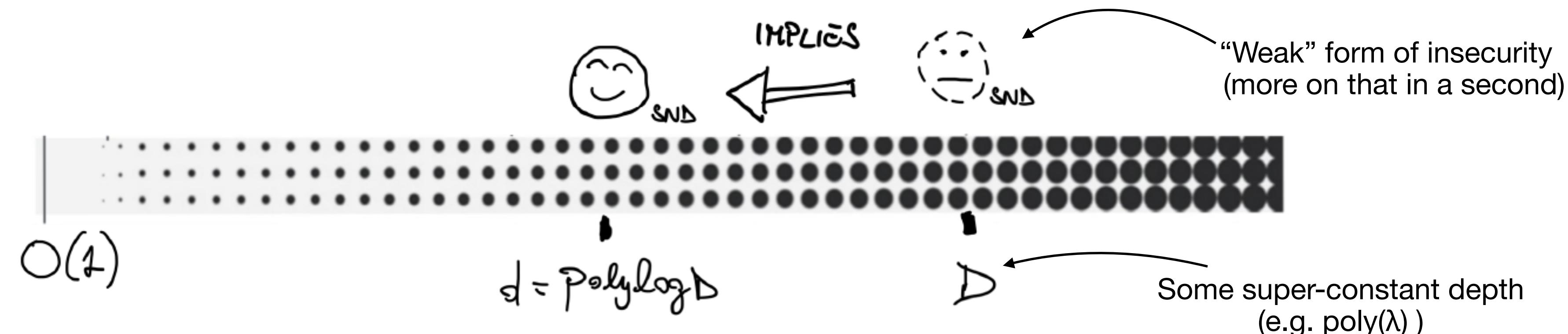
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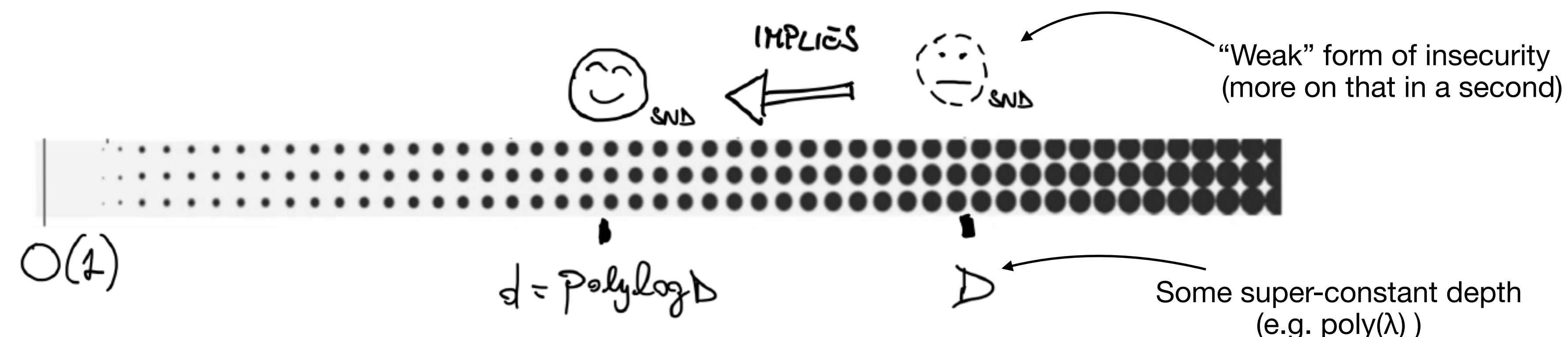
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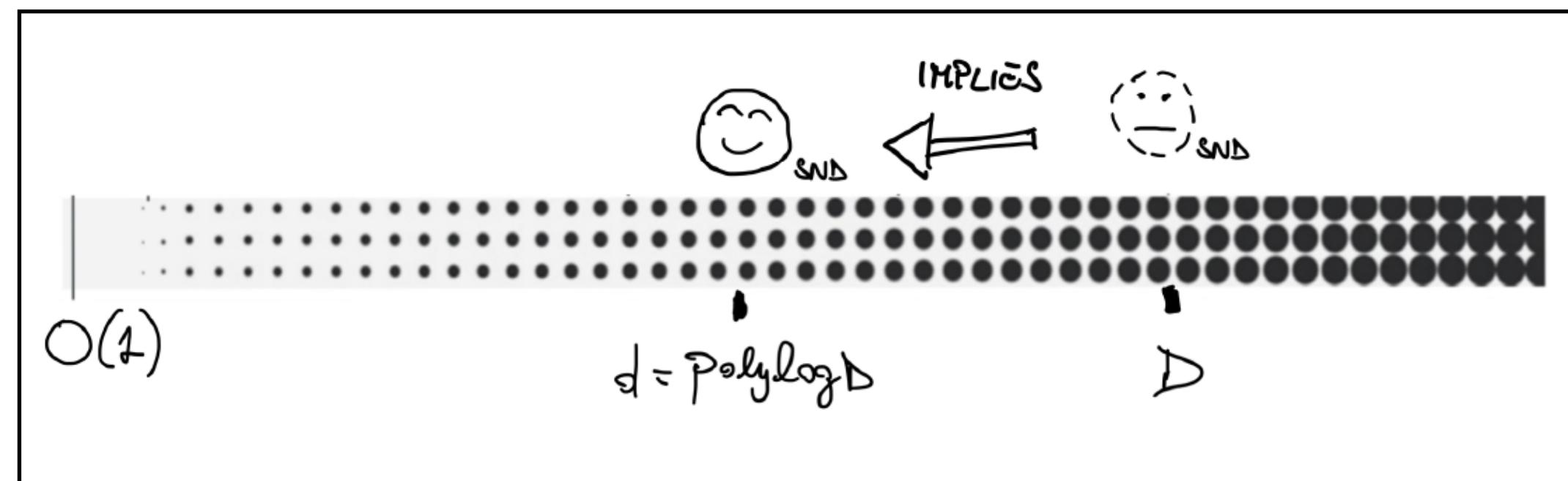
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### Implication:

to prove security at some  $\omega(1)$  depth  $d$ ,  
show some  $\omega(1)$  depth  $D$  where this weak property holds.

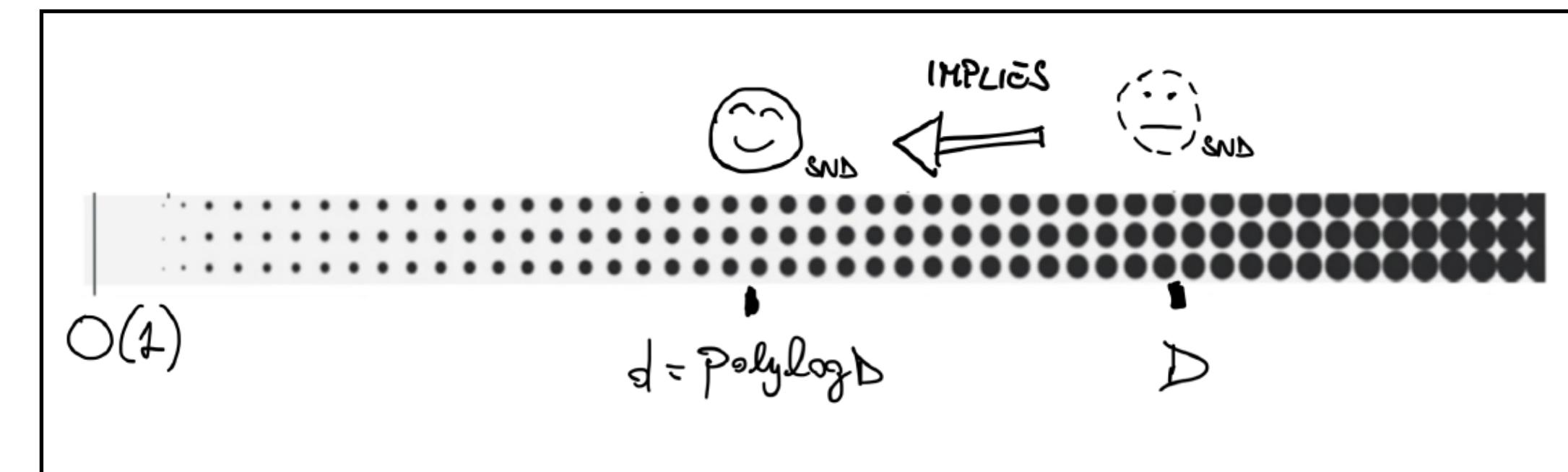
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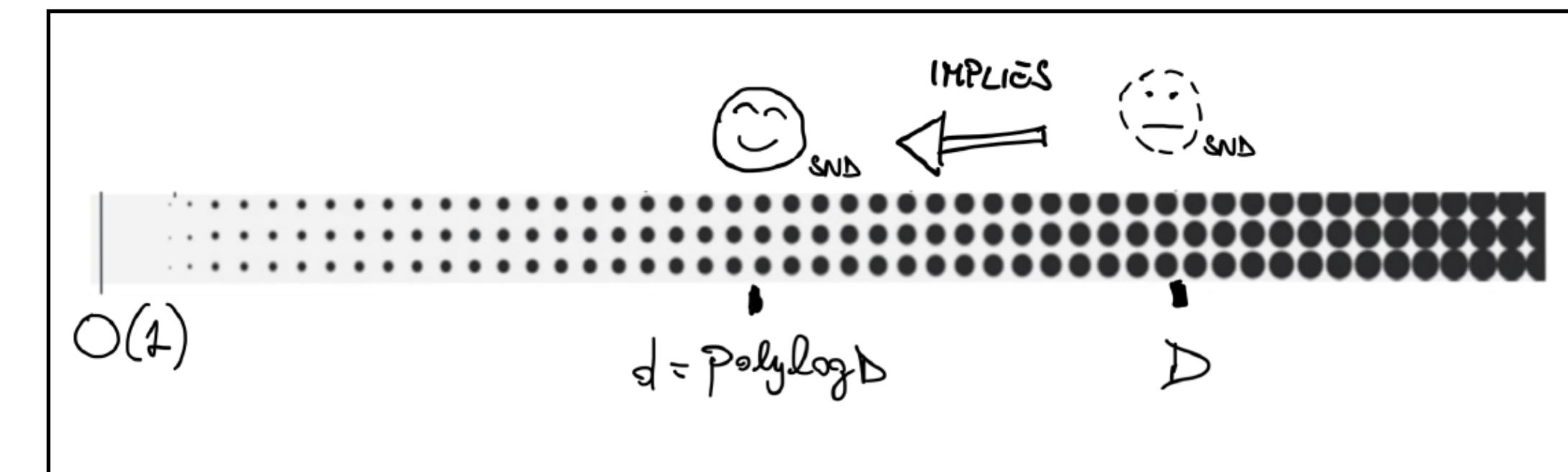
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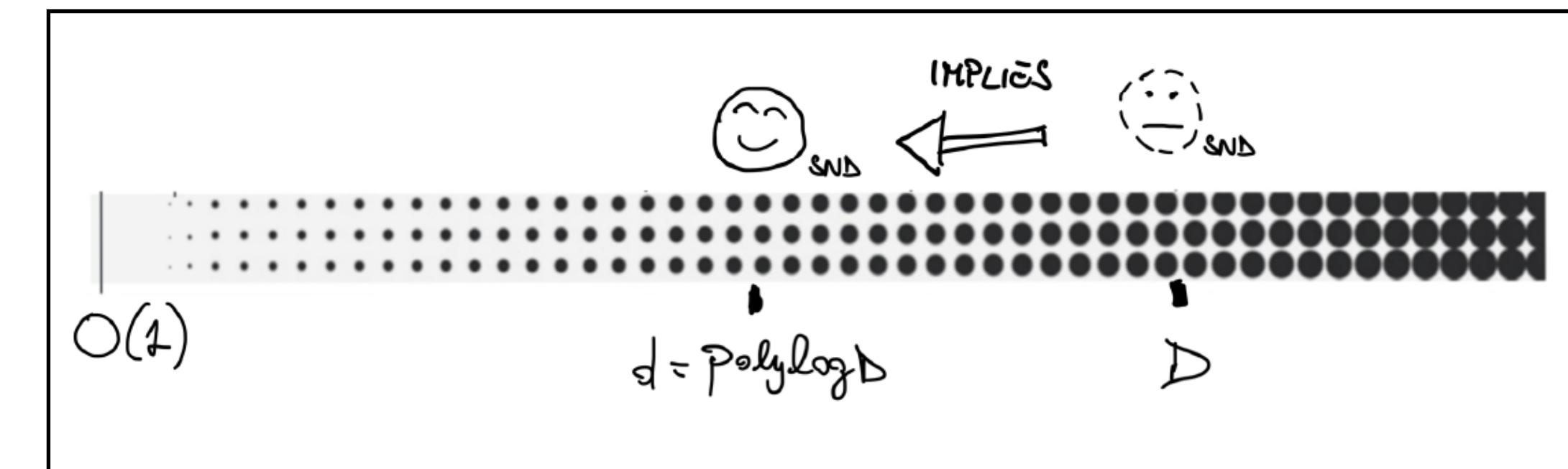
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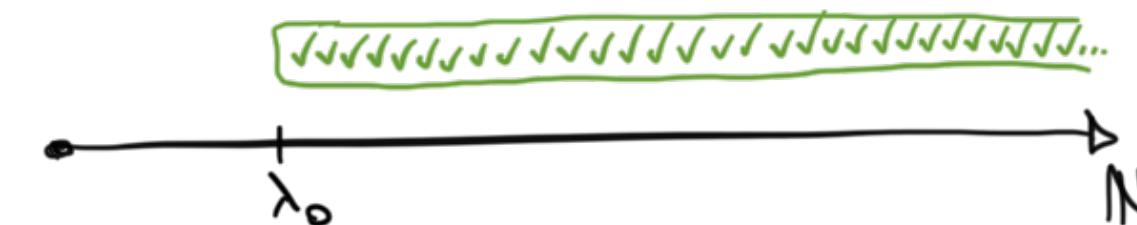


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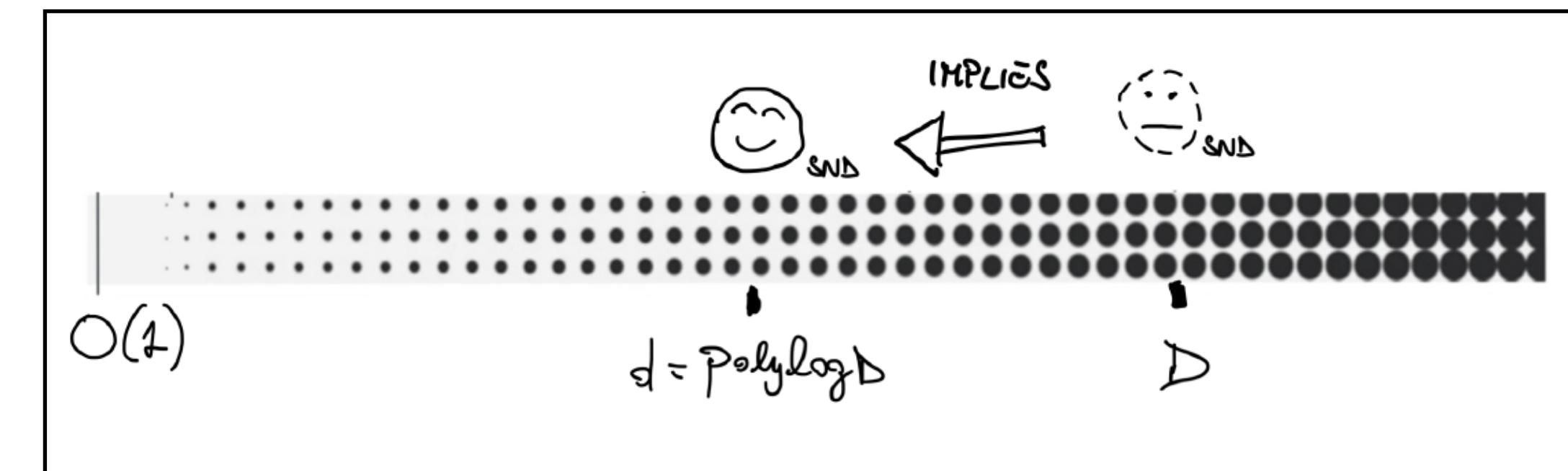
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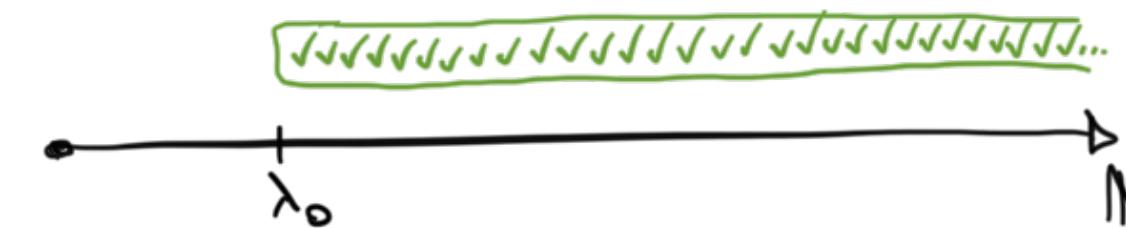


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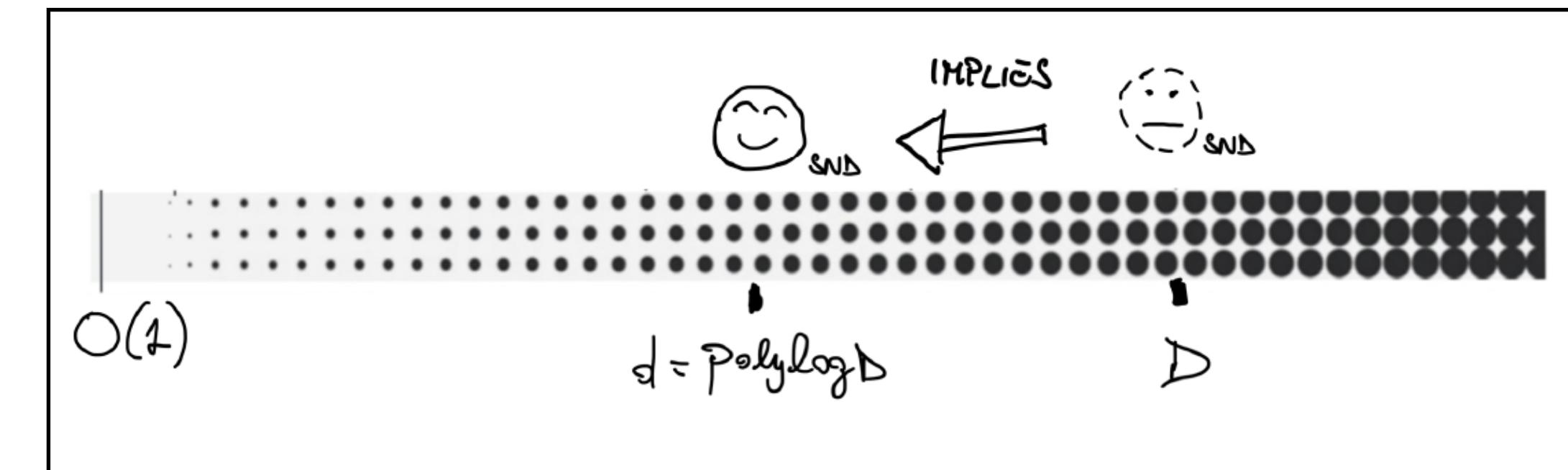
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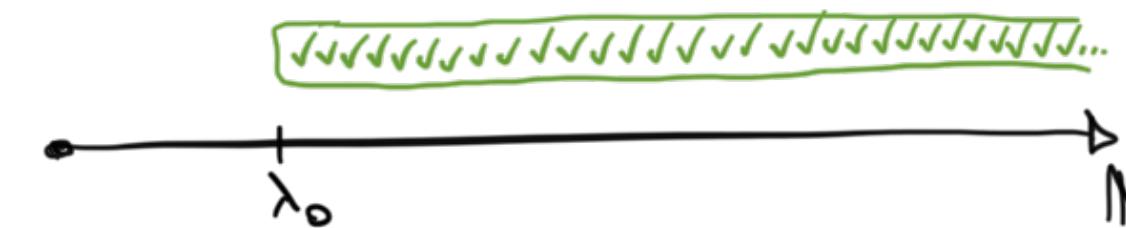


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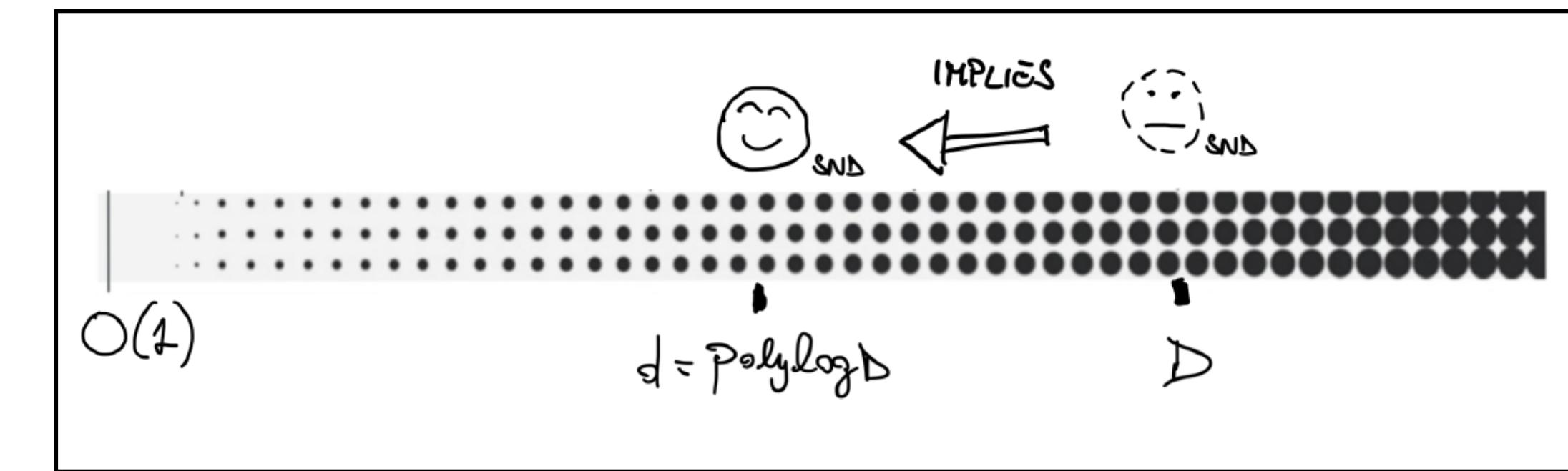
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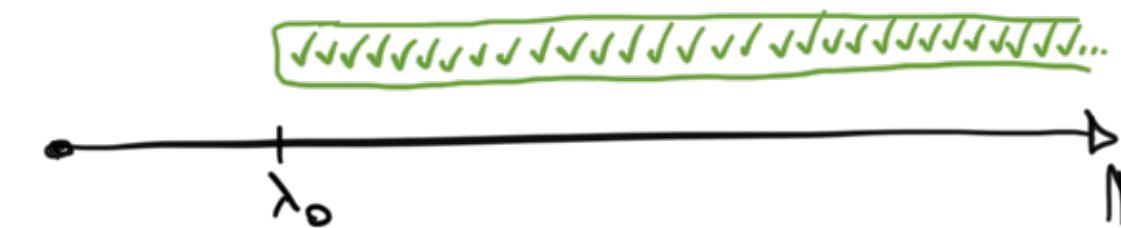


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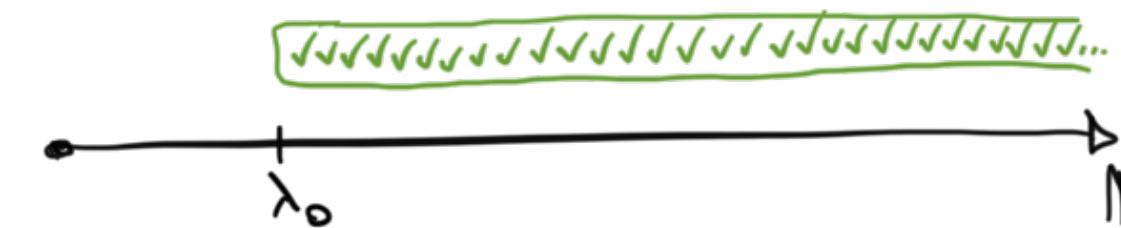
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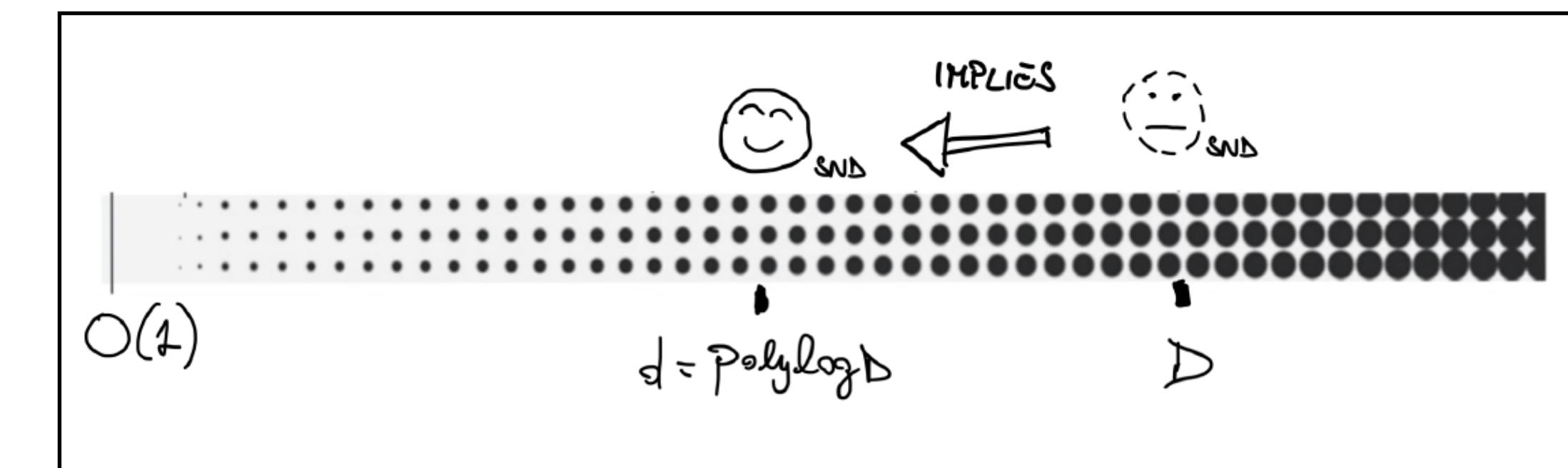
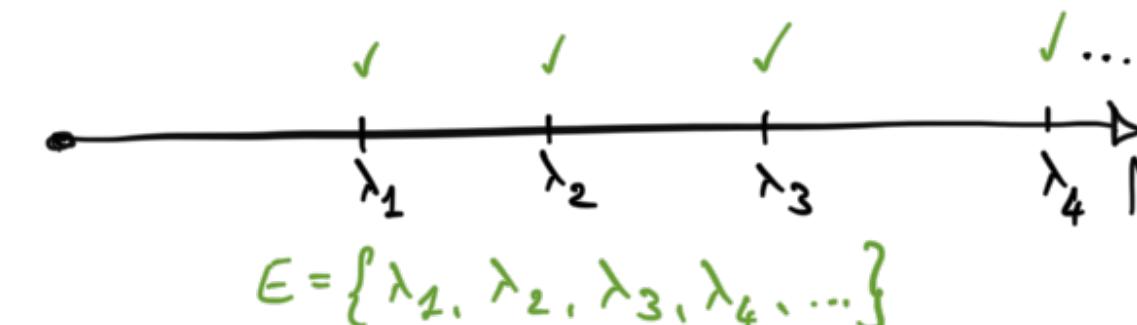
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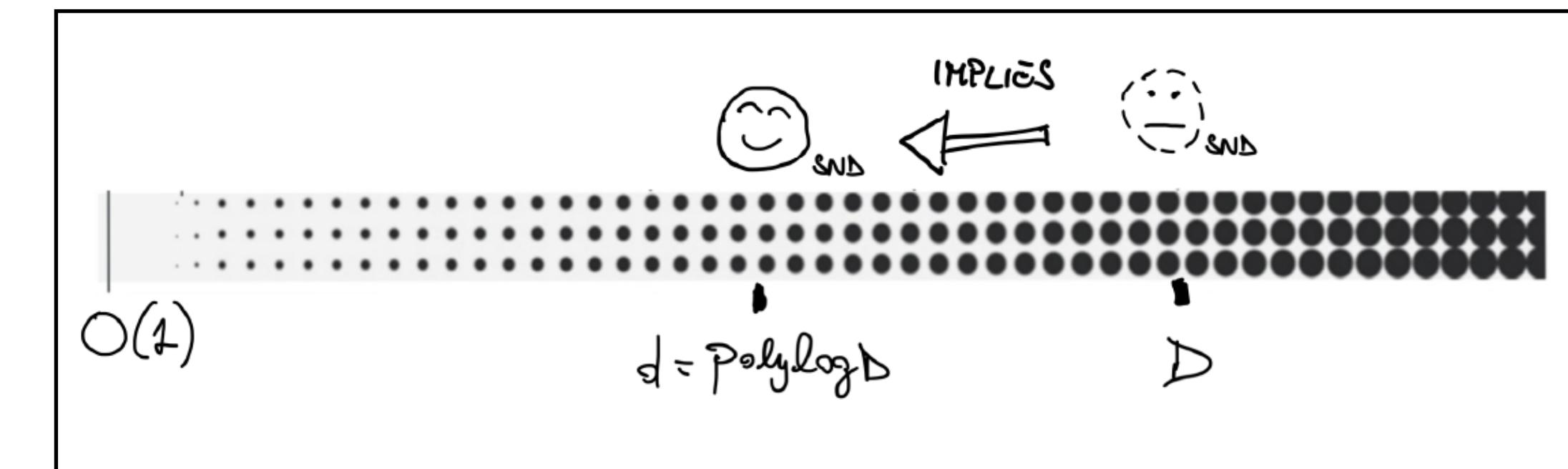


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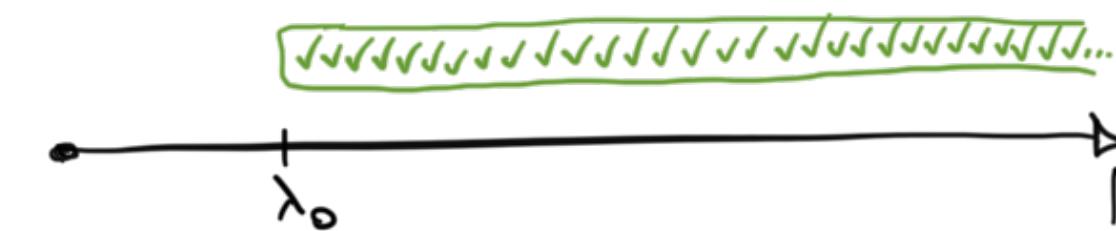


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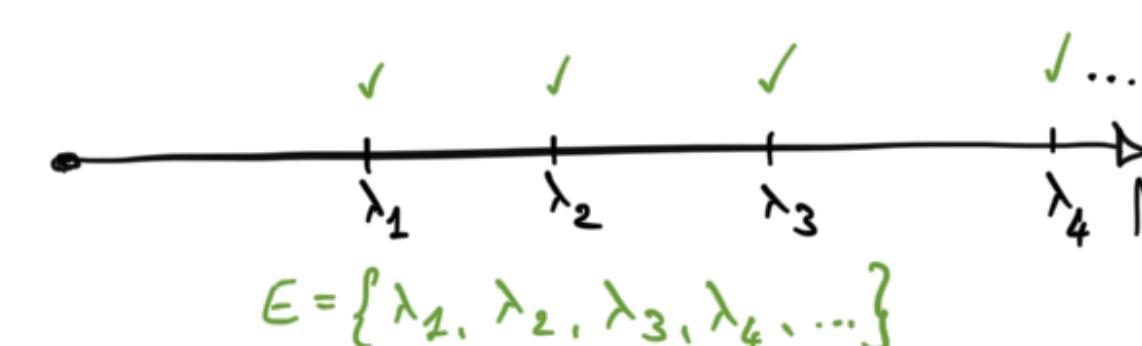
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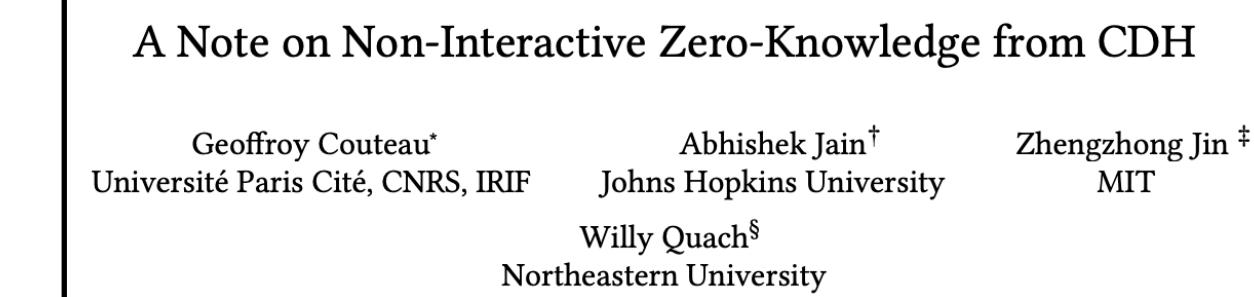
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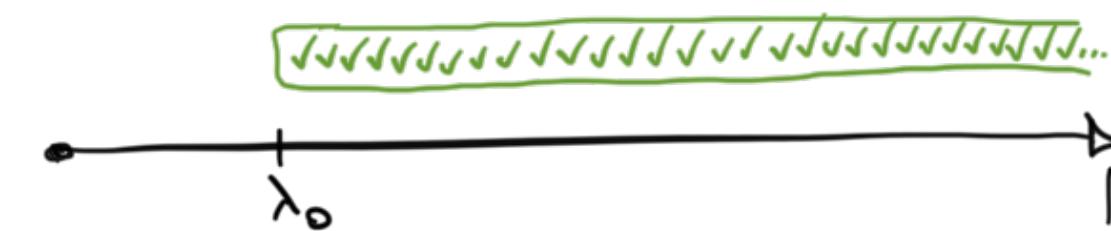
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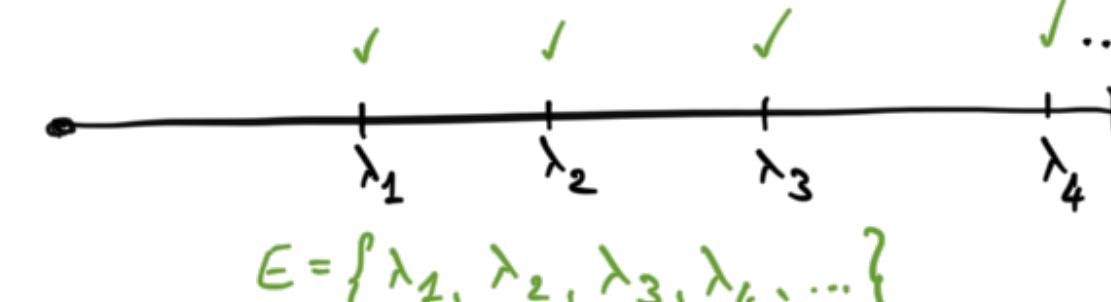
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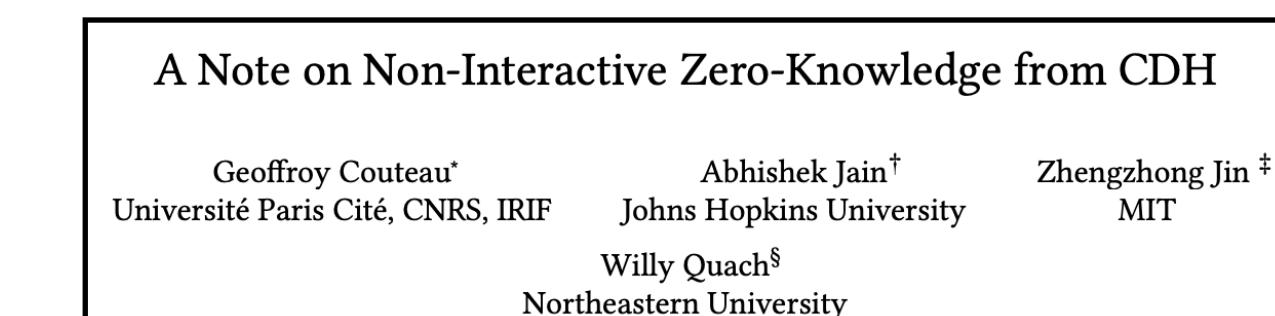
IMPLIES

$O(1)$

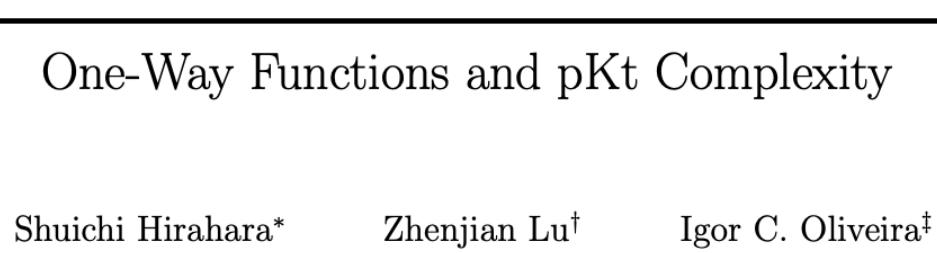
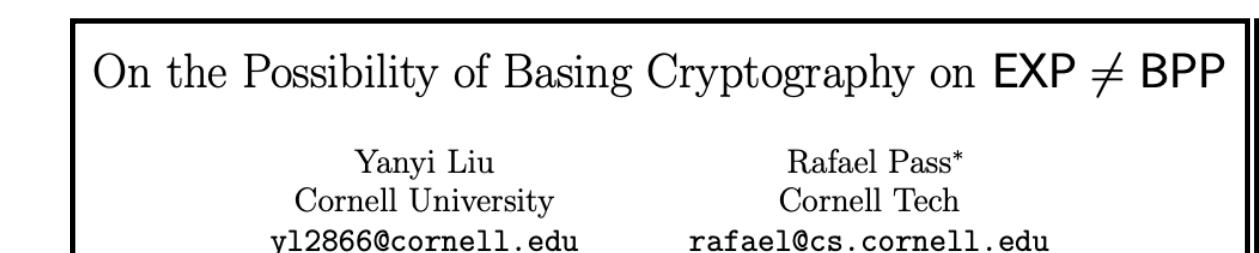
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D

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[CRYPTO '21, TCC '24]:  
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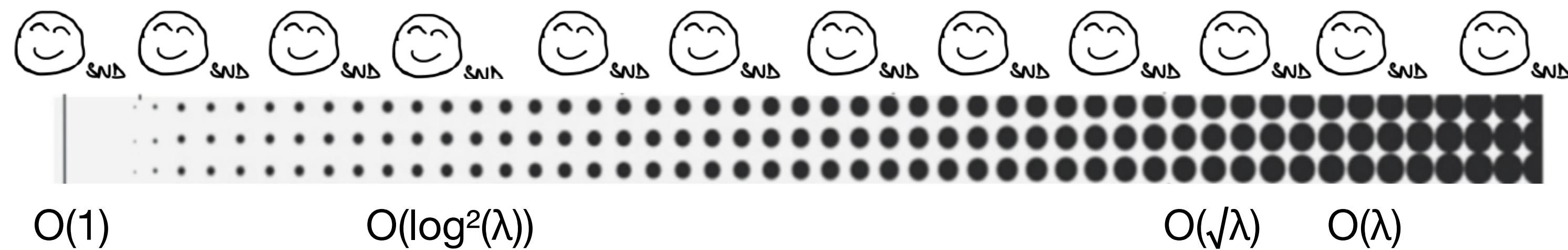
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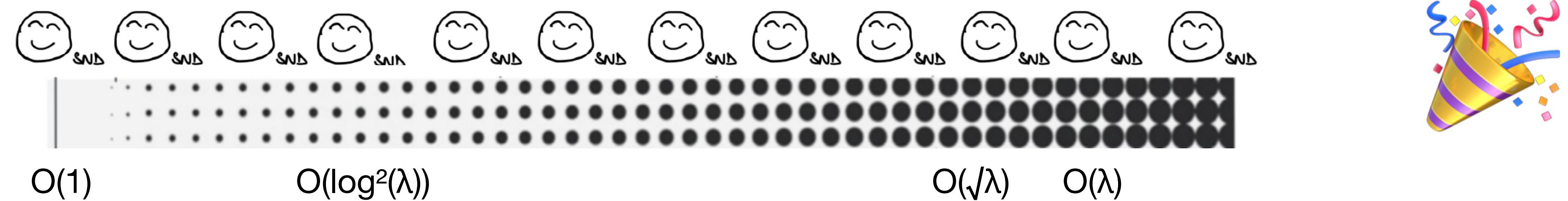


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**Case 2: insecure somewhere.**

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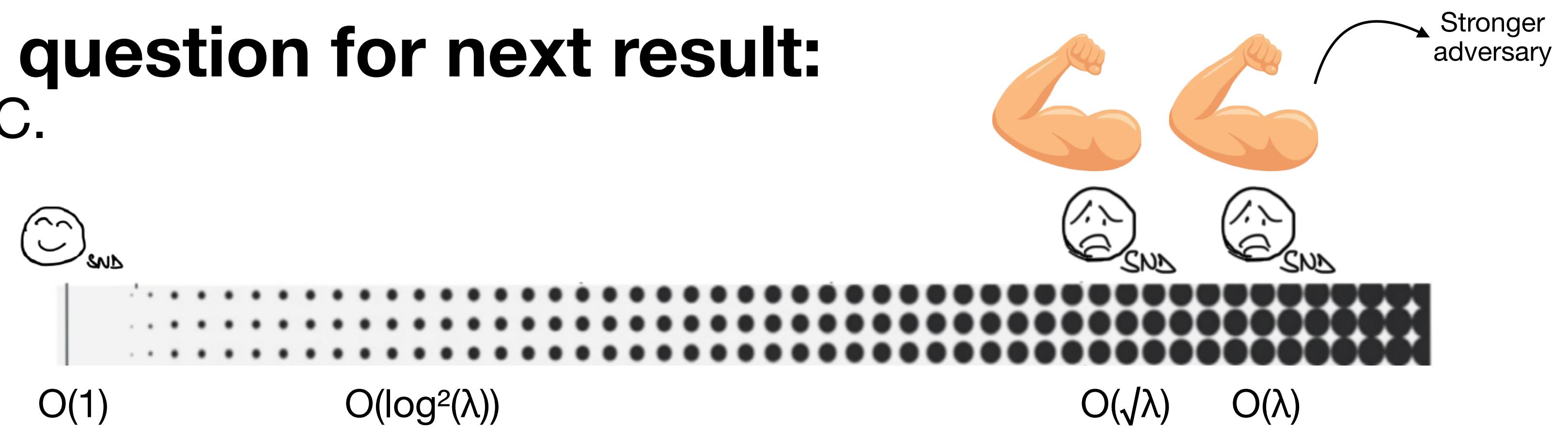


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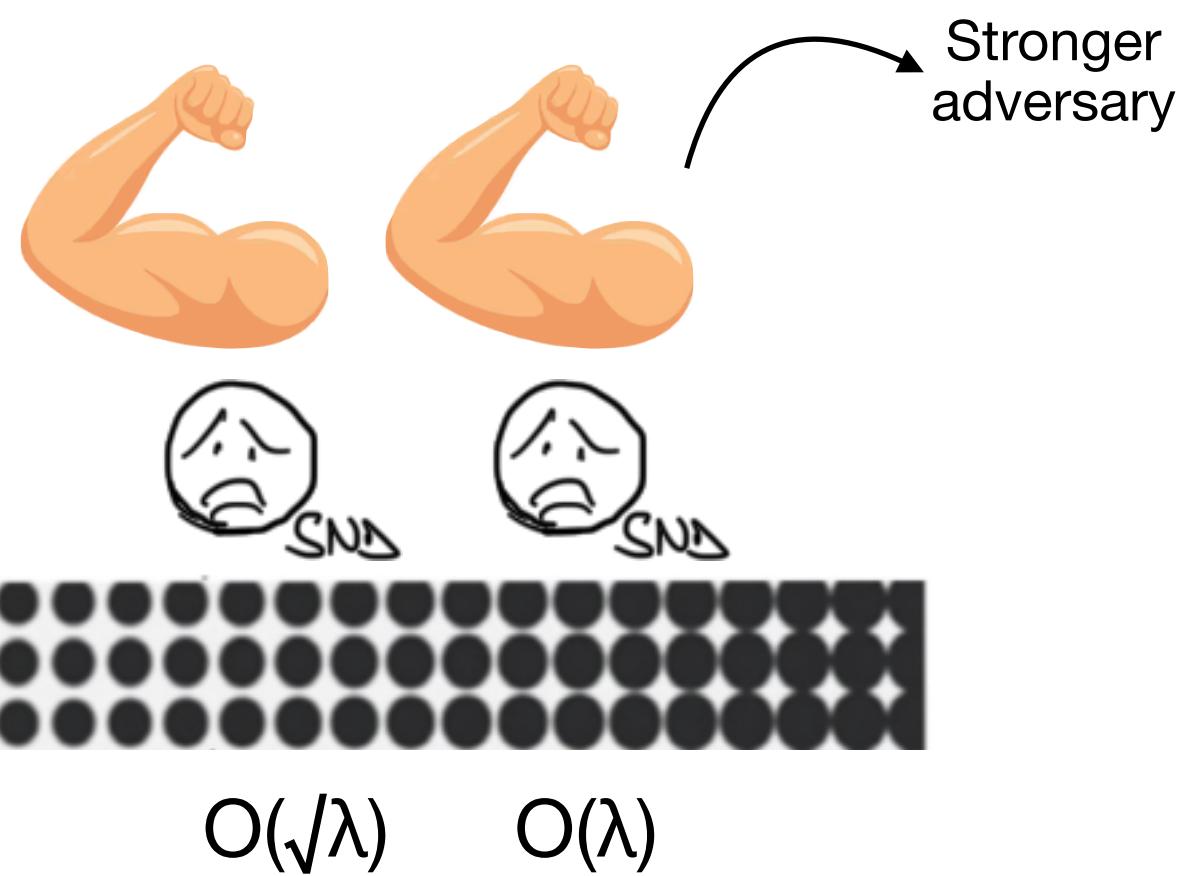
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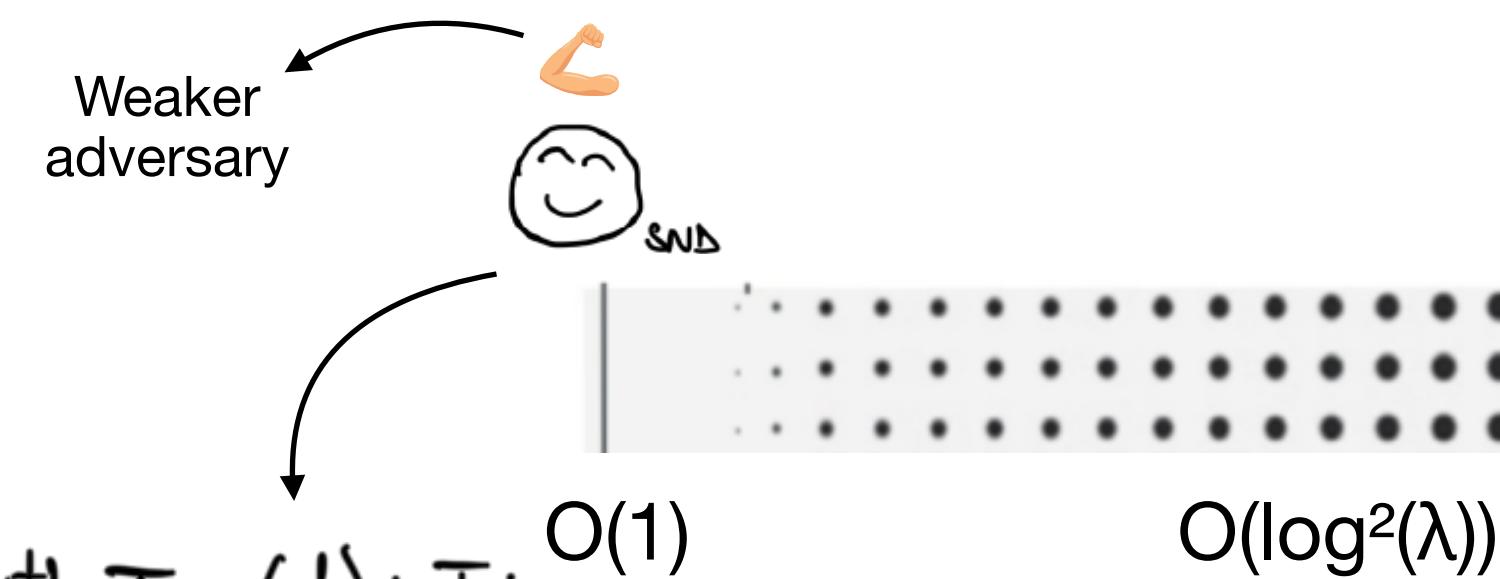
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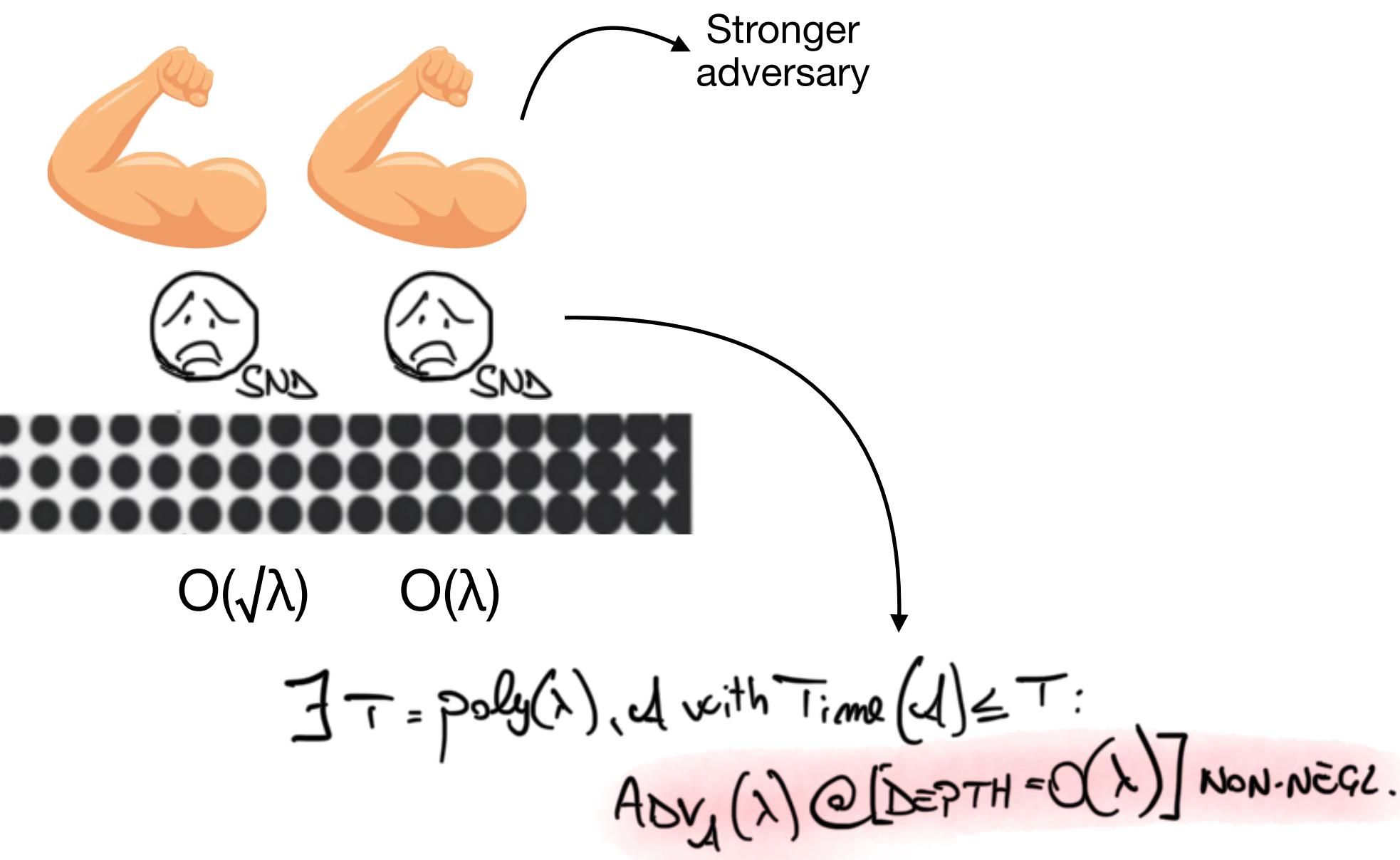
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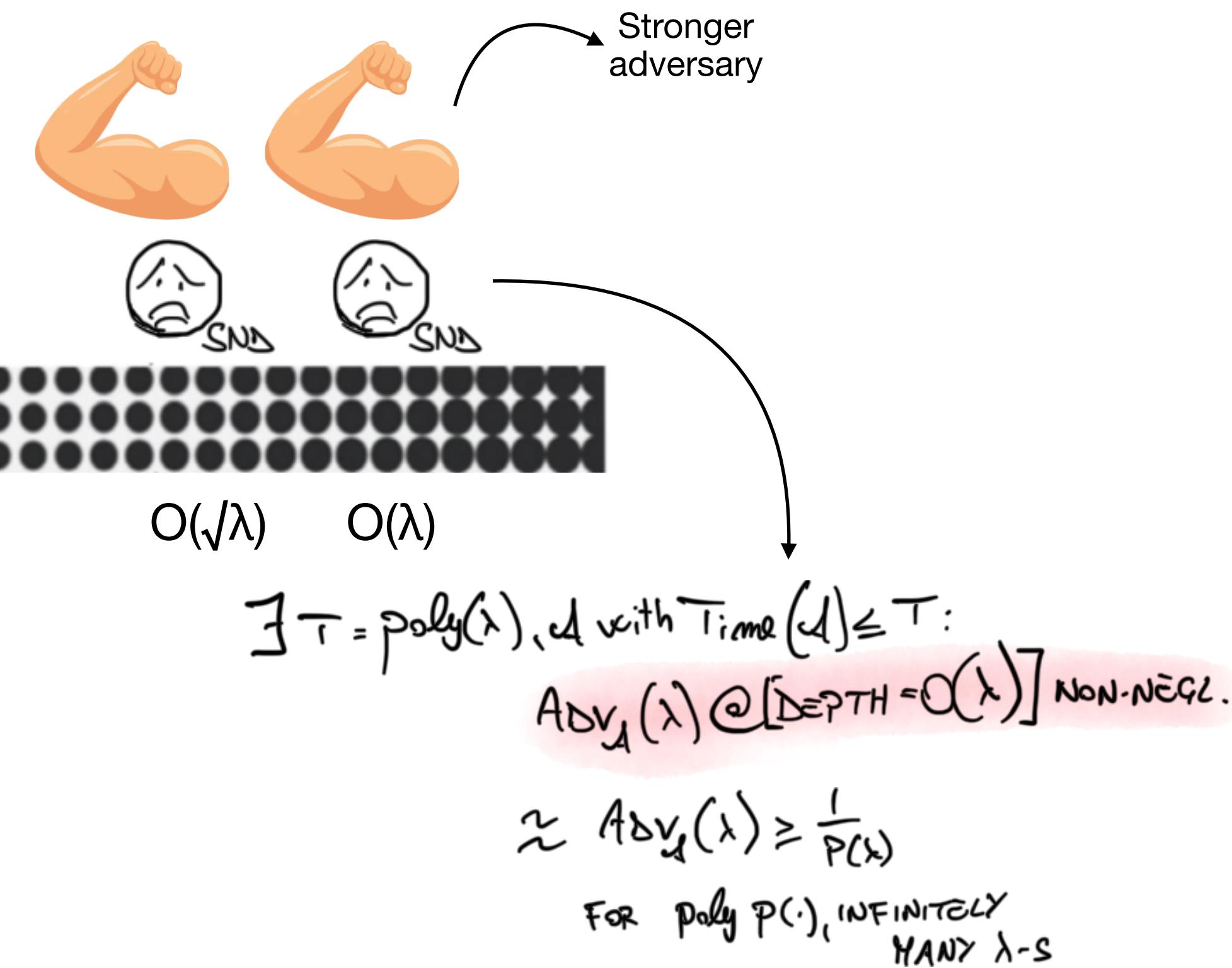
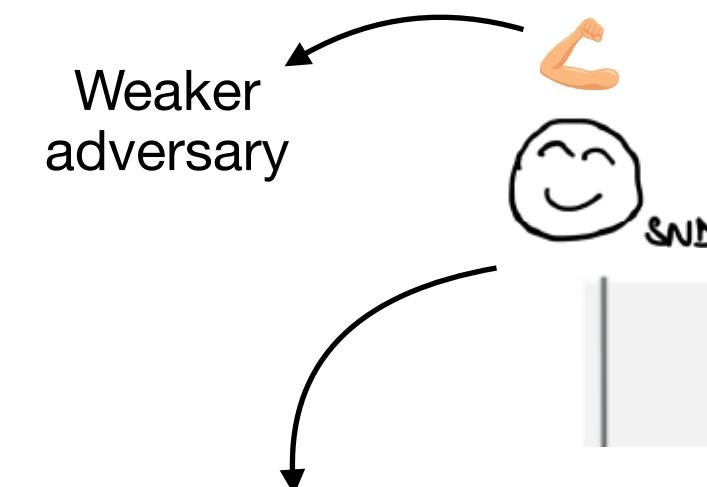
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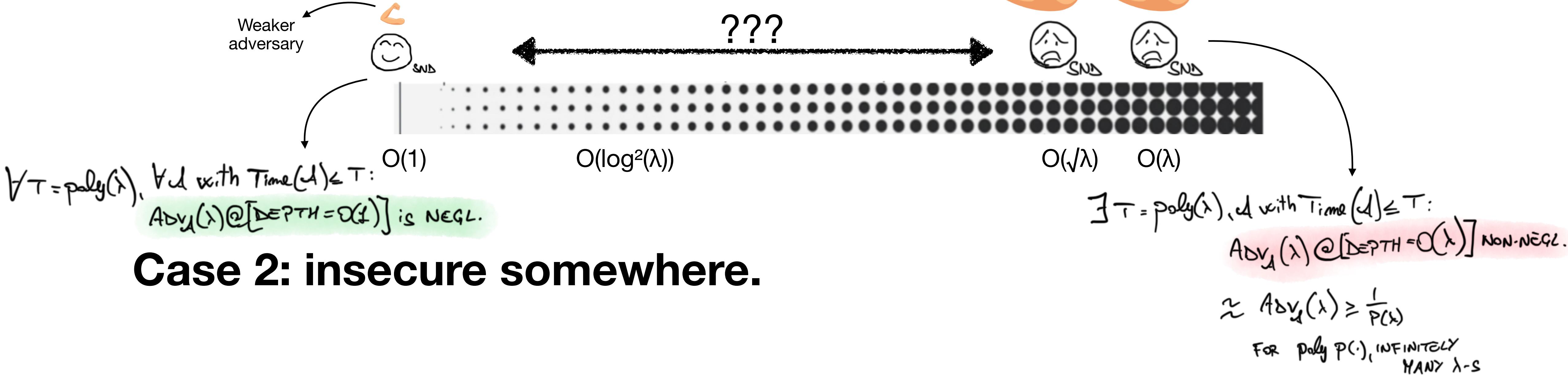


## Case 2: insecure somewhere.

# Our Results (continued)

## Motivating question for next result:

Let  $\Pi$  be an IVC.



**Case 2: insecure somewhere.**

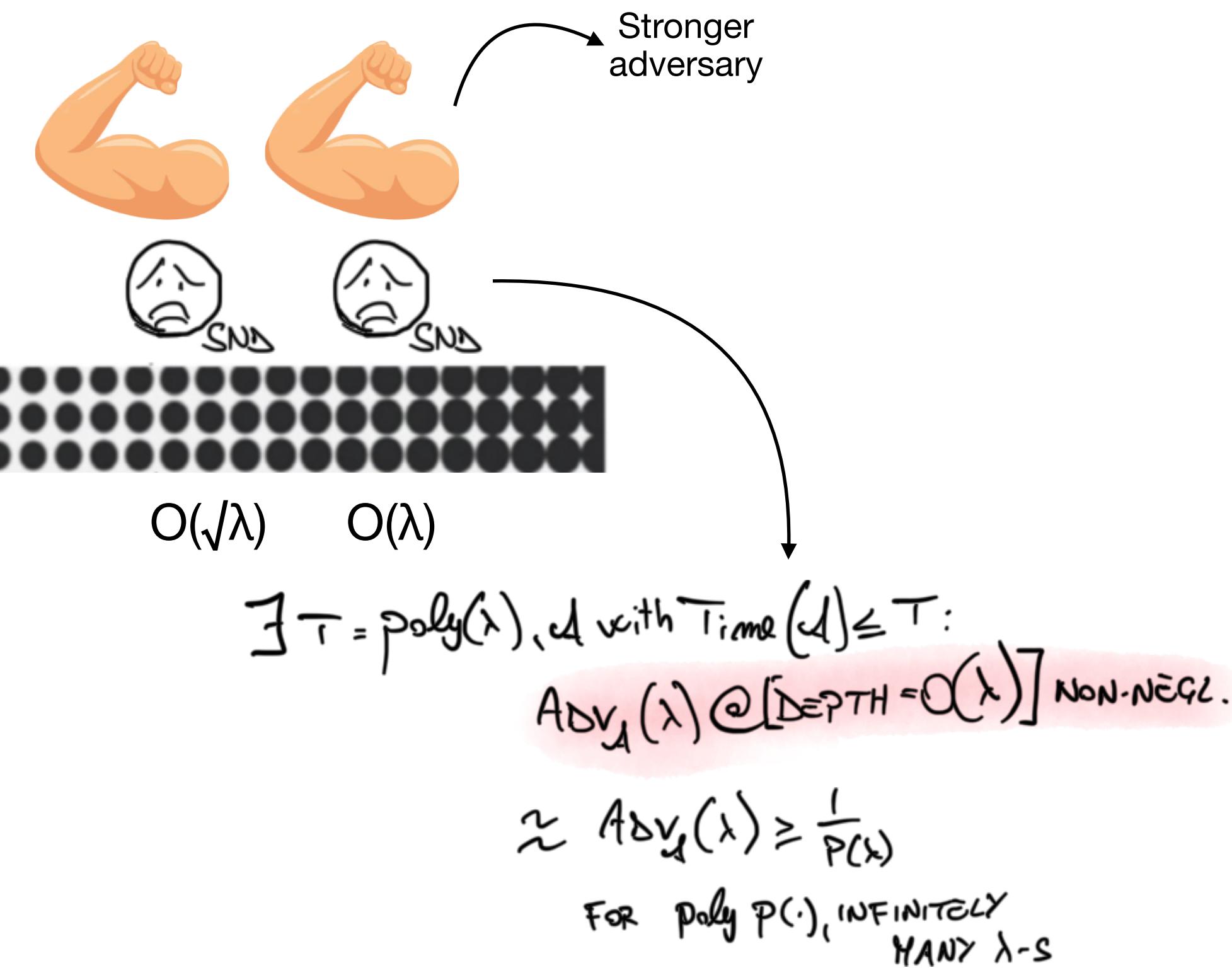
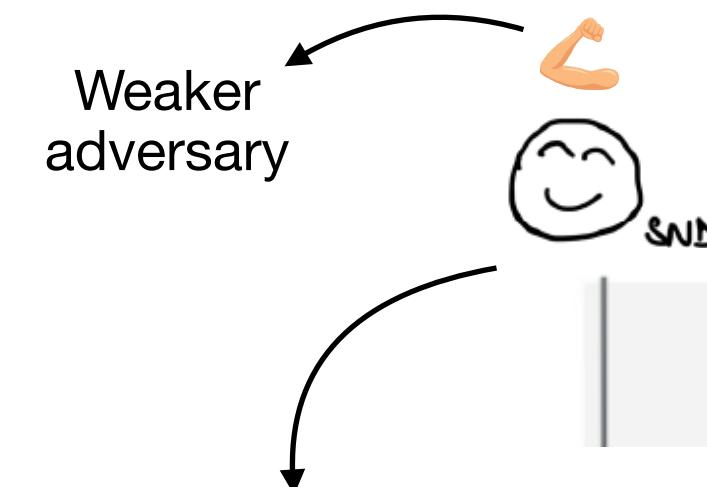
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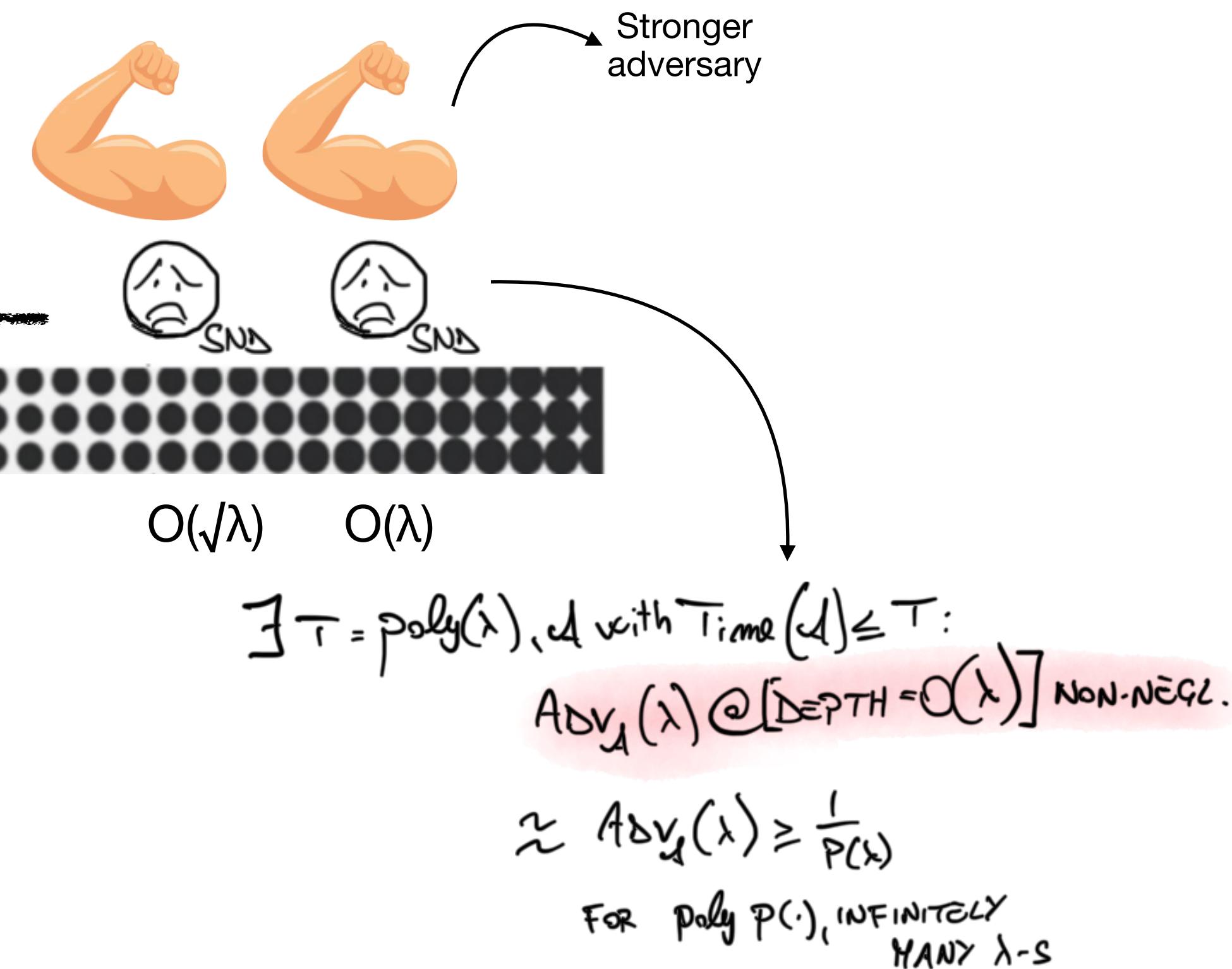
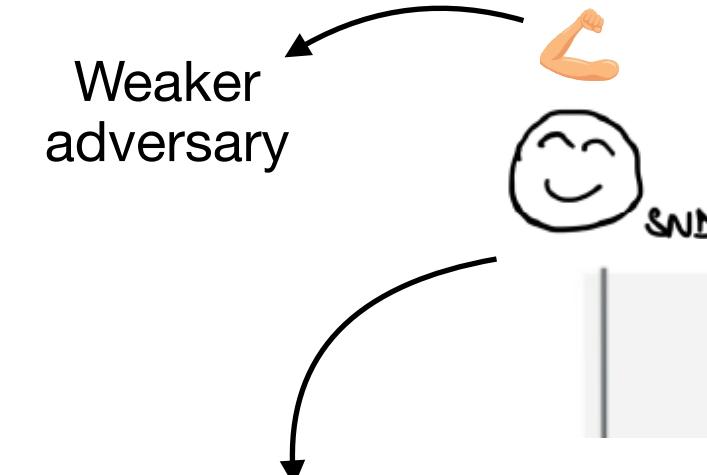
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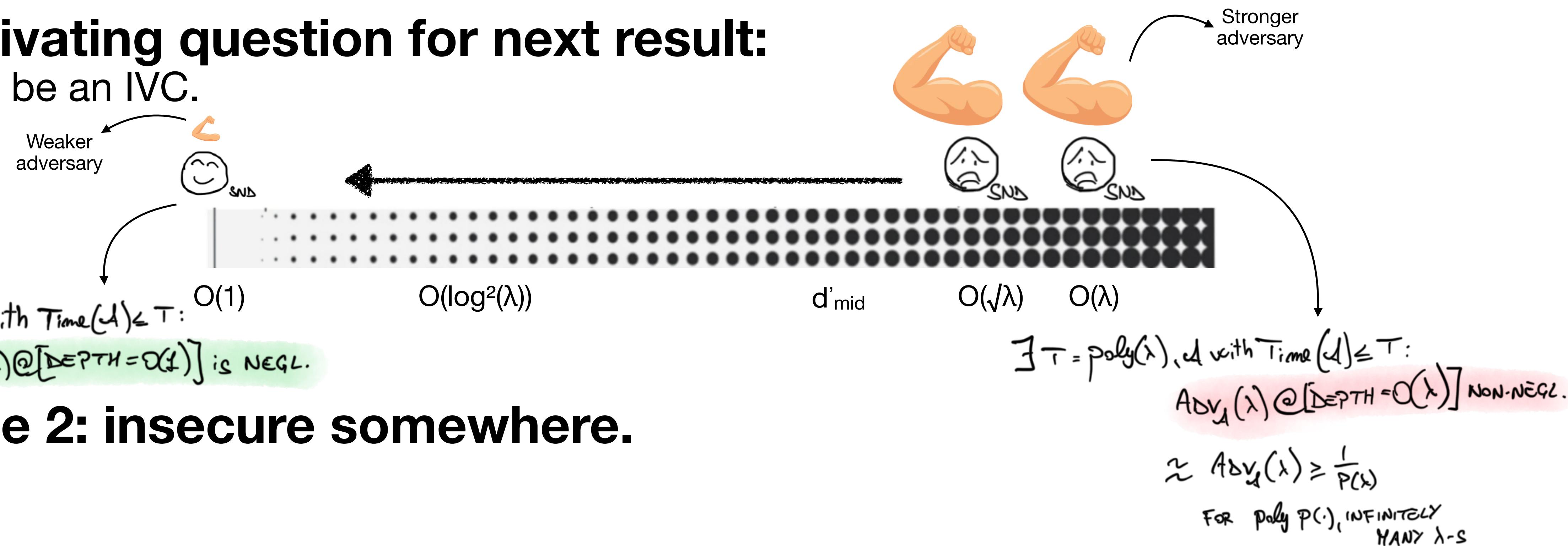
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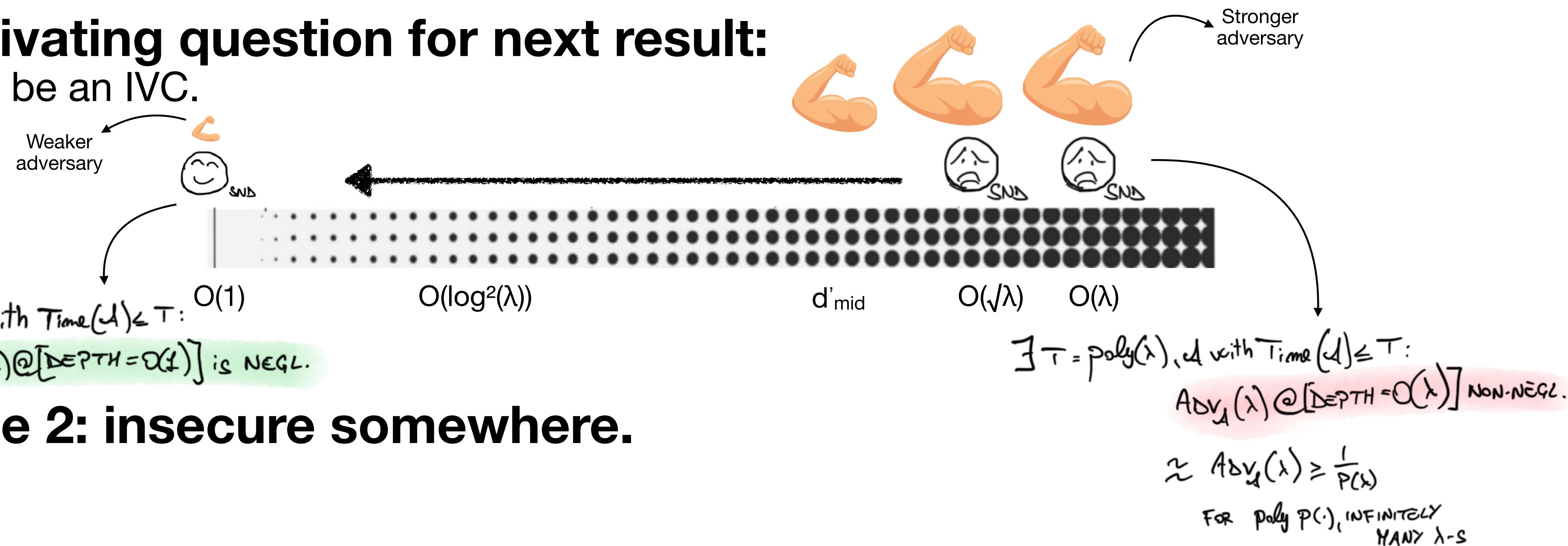
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# Our Results

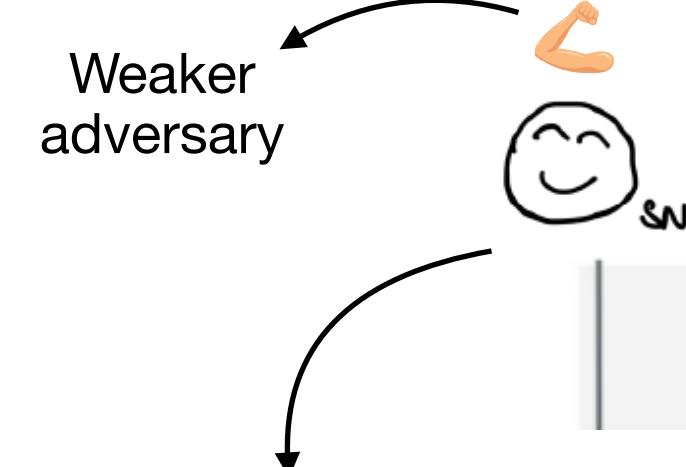
(continued)

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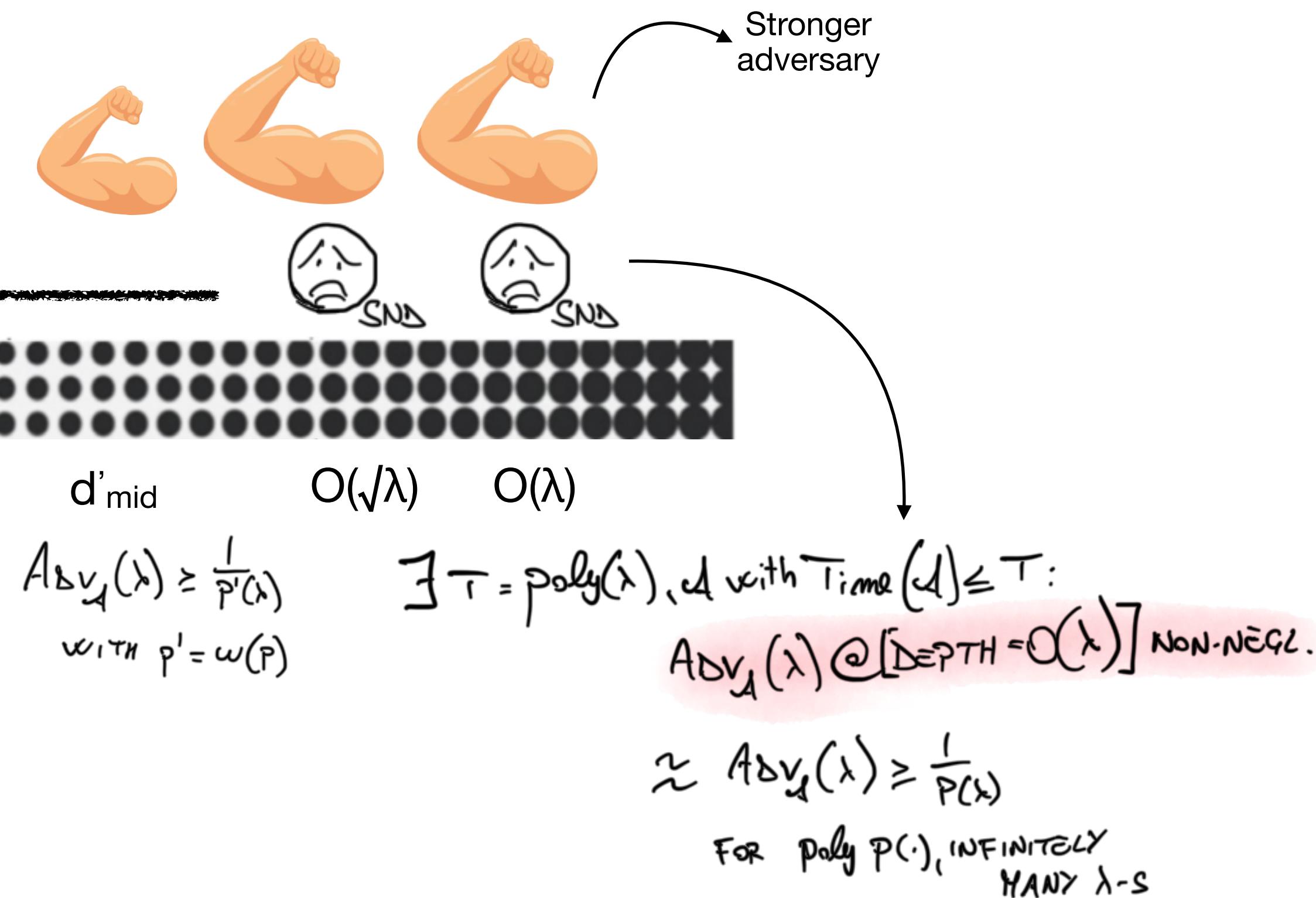
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# Our Results

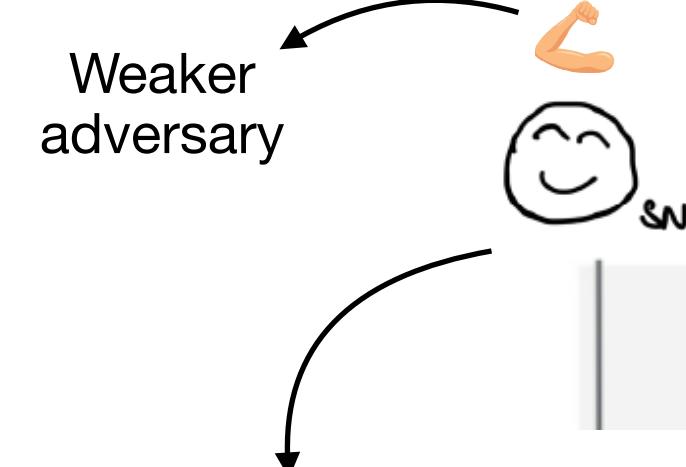
(continued)

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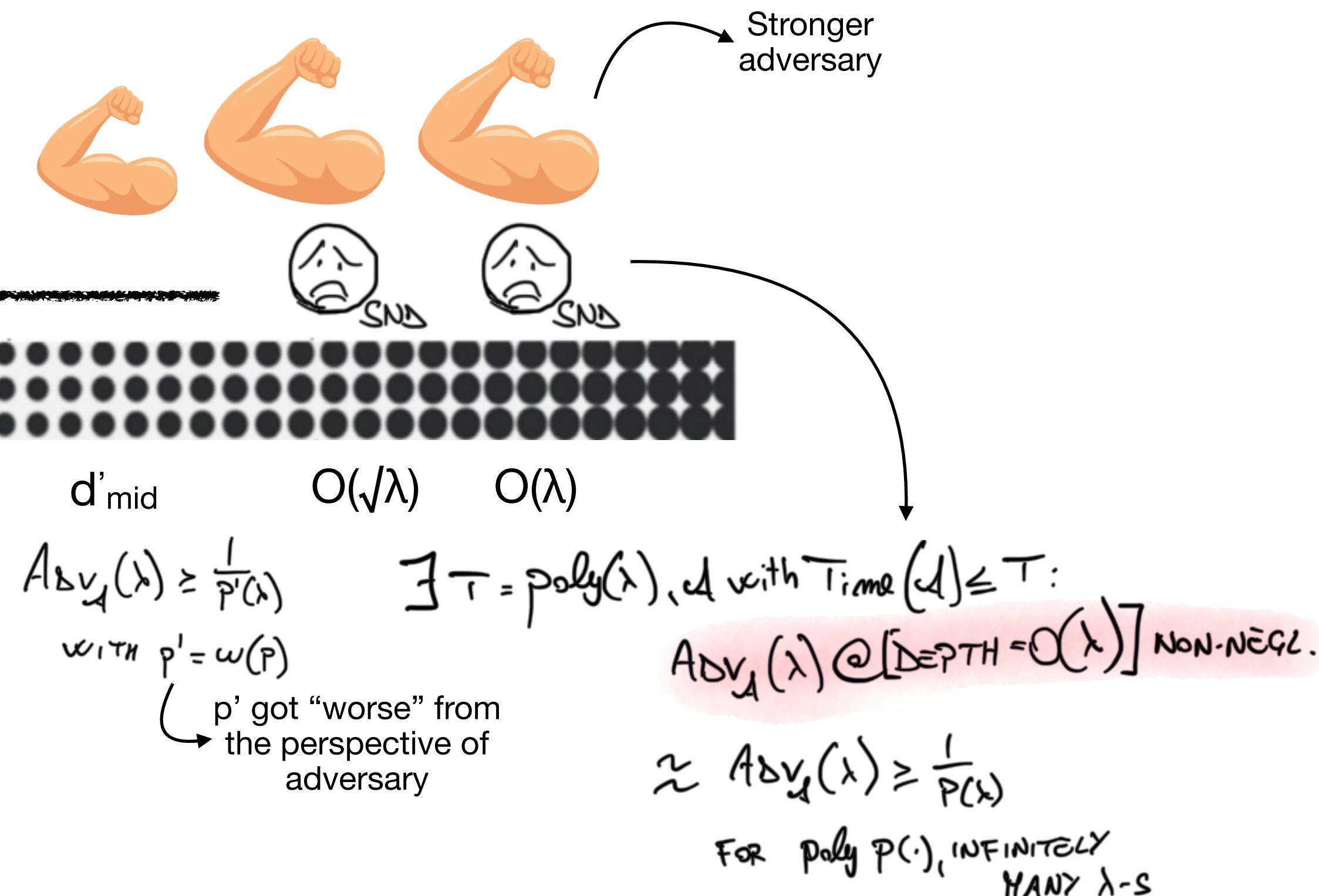
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# Our Results

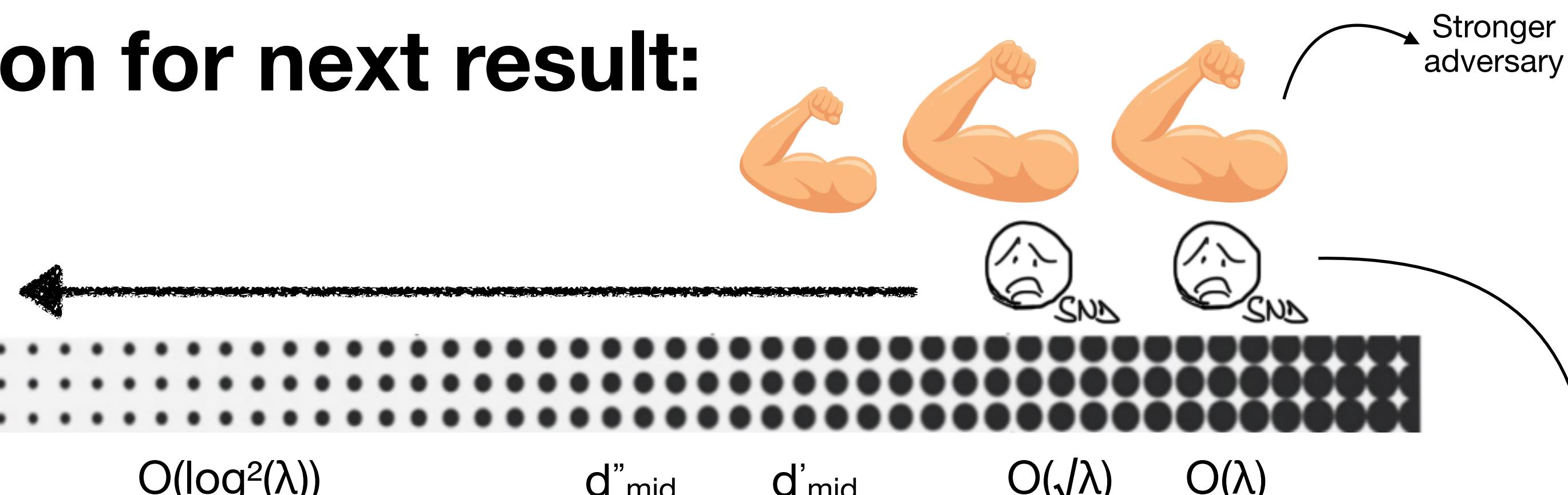
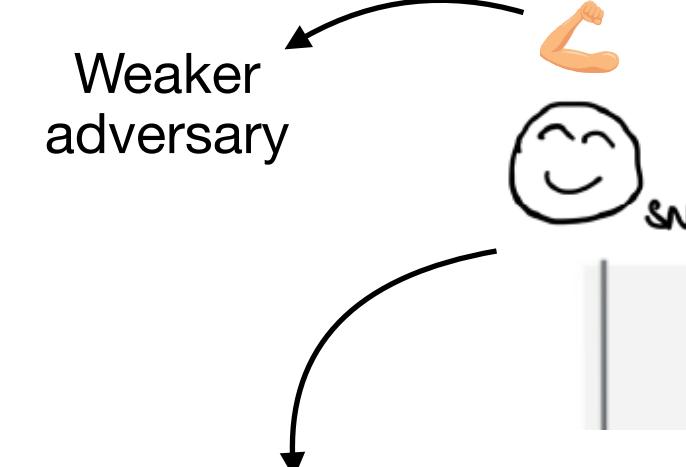
(continued)

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## Case 2: insecure somewhere.



$\text{Adv}_{\mathcal{A}}(\lambda) \geq \frac{1}{P'(\lambda)}$   
 with  $P' = w(P)$   
 p' got "worse" from  
 the perspective of  
 adversary

$\exists T = \text{poly}(\lambda), \mathcal{A} \text{ with } \text{Time}(\mathcal{A}) \leq T:$   
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 FOR poly  $P(\cdot)$ , INFINITELY  
 MANY  $\lambda$ -s

# Our Results

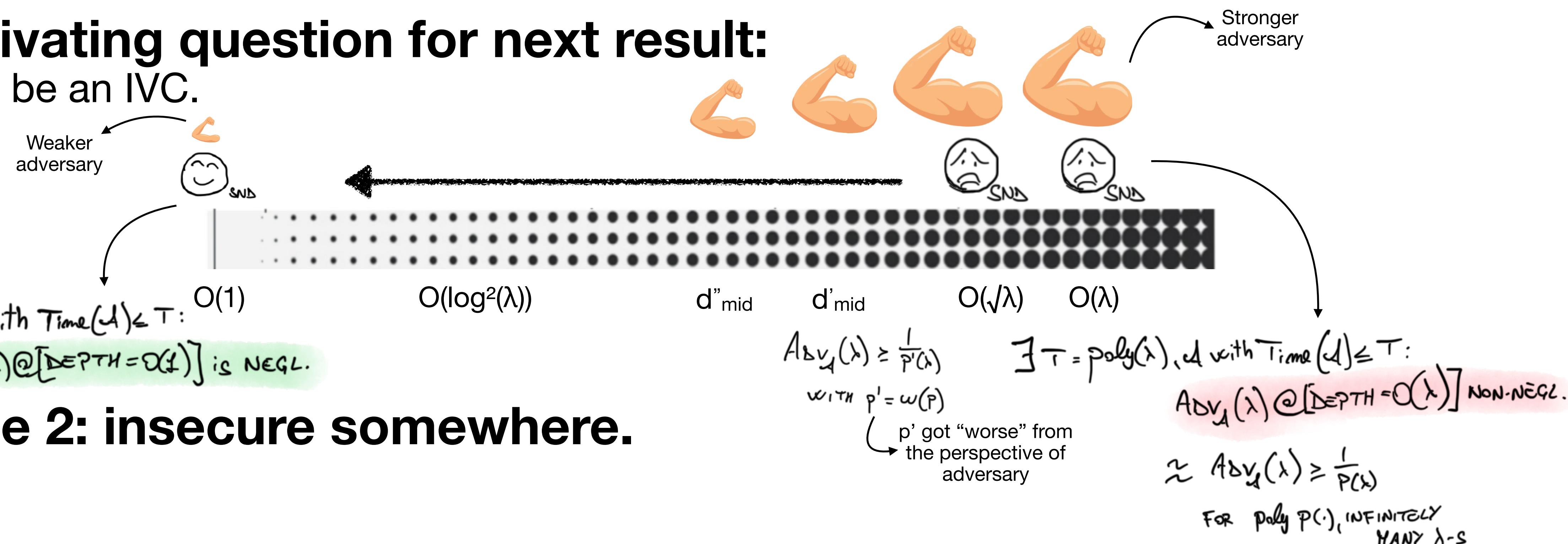
(continued)

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# Our Results

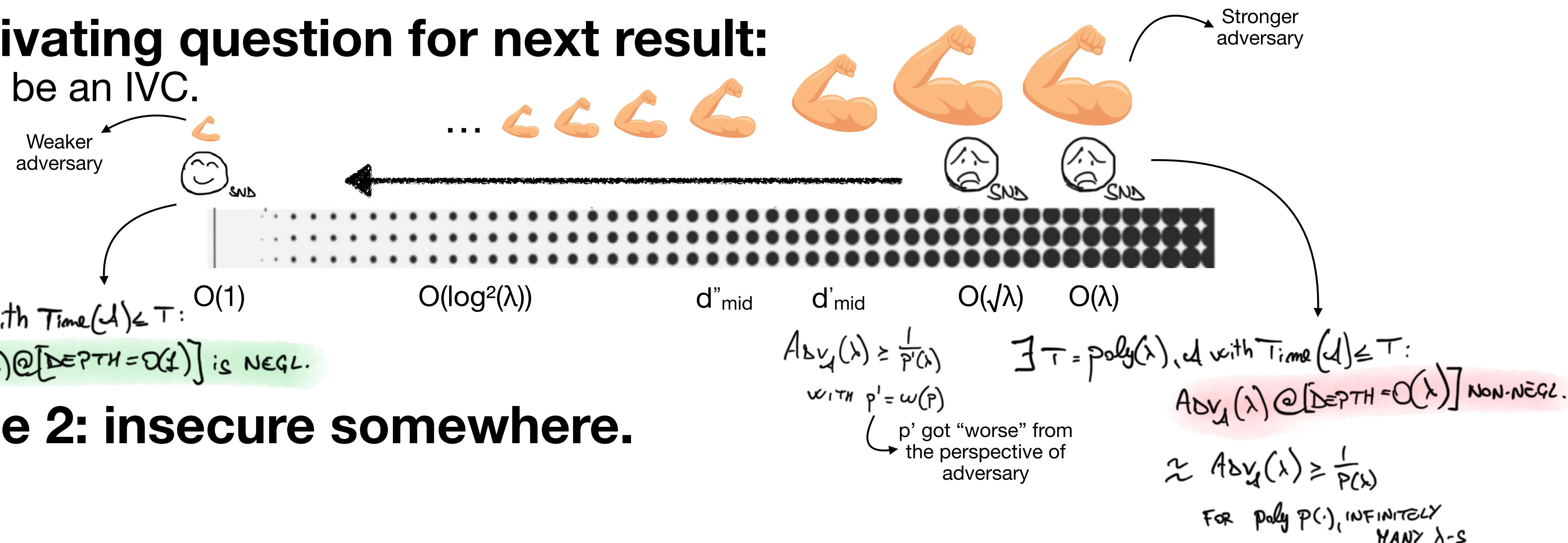
(continued)

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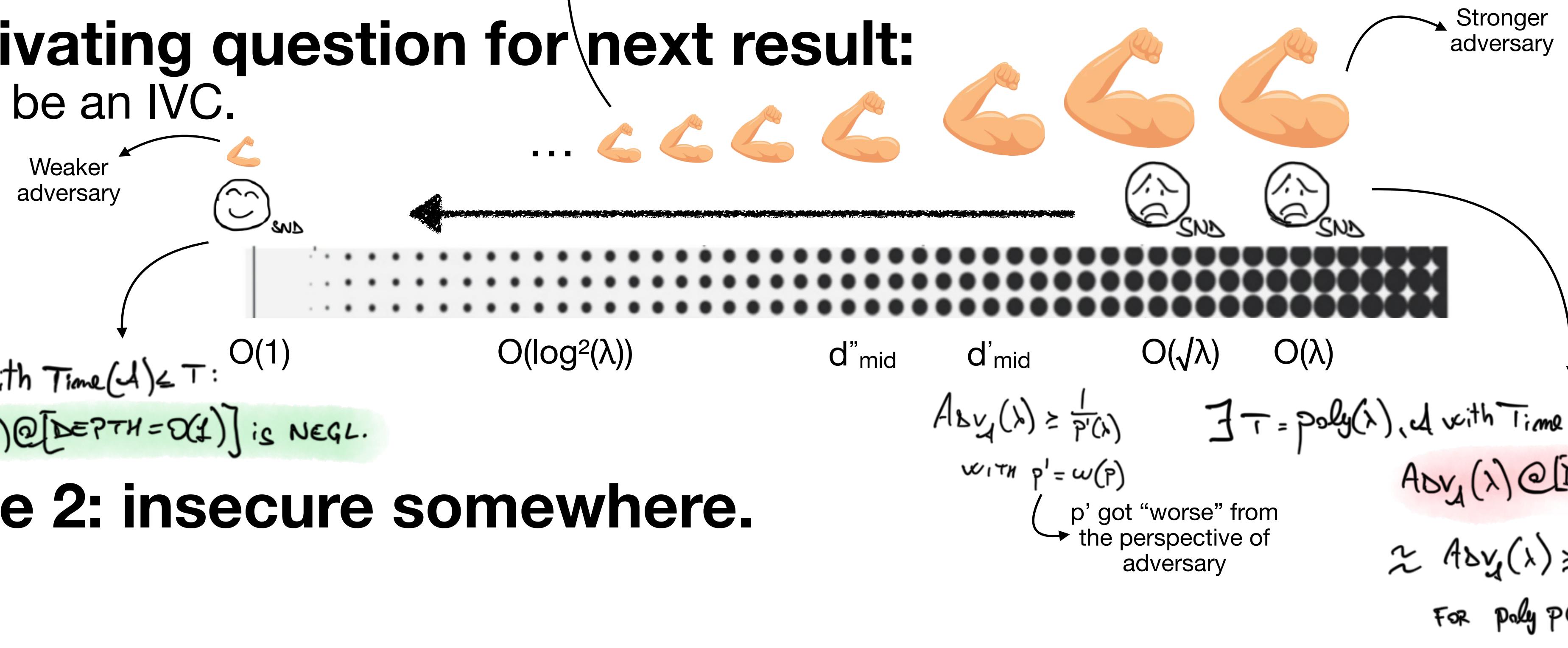
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## Case 2: insecure somewhere.

We call this (potential) pattern in IVC  
graceful security degradation



# Our Results (continued)

Q: Can an IVC exhibit it?

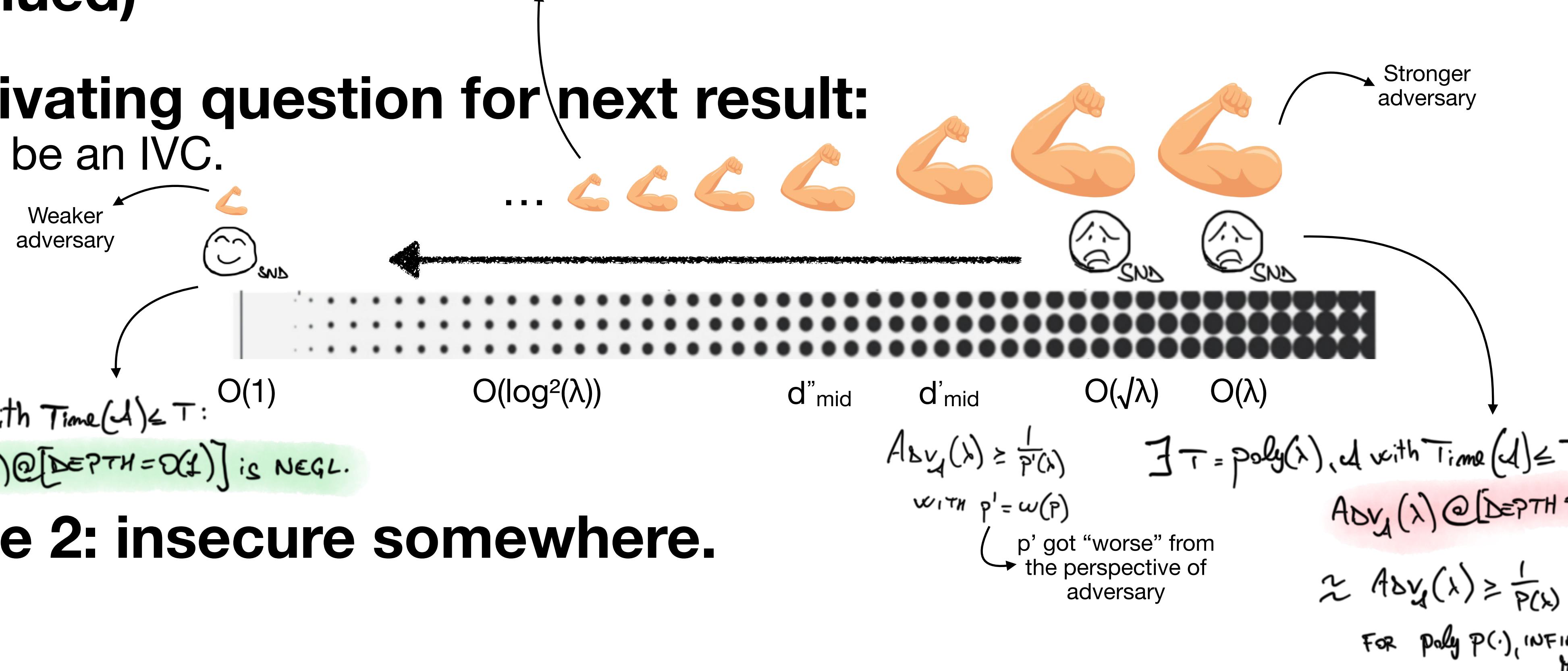
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# Our Results (continued)

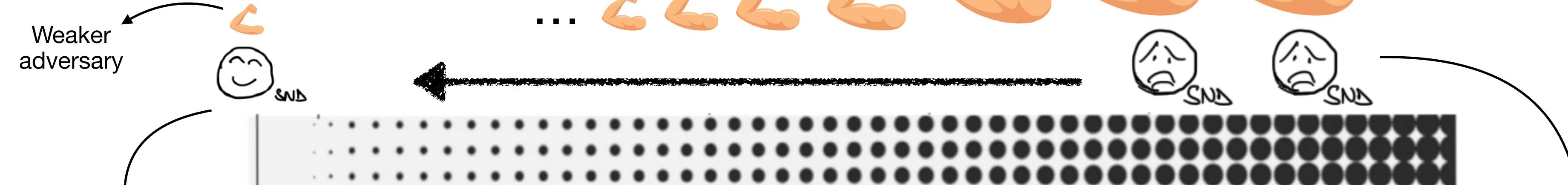
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A practical framing around graceful sec. degradation:

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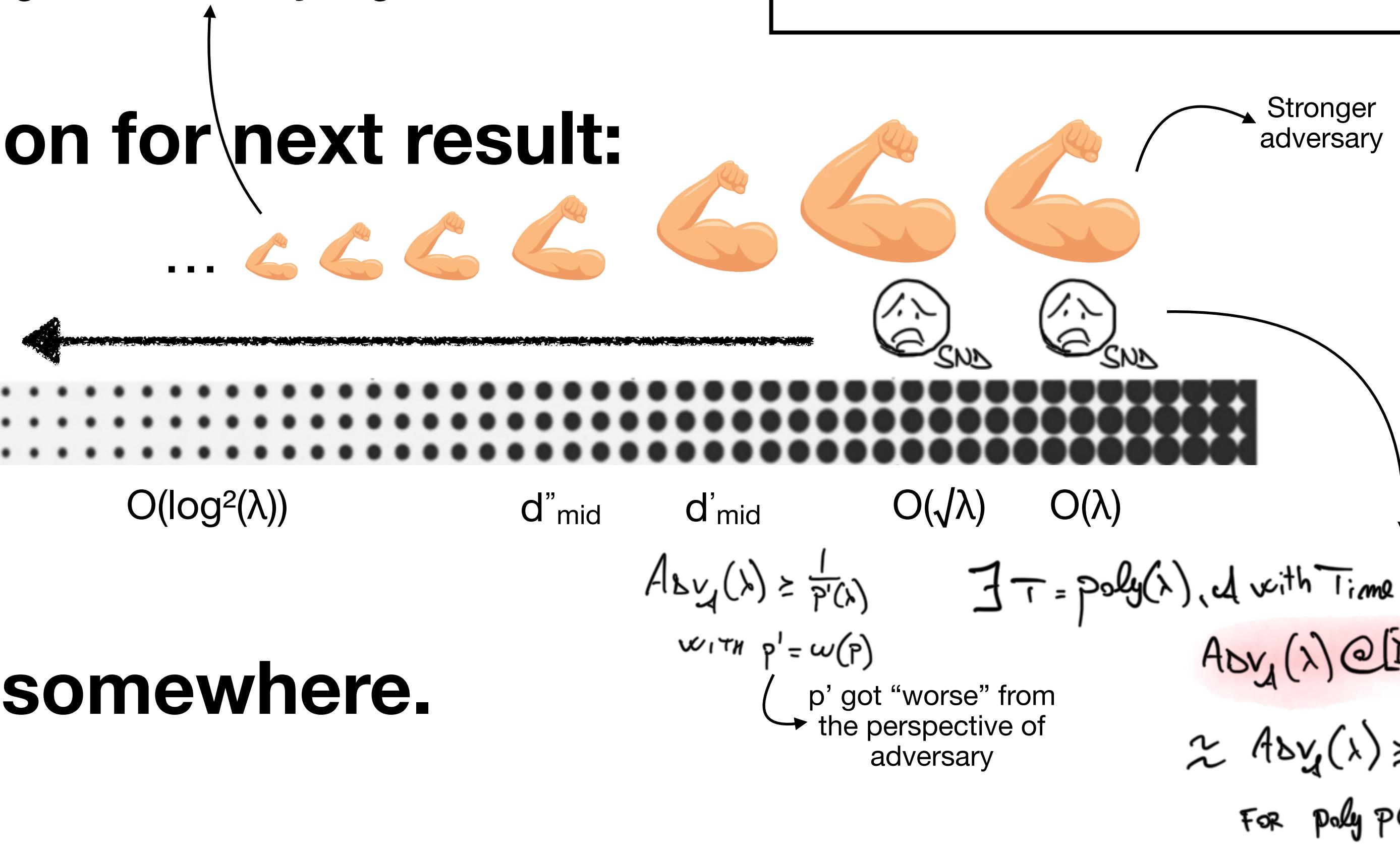
CCCCCCCC  $\approx$  better and better inverse poly-s

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A practical framing around graceful sec. degradation:



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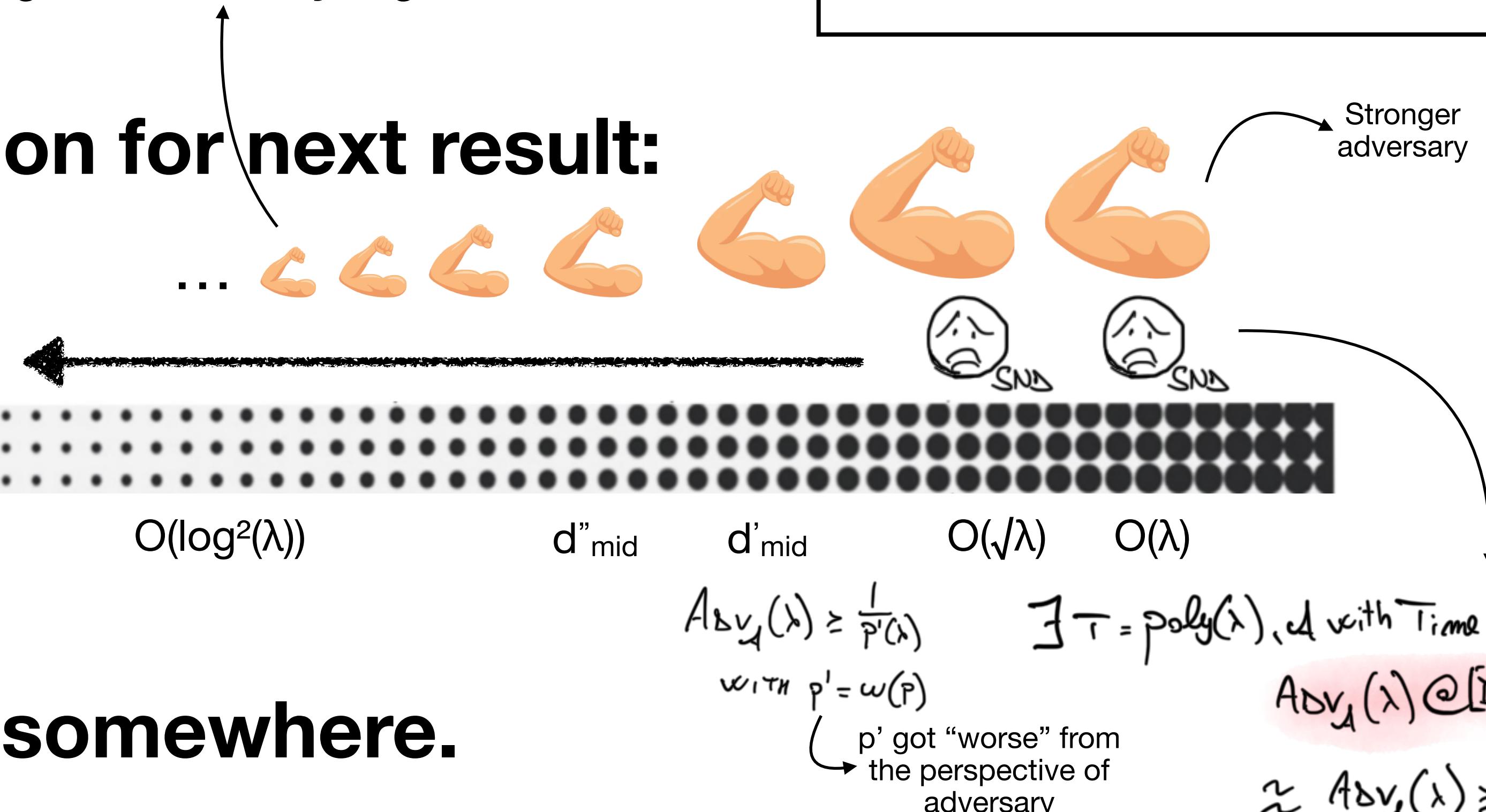
And cryptographers do sometimes work with inverse poly sec.

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# Our Results (continued)

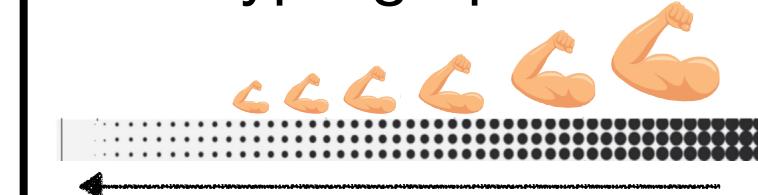
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A practical framing around graceful sec. degradation:

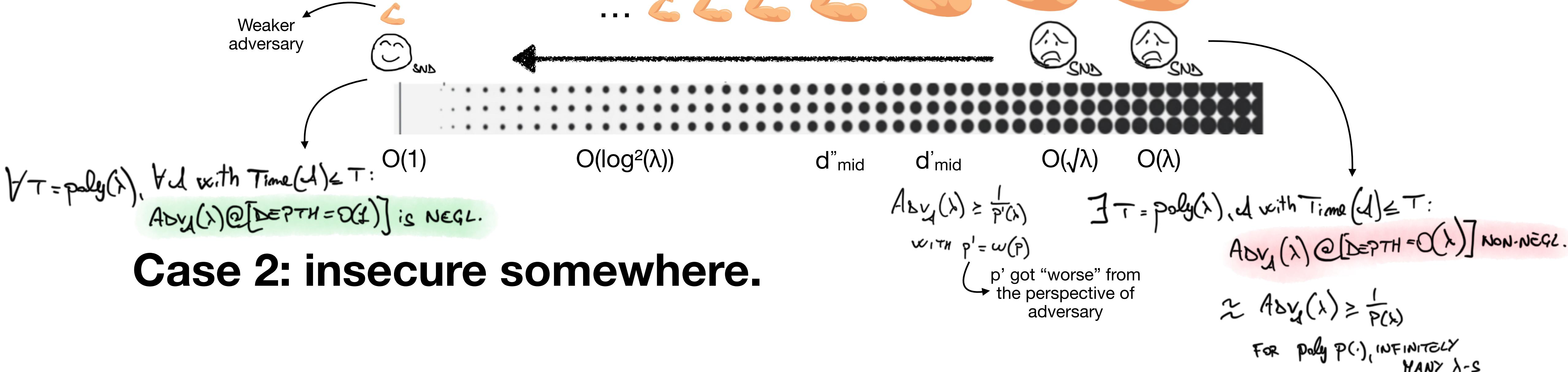
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Let  $\Pi$  be an IVC.



## Case 2: insecure somewhere.

# Our Results (continued)

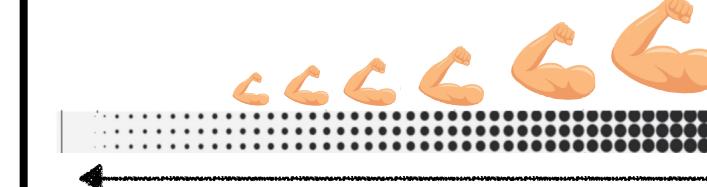
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A practical framing around graceful sec. degradation:

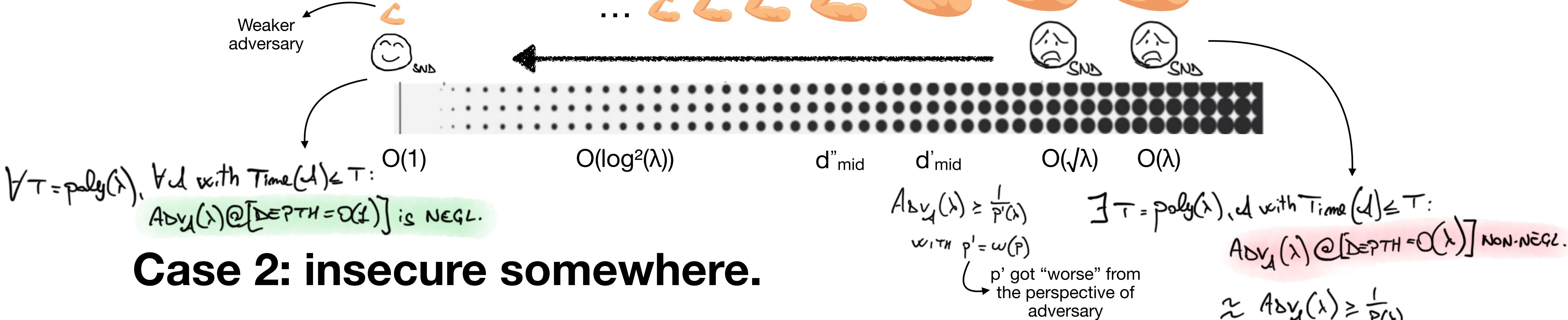
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Let  $\Pi$  be an IVC.



## Result (“no free snack” theorem):

Let  $\Pi$  be an IVC. Then:

# Our Results (continued)

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We call this (potential) pattern in IVC graceful security degradation

A practical framing around graceful sec. degradation:

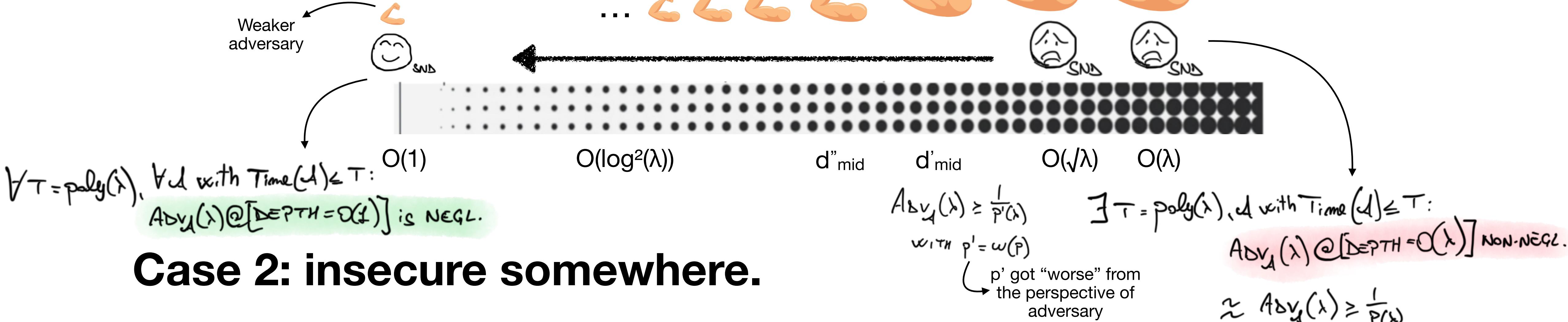
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## Case 2: insecure somewhere.

## Result (“no free snack” theorem):

Let  $\Pi$  be an IVC. Then:

- either  $\Pi$  is secure at arbitrary polynomial depths,

# Our Results (continued)

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A practical framing around graceful sec. degradation:

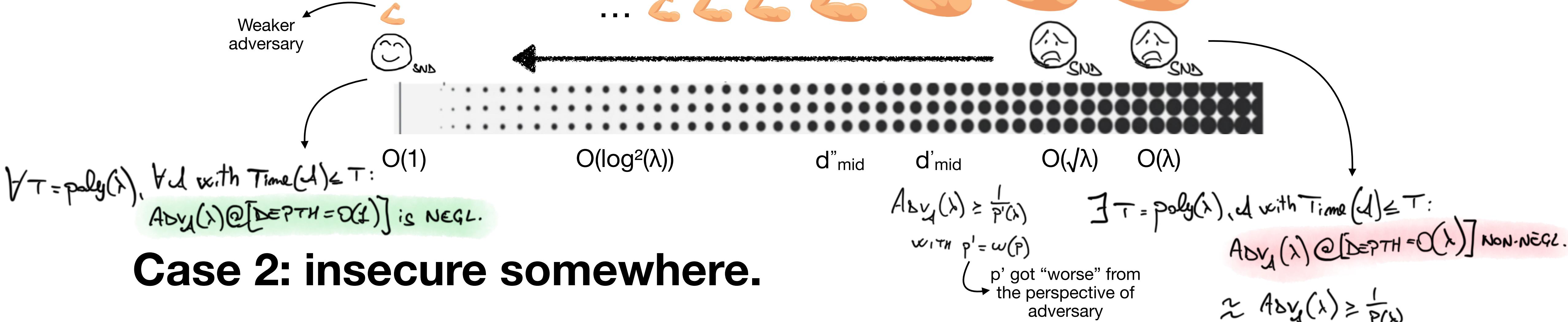
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Let  $\Pi$  be an IVC.



## Result ("no free snack" theorem):

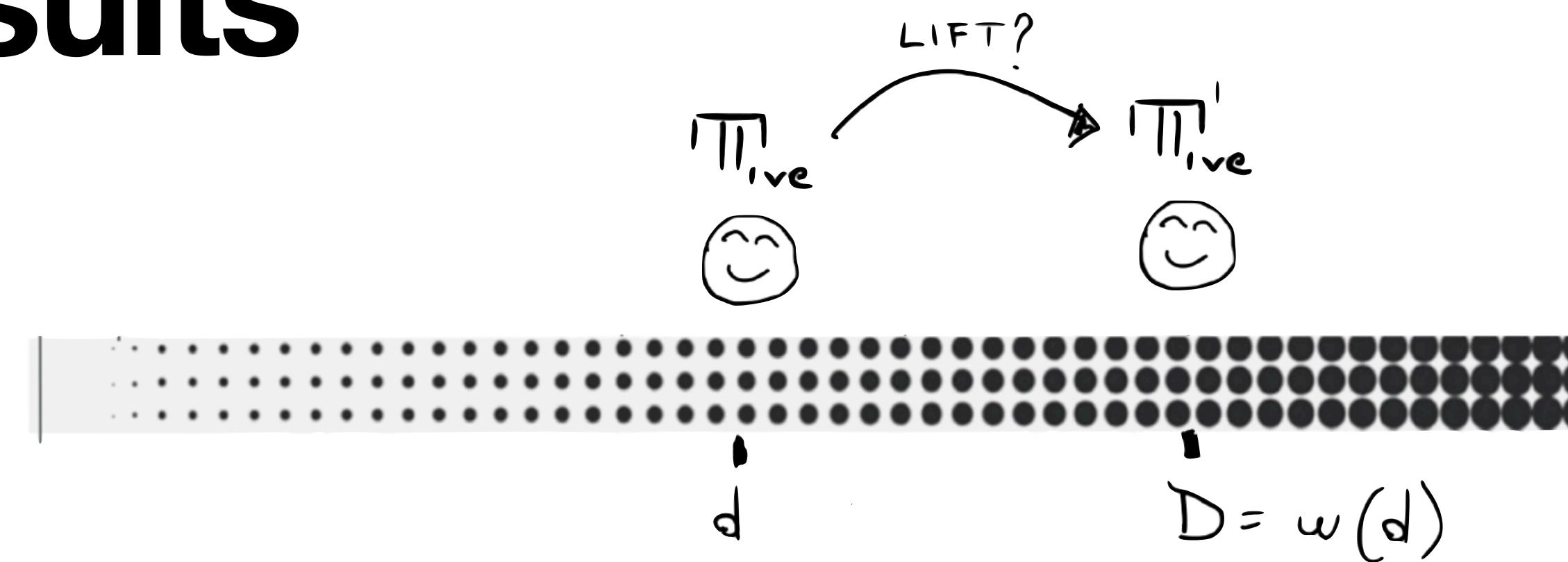
Let  $\Pi$  be an IVC. Then:

- either  $\Pi$  is secure at arbitrary polynomial depths,
- or  $\Pi$  cannot exhibit graceful security degradation.

# **Our Results**

## **(continued)**

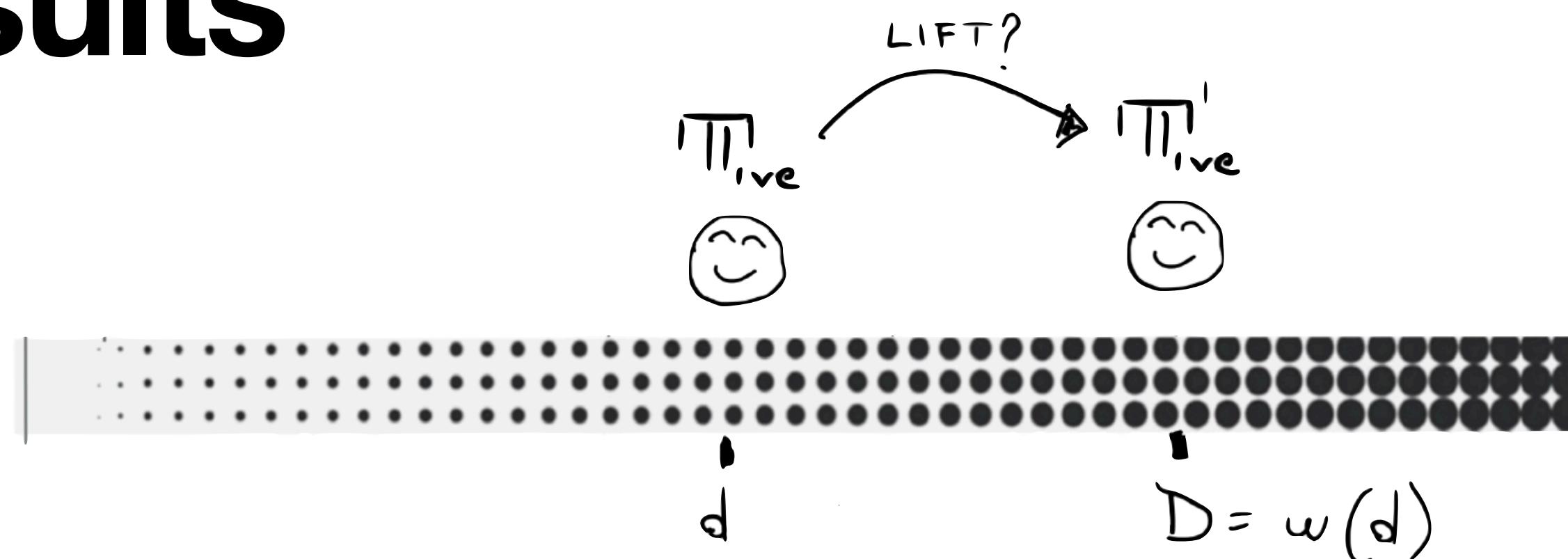
# Our Results (continued)



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**NB:** We are interested in:

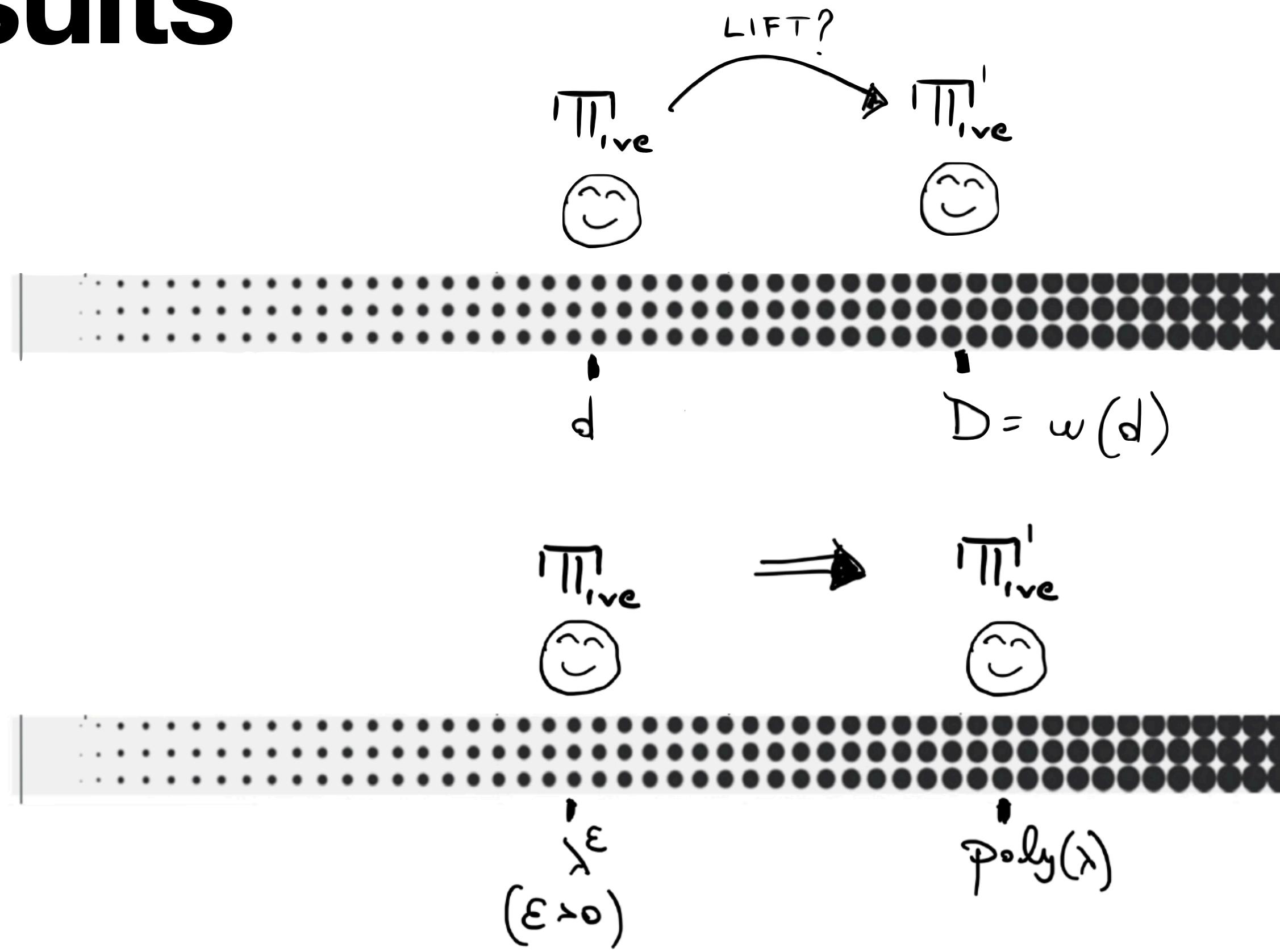
- black-box lifting results,
- and that preserve performance.



# Our Results (continued)

**NB:** We are interested in:

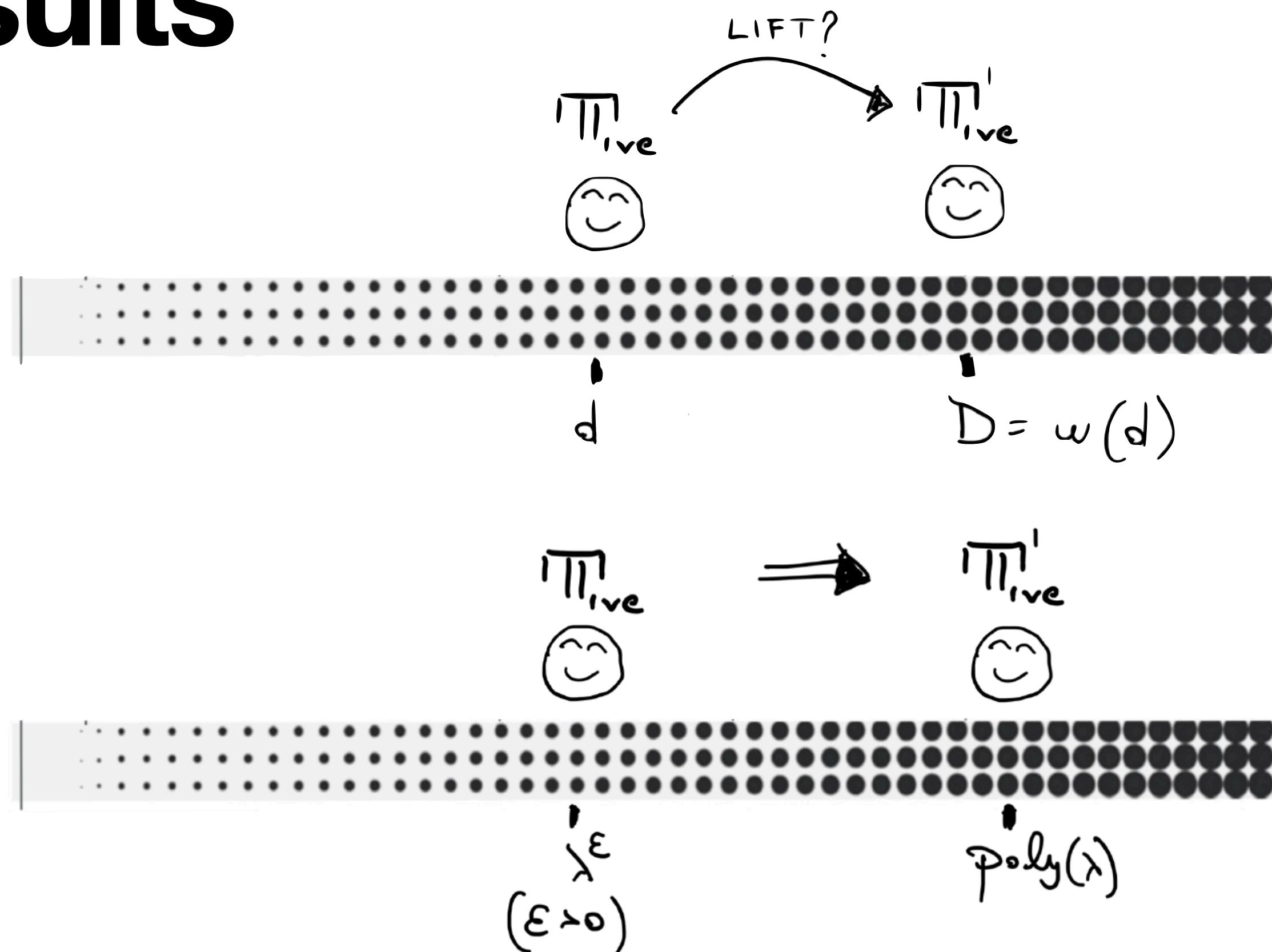
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# Our Results (continued)

**NB:** We are interested in:

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## Theorem (sublinear depths):

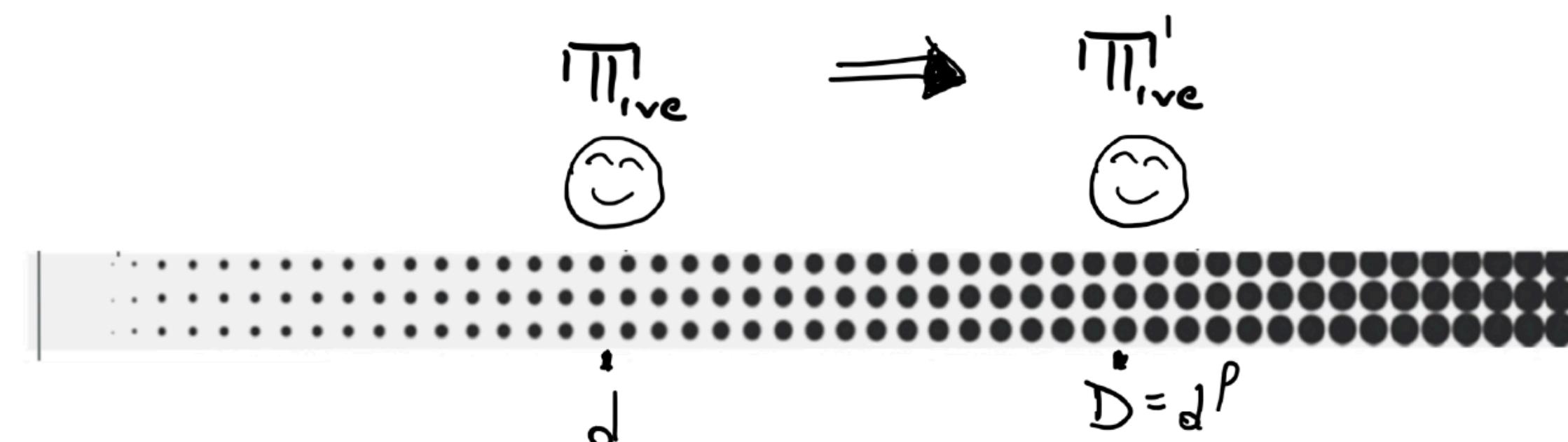
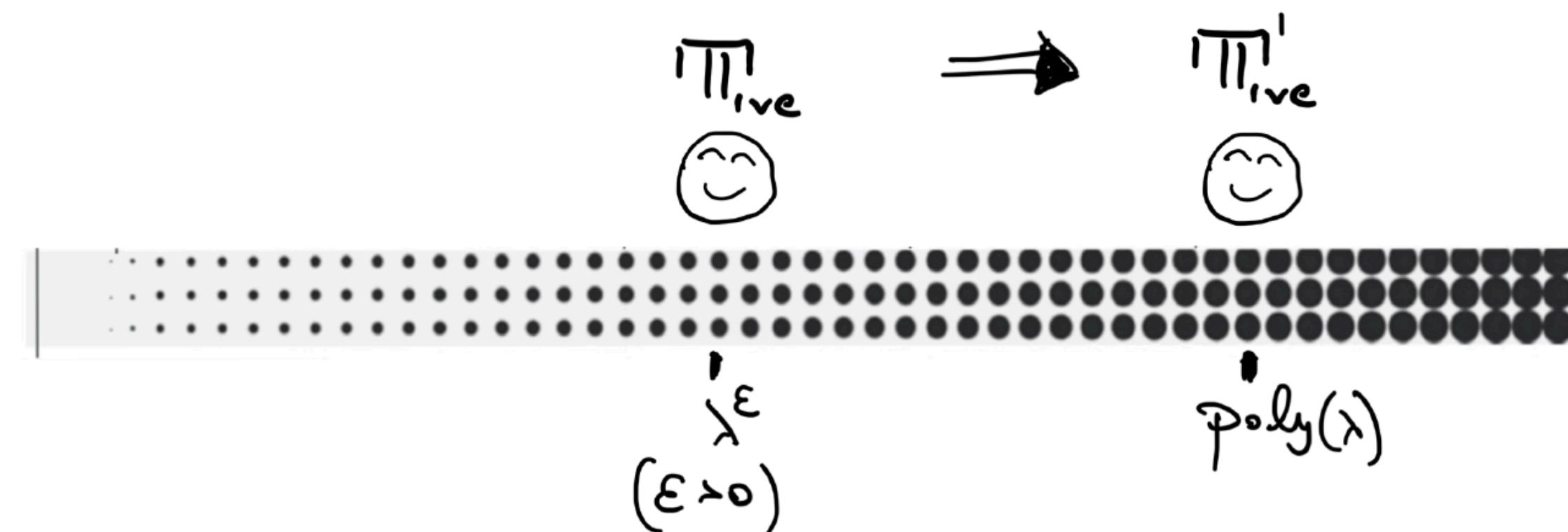
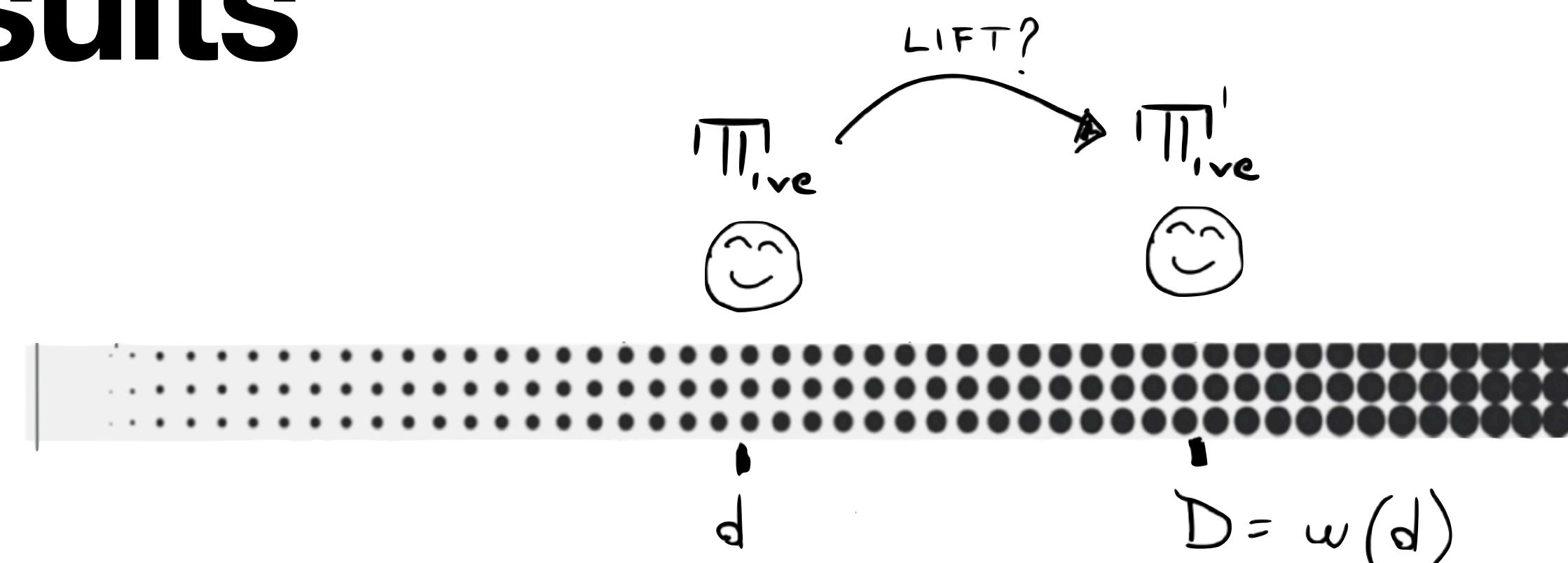
$\exists$  IVC  $\Pi$  SND at depth  $\lambda^\varepsilon$  (for some  $\varepsilon > 0$ )  
 $\Rightarrow \exists$  IVC  $\Pi'$  SND at arbitrary depth.

Overhead for P/V/proof size in  $\Pi'$  is  $O_\lambda(1)$ .

# Our Results (continued)

**NB:** We are interested in:

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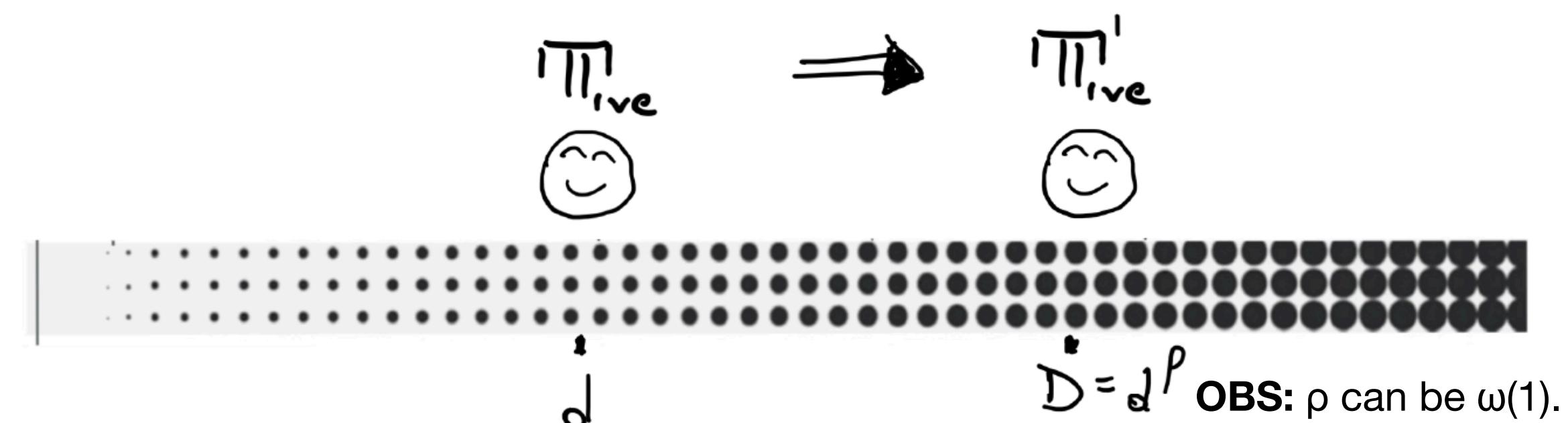
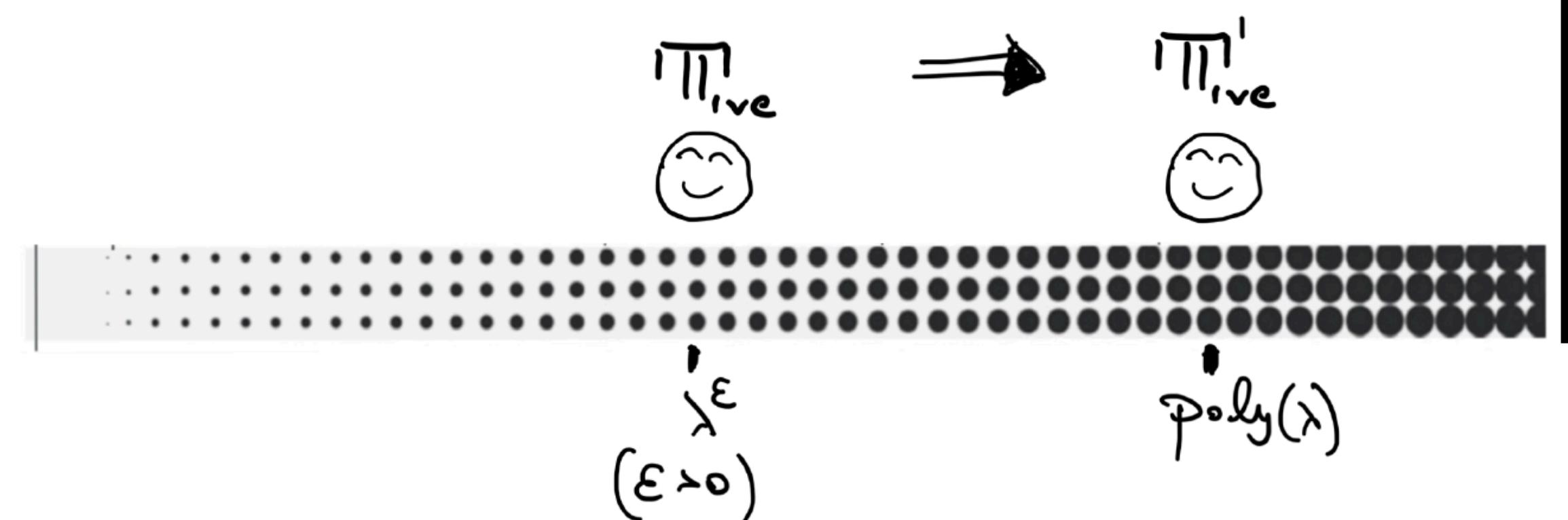
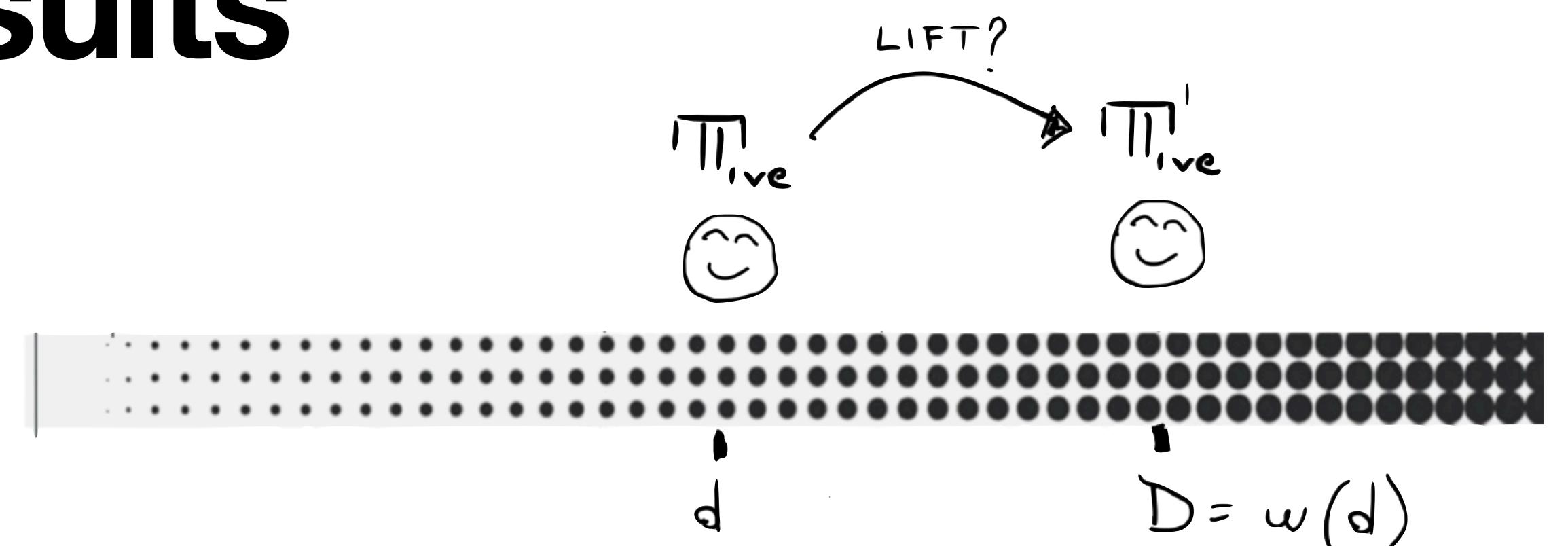


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## Theorem (sublinear depths):

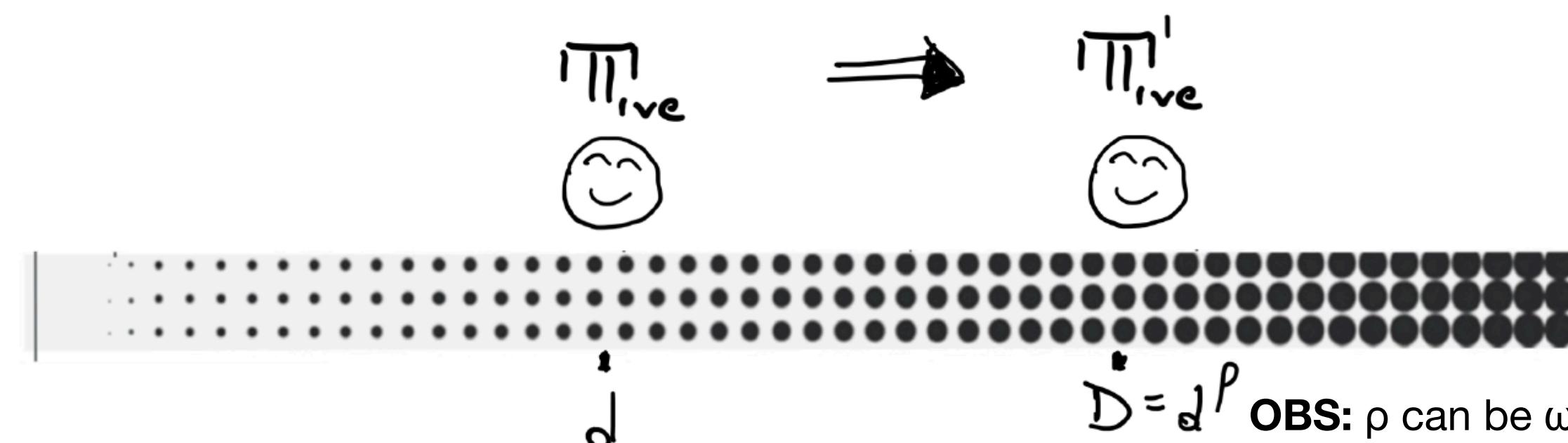
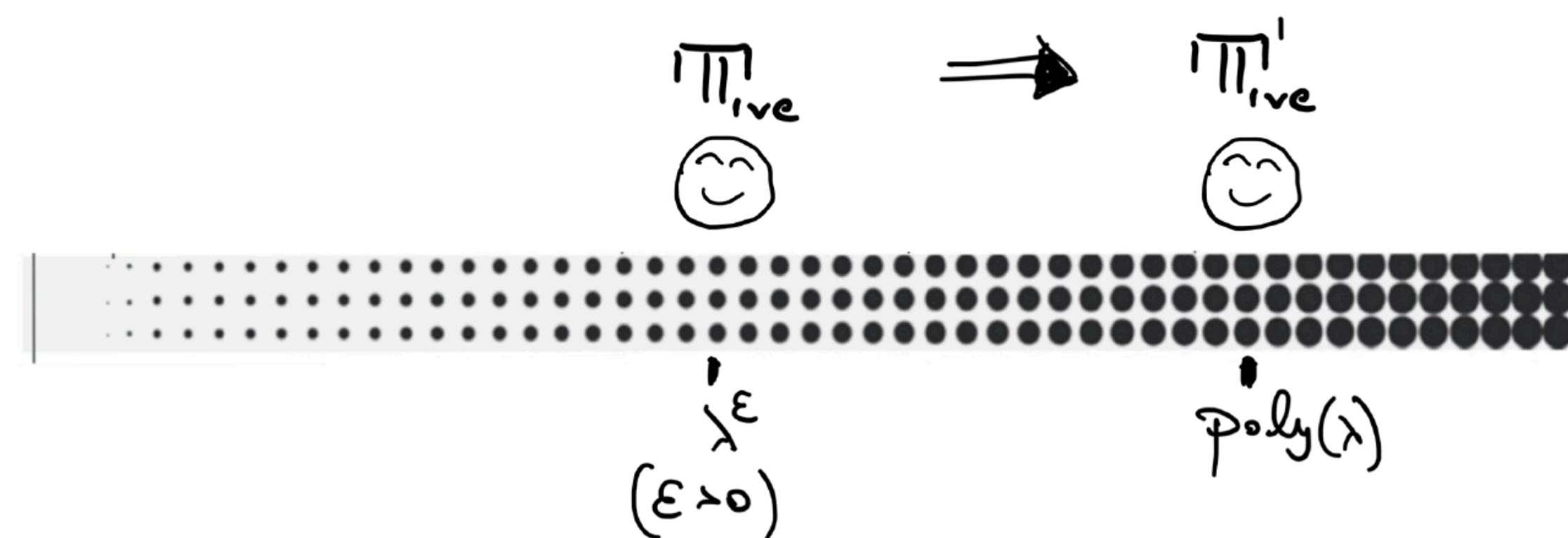
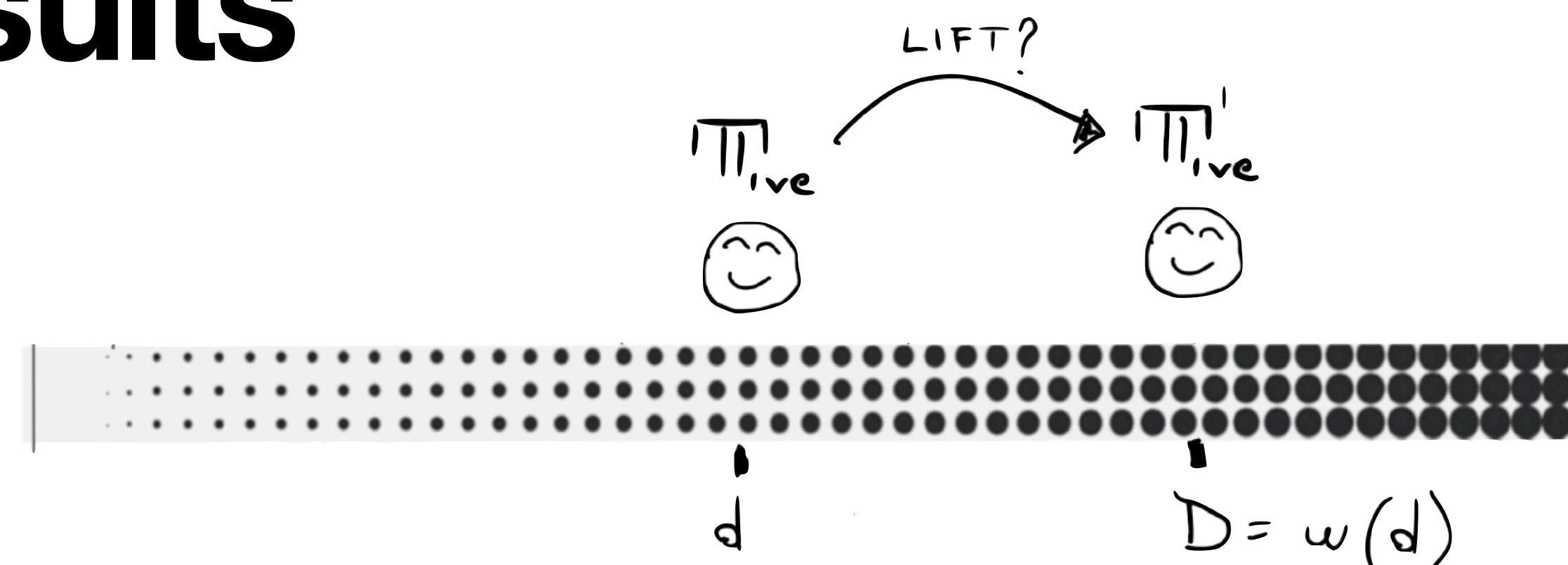
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# Our Results (continued)

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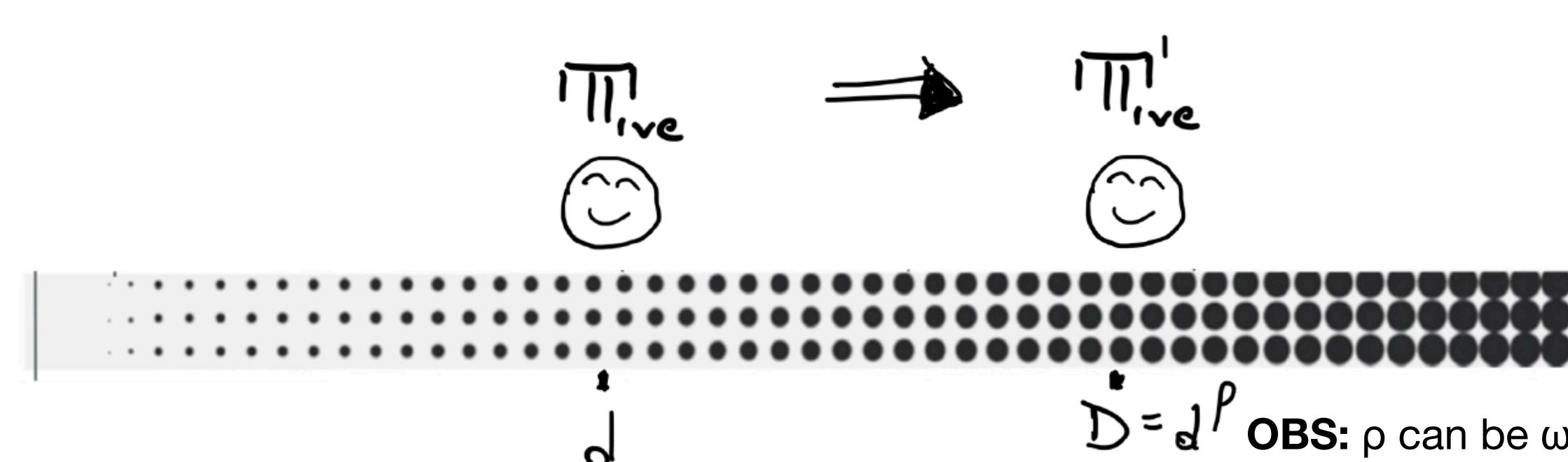
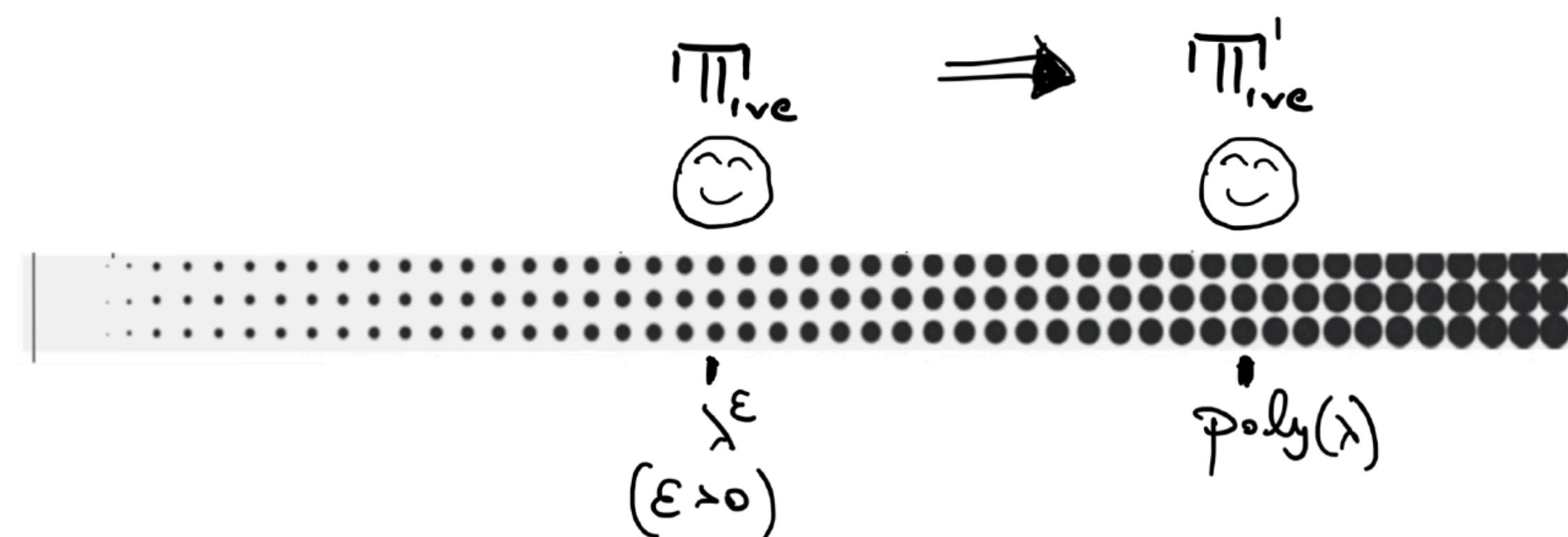
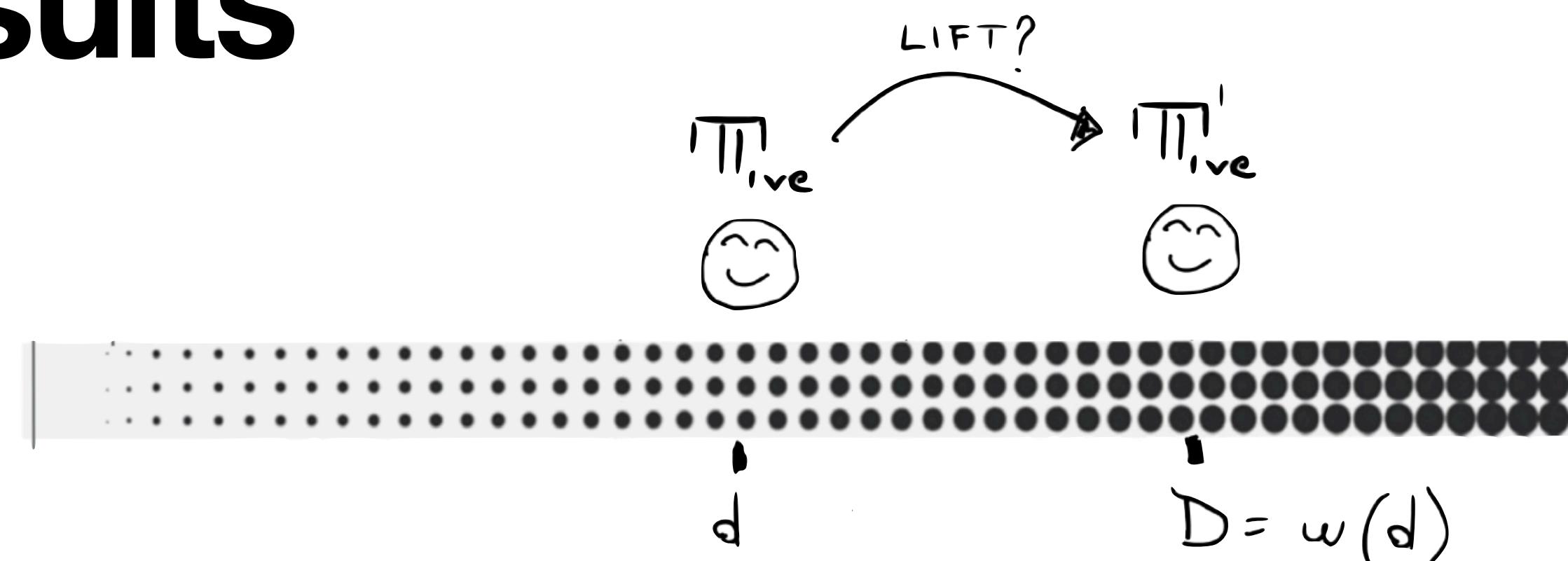
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**Theorem (general lifting):**  
 $\exists$  IVC  $\Pi$  SND at depth  $d$   
 $\Rightarrow \exists$  IVC  $\Pi'$  SND at depth  $D = d^\rho$ .  
 Overhead\* for P/V/proof size in  $\Pi'$  is  $O_\lambda(\rho)$

# Our Results (continued)

**NB:** We are interested in:

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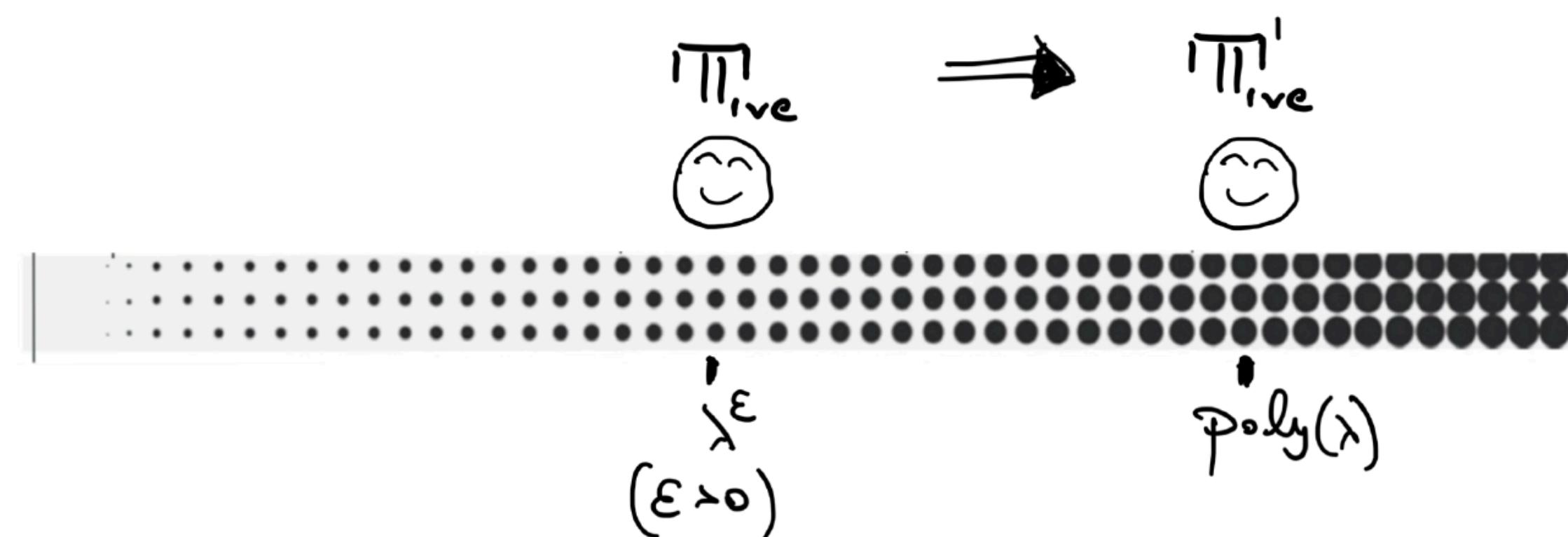
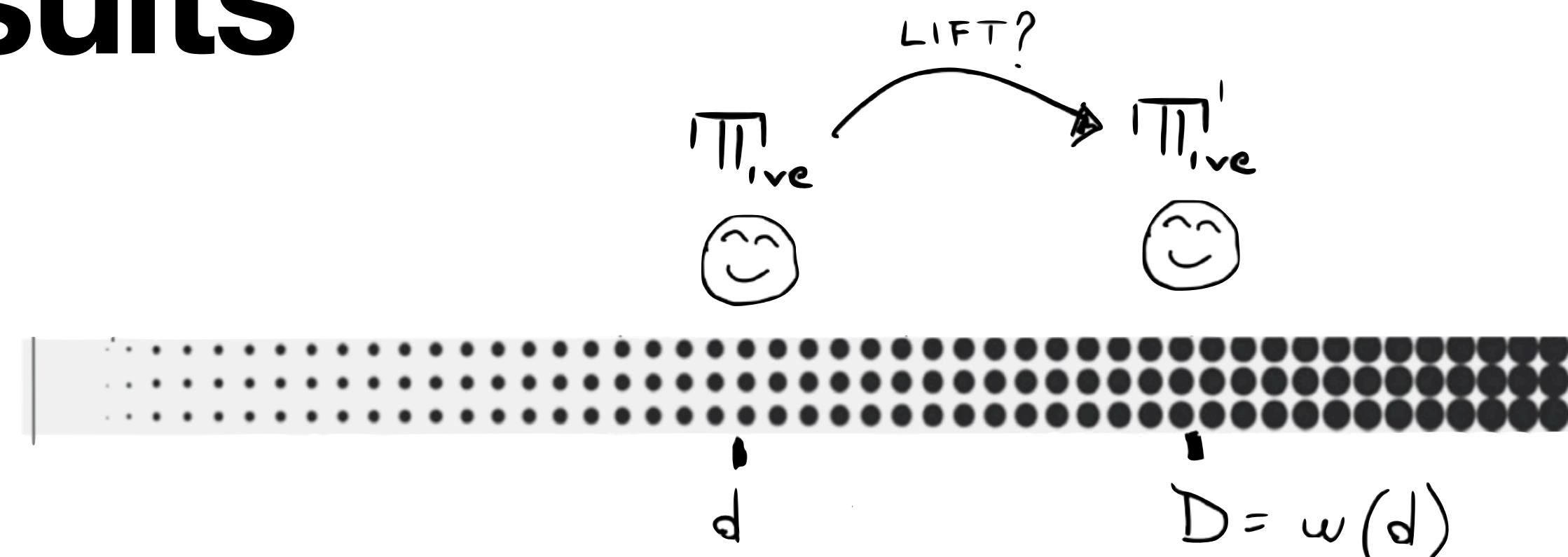
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\*: amortized prover time; applies for linear  $T_P(\Pi)$   
 (if not linear additional polylog overhead).

# Our Results (continued)

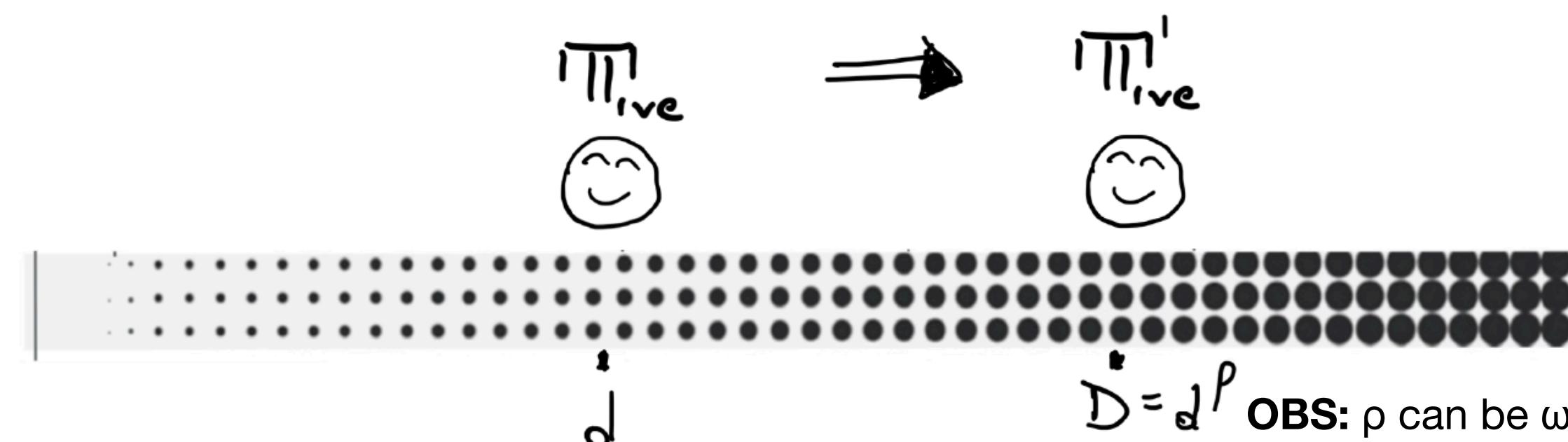
**NB:** We are interested in:

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**Corollary:**

- IVC SND at  $O(1) \Rightarrow$
- IVC  $\Pi'$  SND at depth  $D = \text{poly.}$



**Theorem (sublinear depths):**

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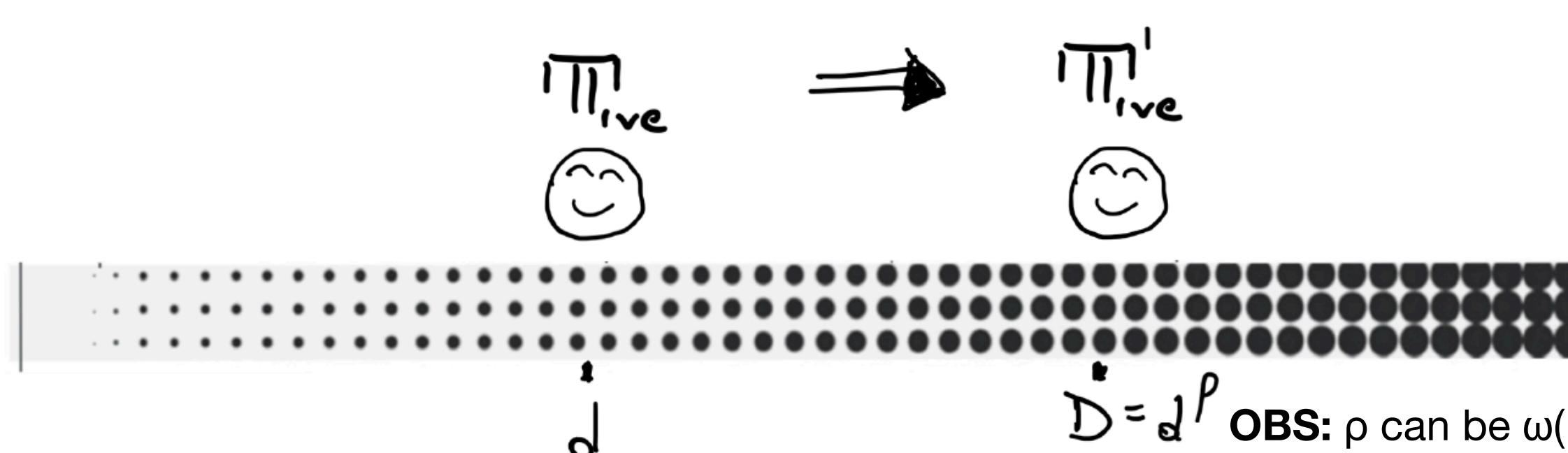
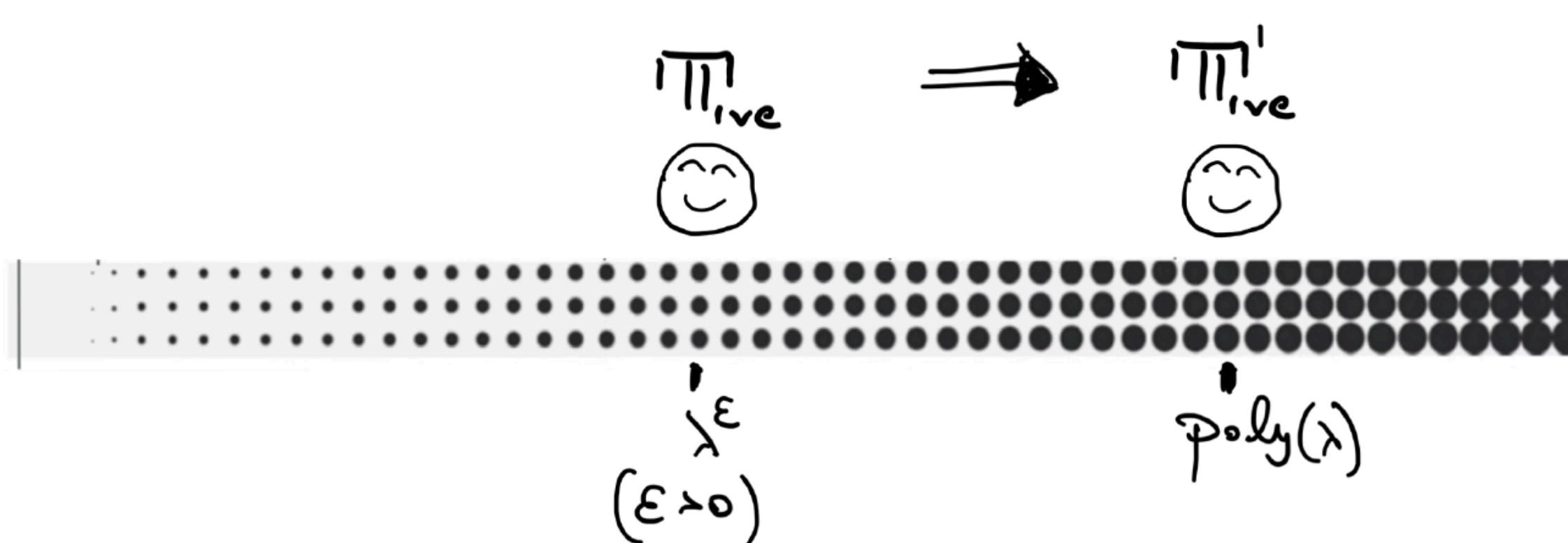
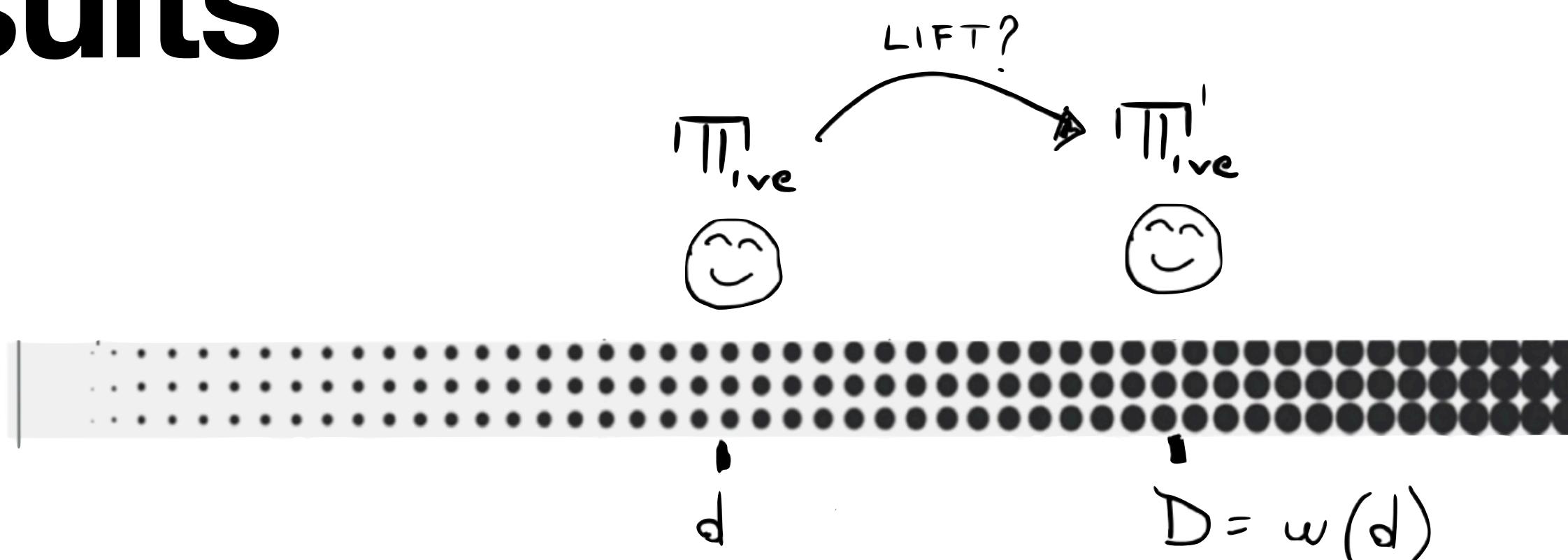
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\*: amortized prover time; applies for linear  $T_P(\Pi)$   
(if not linear additional polylog overhead).

# Our Results (continued)

**NB:** We are interested in:

- black-box lifting results,
- and that preserve performance.



**Theorem (sublinear depths):**  
 $\exists$  IVC  $\Pi$  SND at depth  $\lambda^\varepsilon$  (for some  $\varepsilon > 0$ )  
 $\Rightarrow \exists$  IVC  $\Pi'$  SND at arbitrary depth.  
 Overhead for P/V/proof size in  $\Pi'$  is  $O_\lambda(1)$ .

**Corollary:**

- $\exists$  IVC SND at  $O(1) \Rightarrow$
- $\exists$  IVC  $\Pi'$  SND at depth  $D = \text{poly.}$



Special case:  $d = O(1); \rho = O(\log \lambda)$

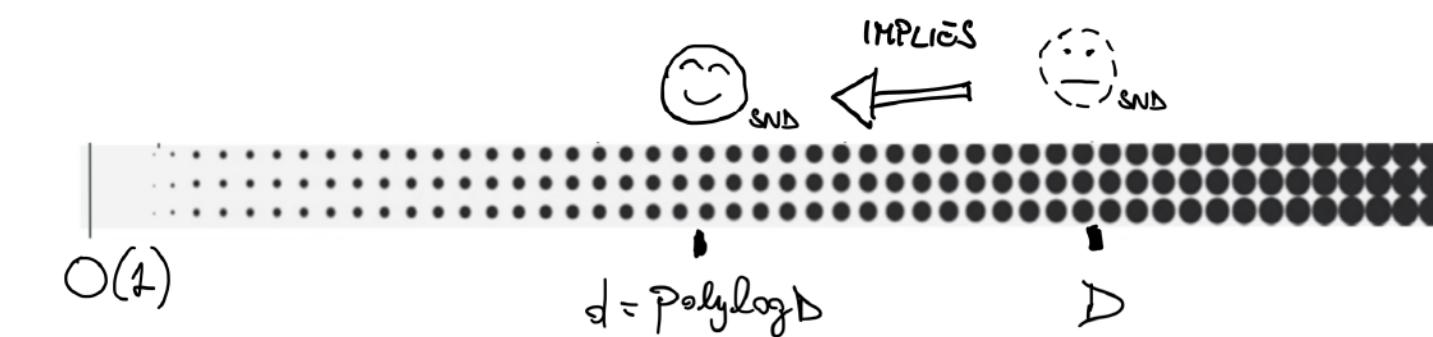
**Theorem (general lifting):**  
 $\exists$  IVC  $\Pi$  SND at depth  $d$   
 $\Rightarrow \exists$  IVC  $\Pi'$  SND at depth  $D = d^\rho$ .  
 Overhead\* for P/V/proof size in  $\Pi'$  is  $O_\lambda(\rho)$

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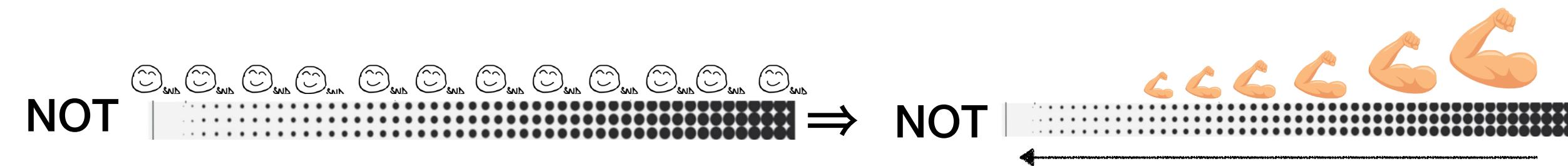
# When Do Our Results Apply?

## For what security notions do they hold?

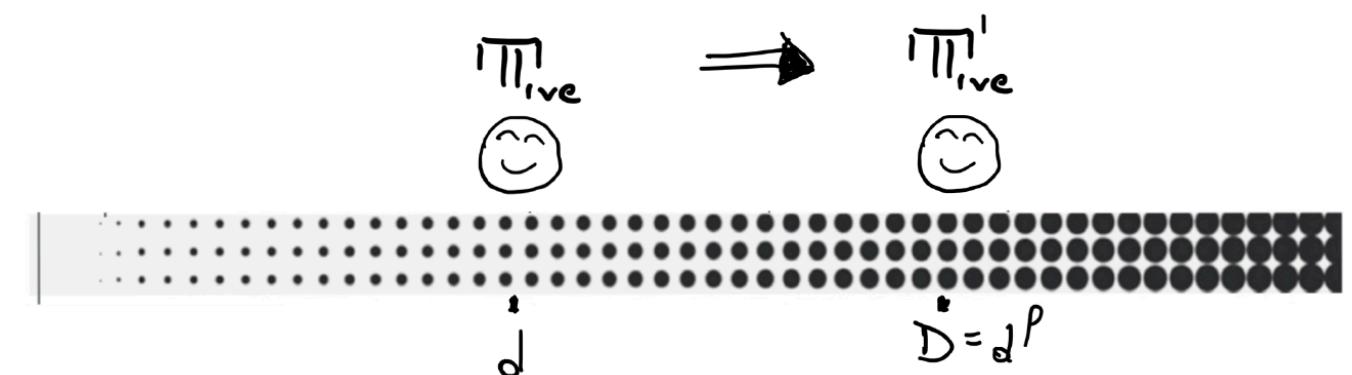
io-sec at  $D \Rightarrow$  sec at  $d = o(D)$



Insecure IVCs cannot exhibit graceful sec. degradation



Black-box lifting with low overhead

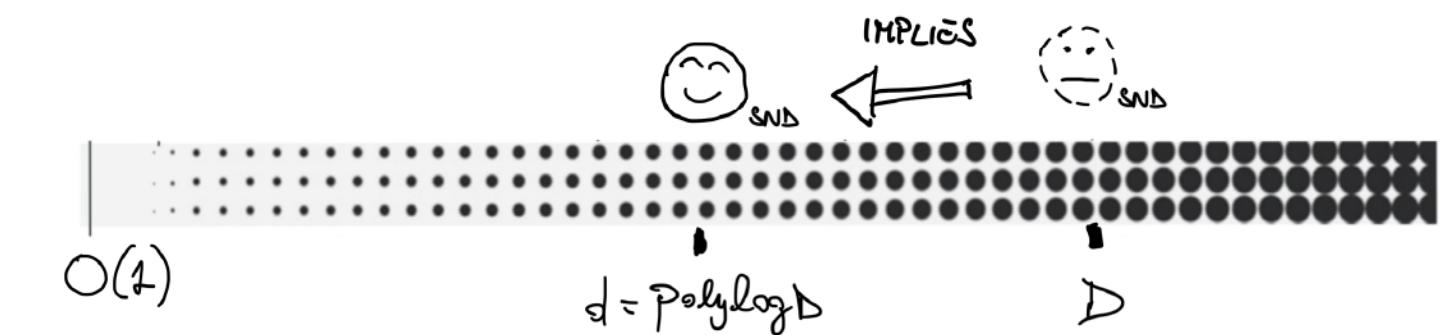


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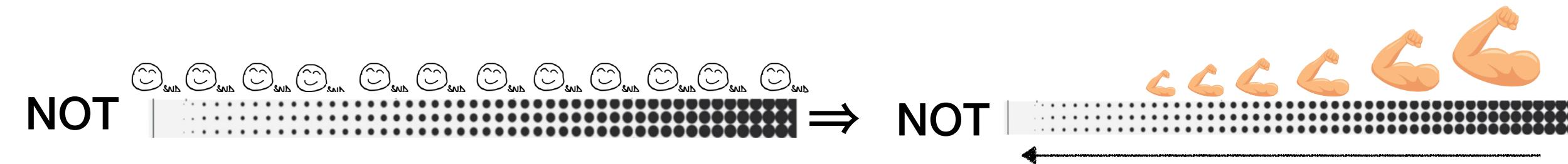
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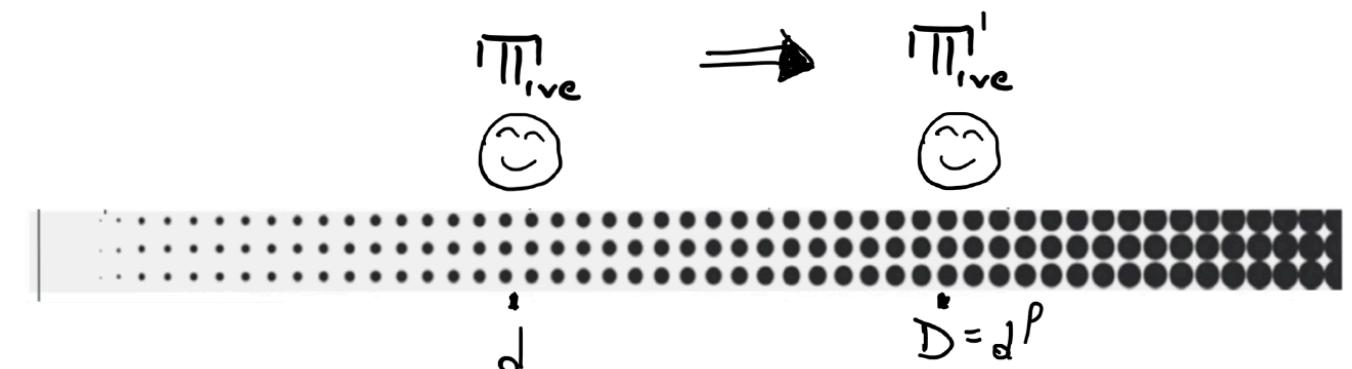
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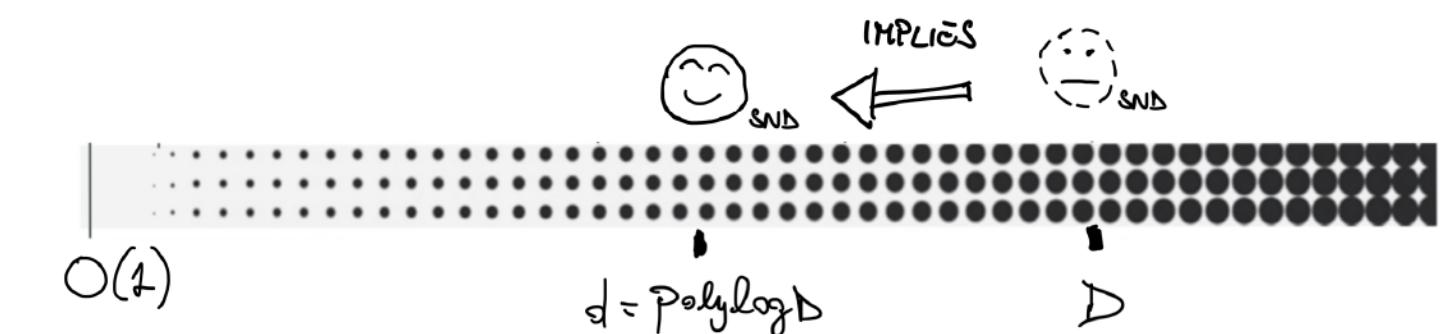


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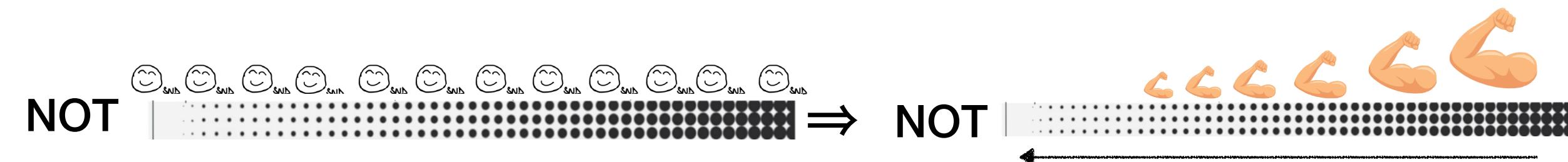
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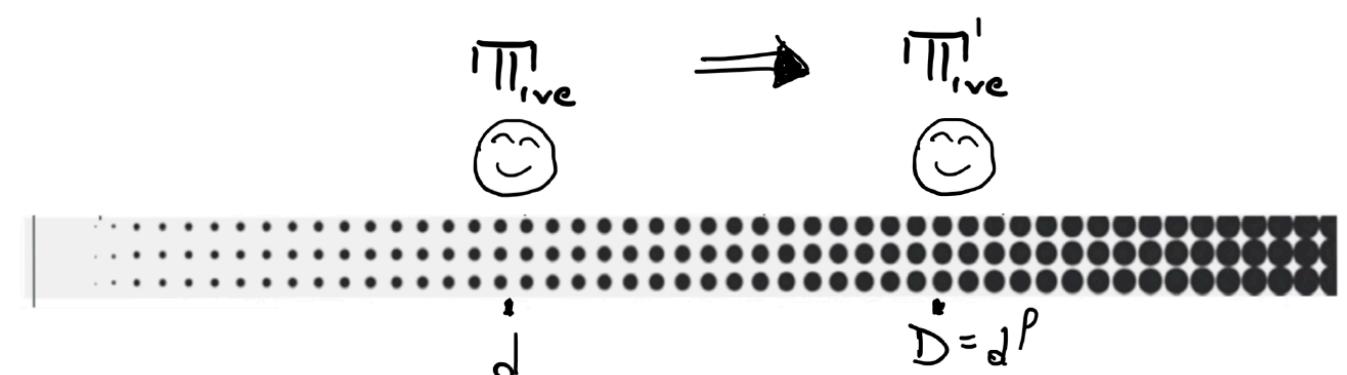
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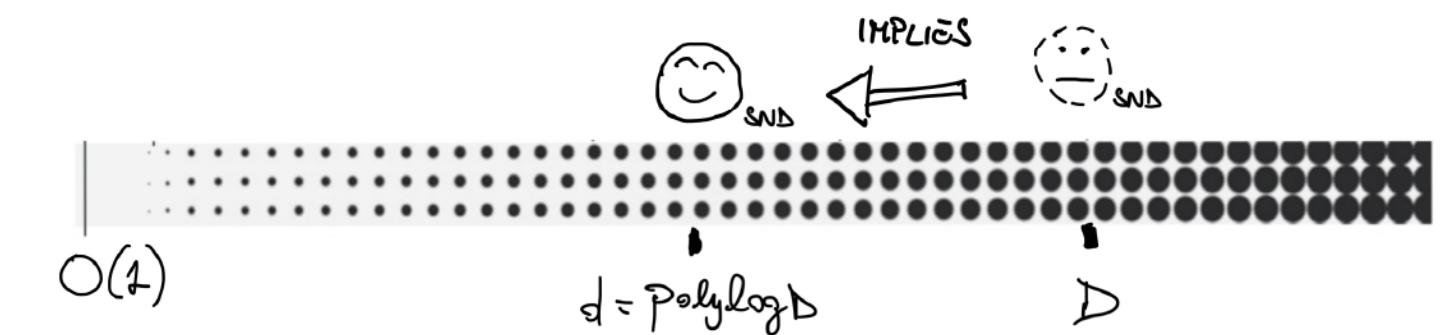


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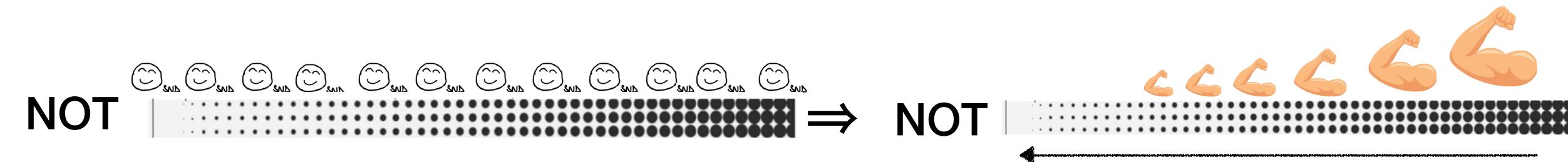
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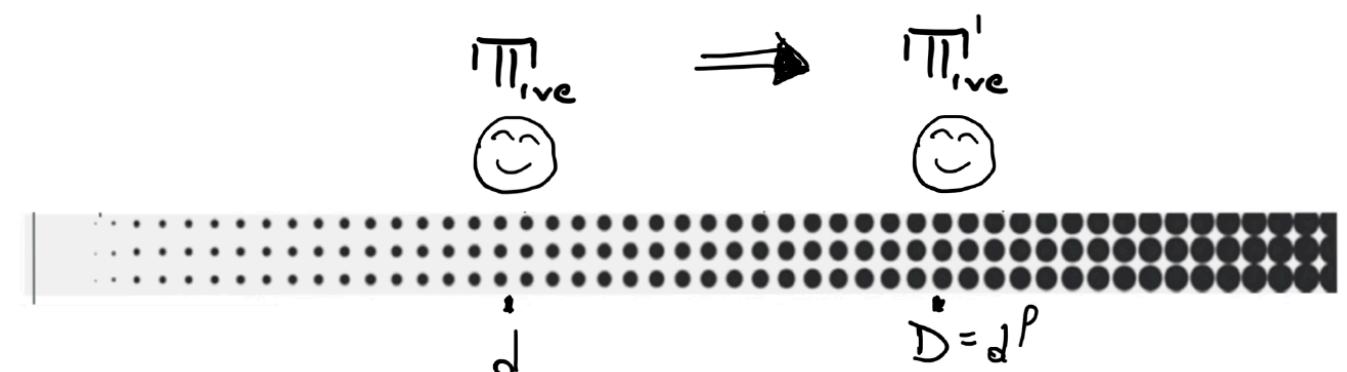
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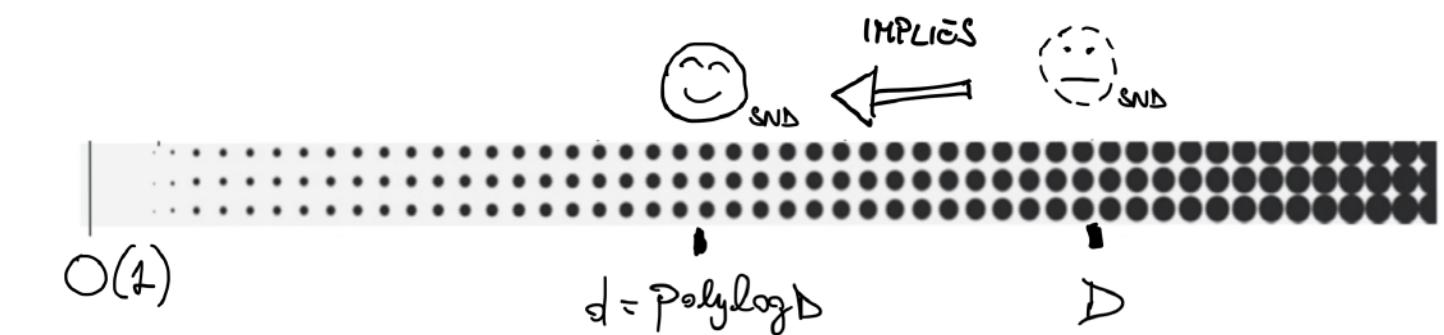


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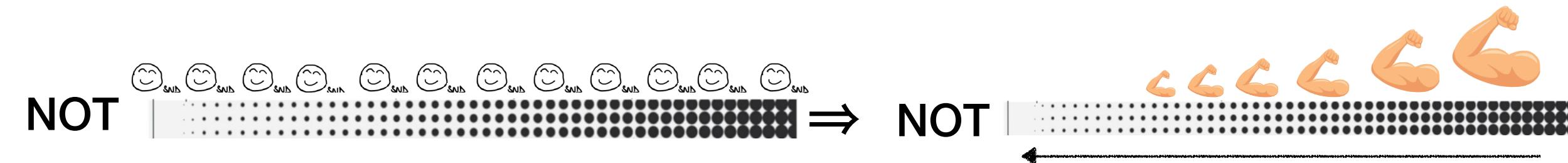
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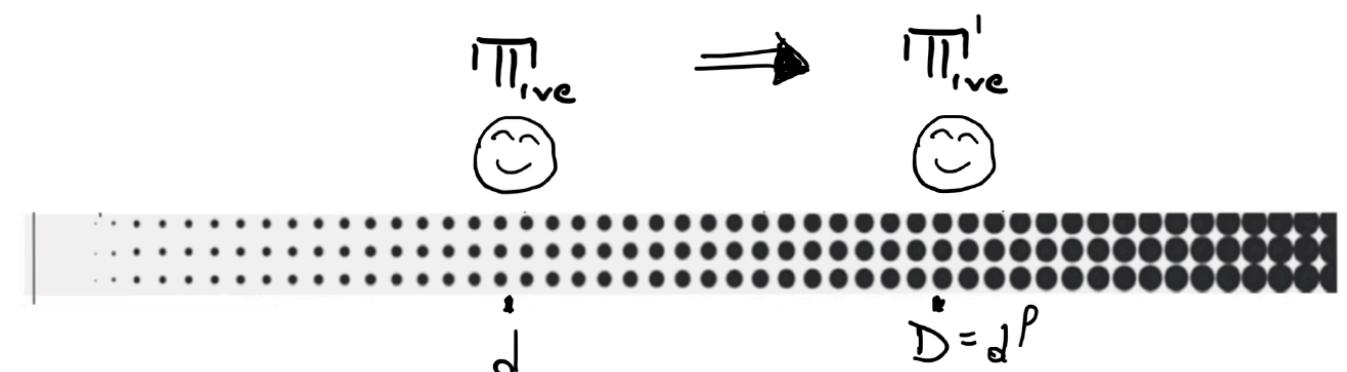
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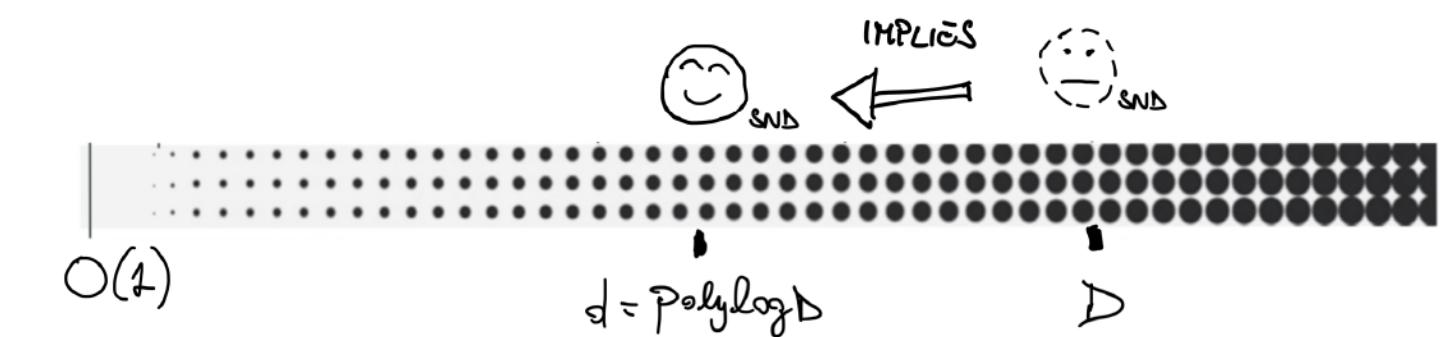


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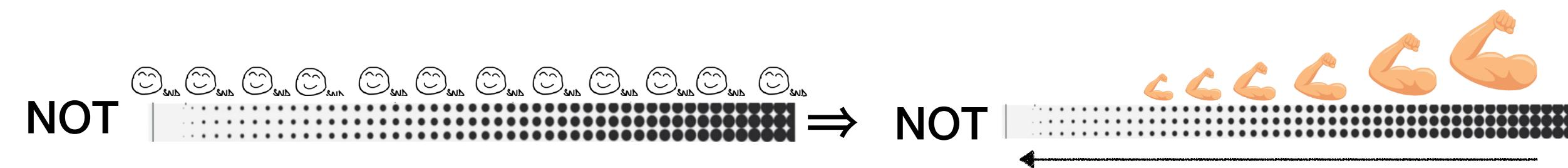
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- Extractability

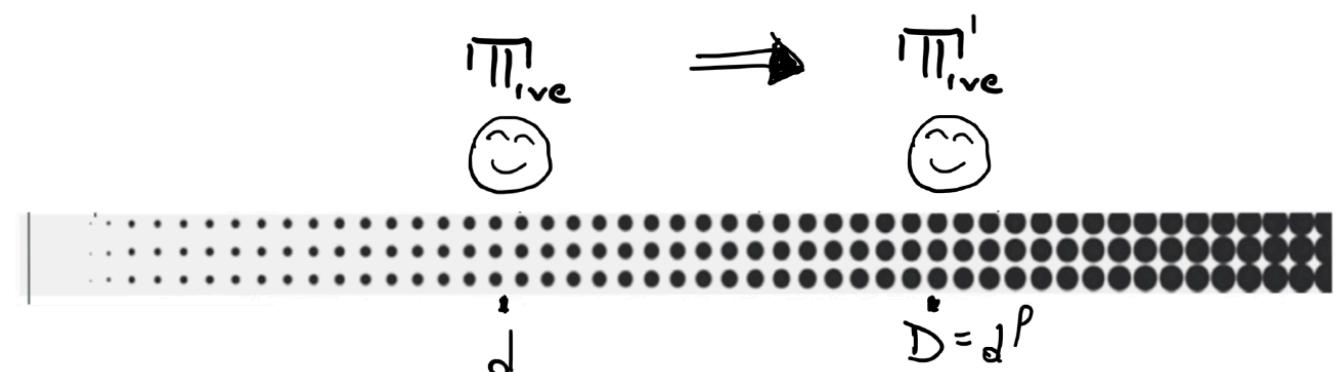
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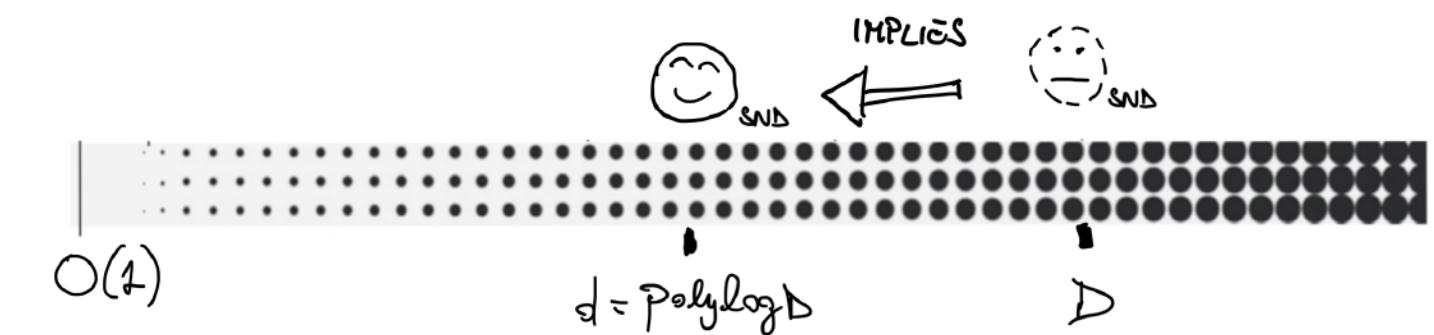


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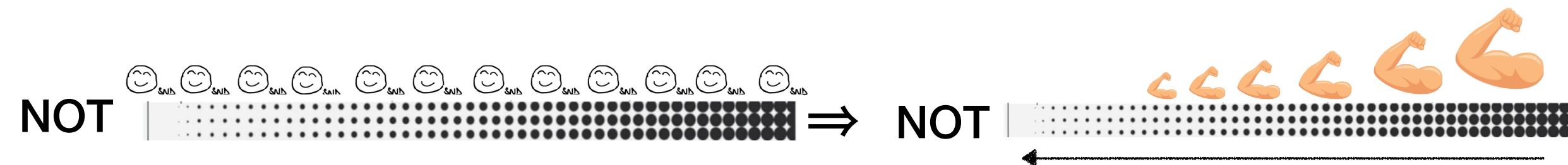
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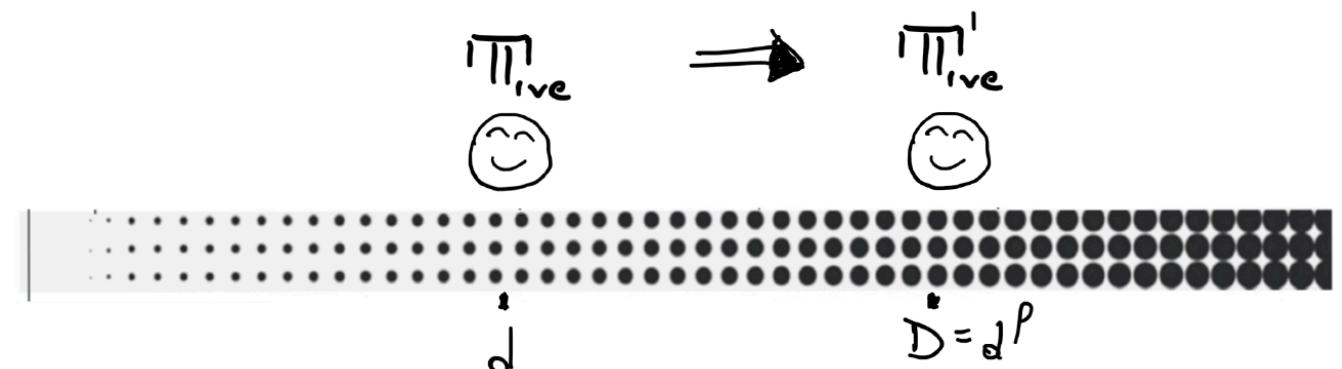
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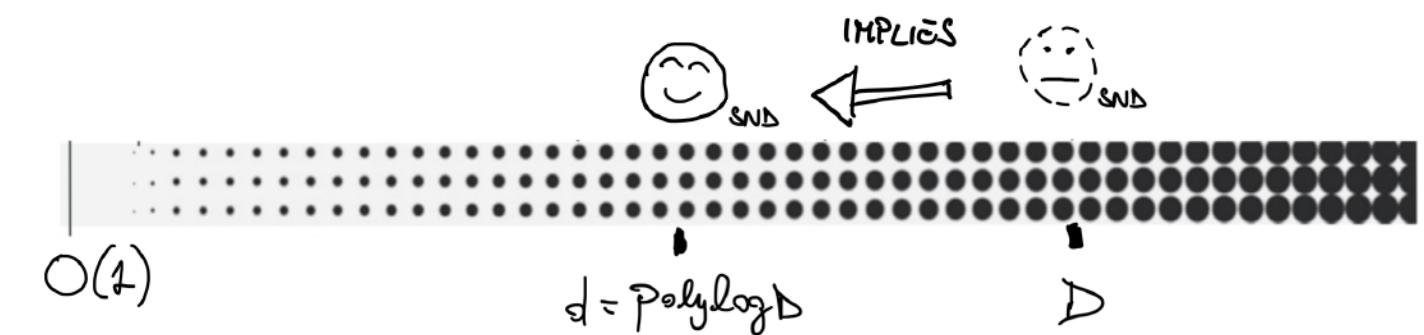


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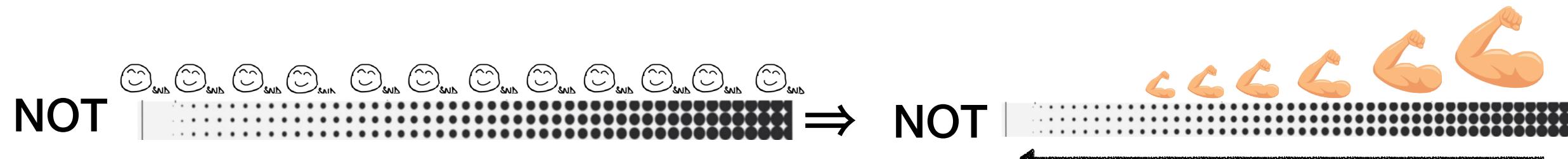
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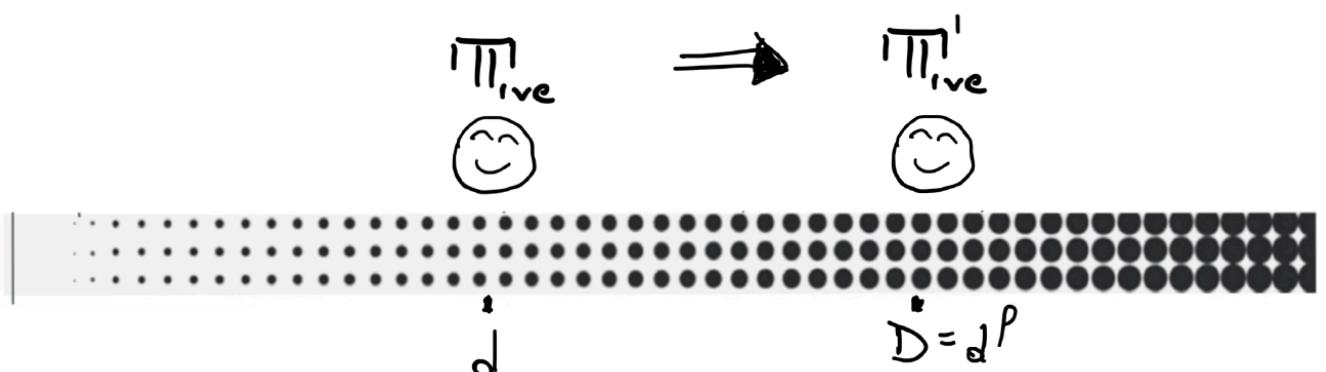
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# **New Notion: Incremental Functional Commitments**

# What are Functional Commitments? (FC)

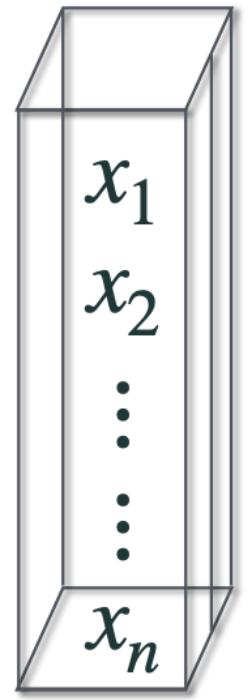


**Server (Prover)**



**Client (Verifier)**

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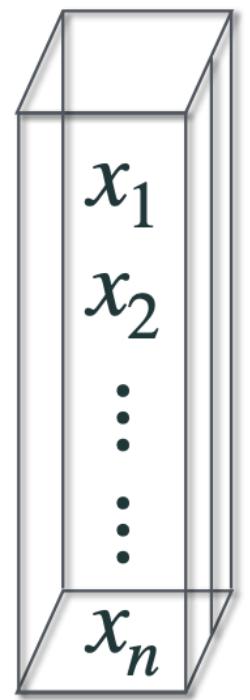


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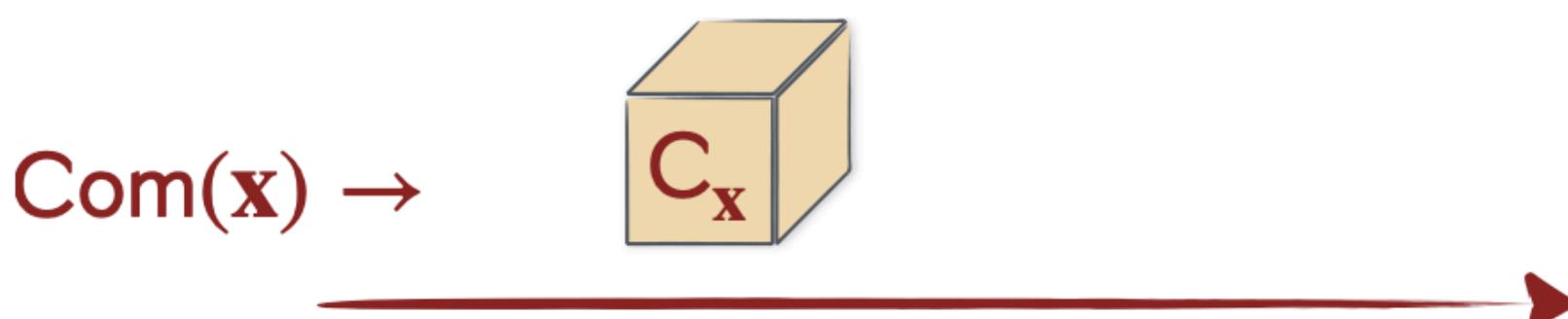


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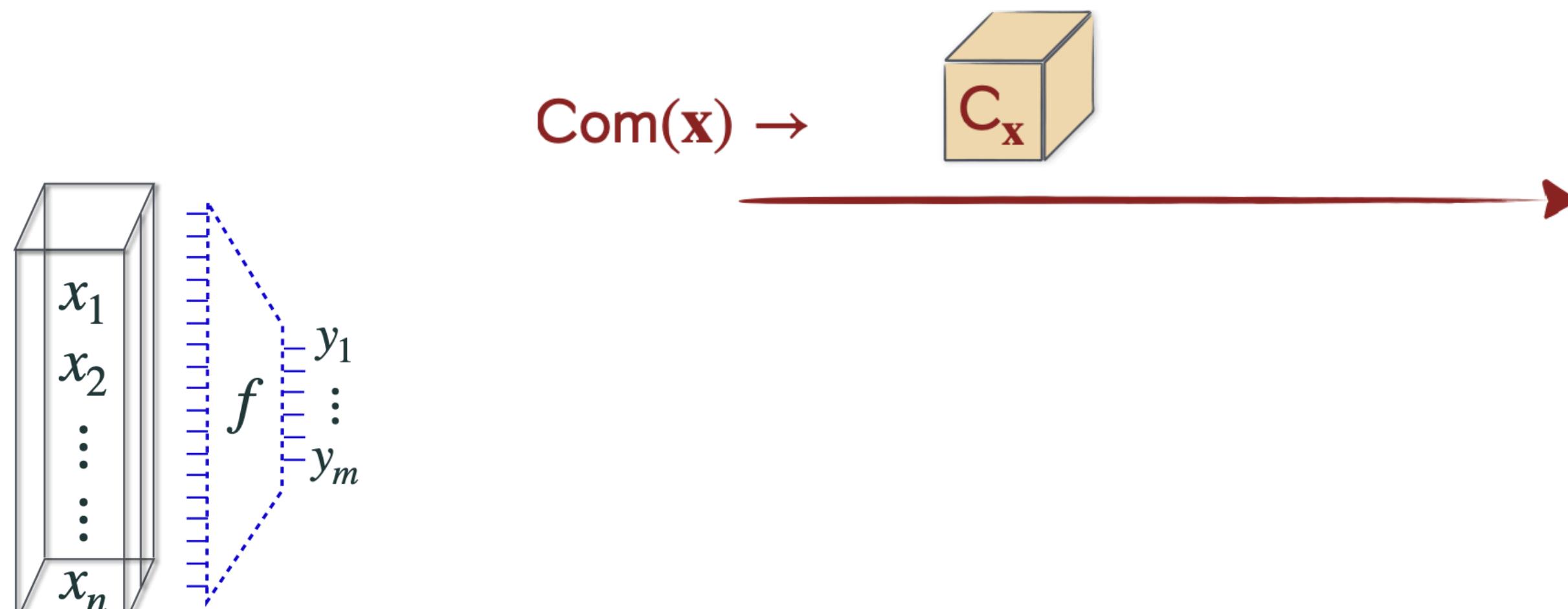


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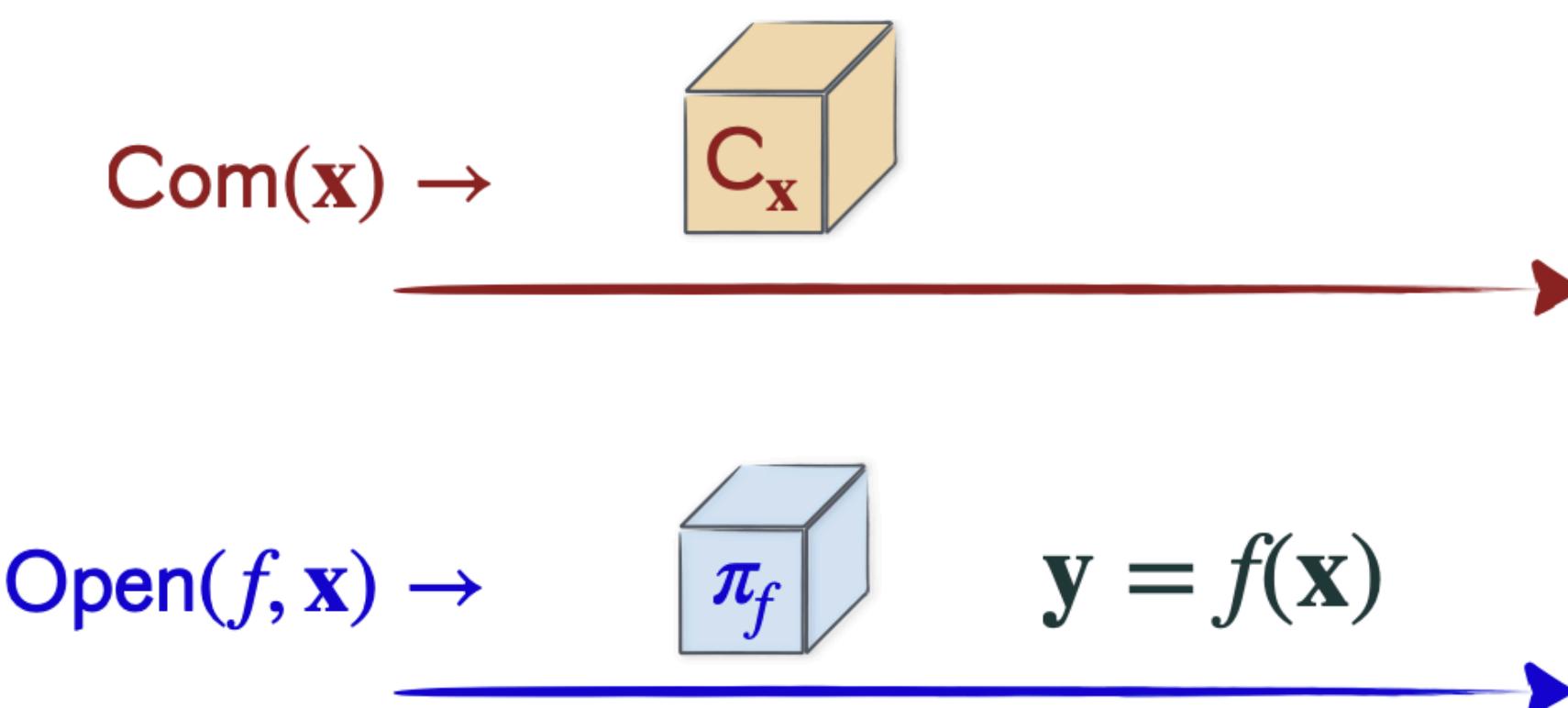
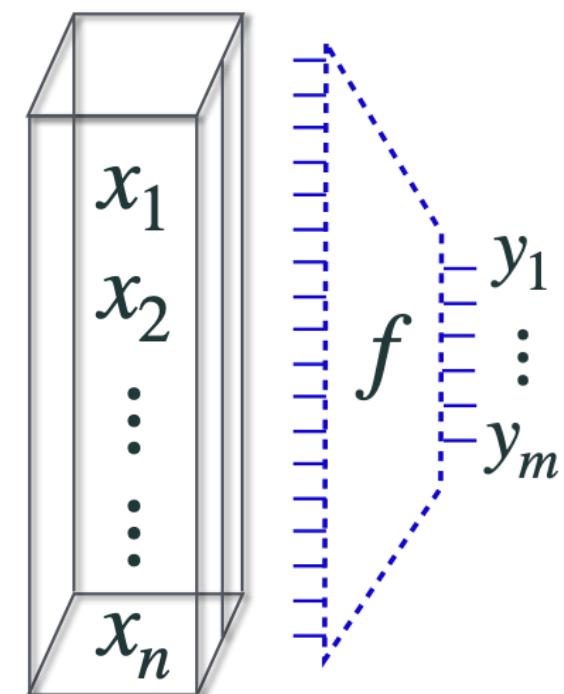


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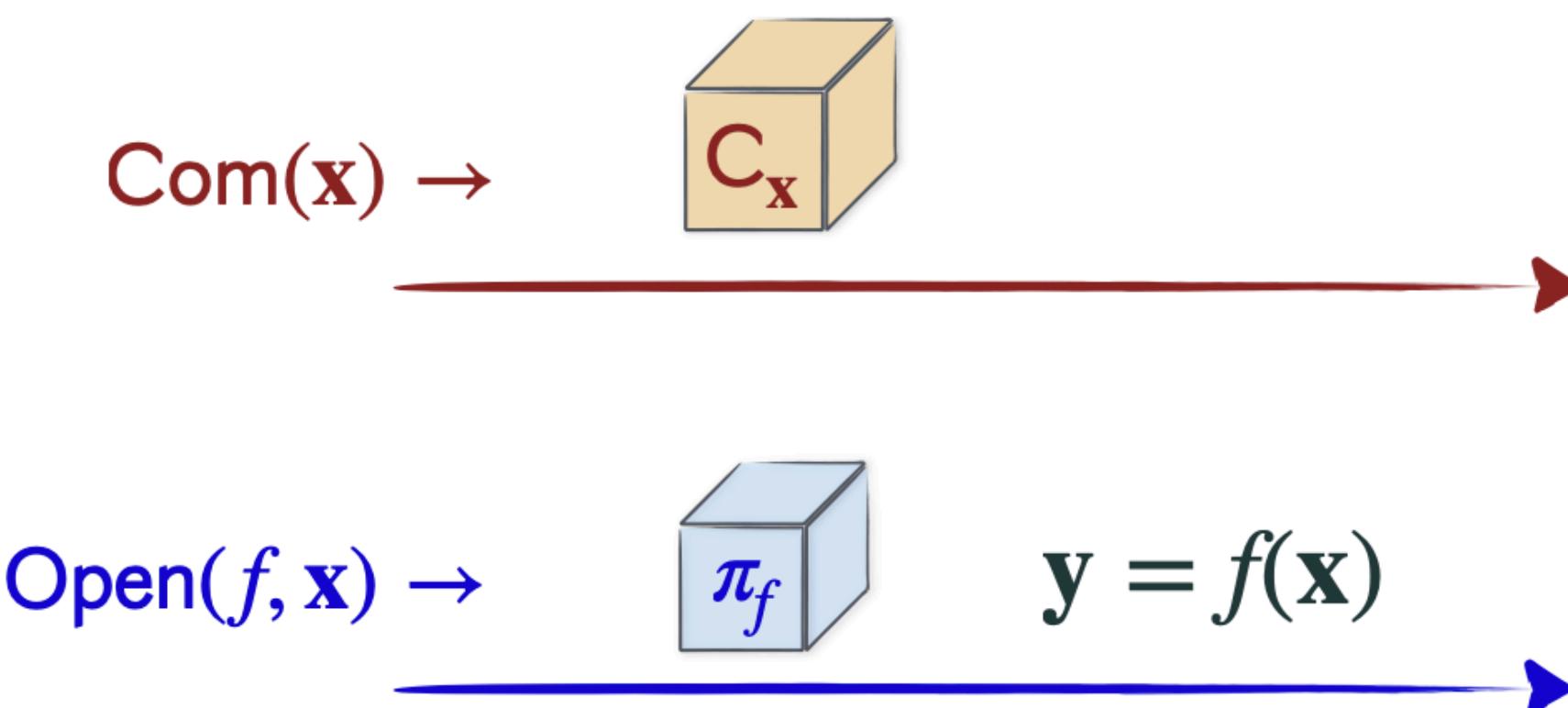
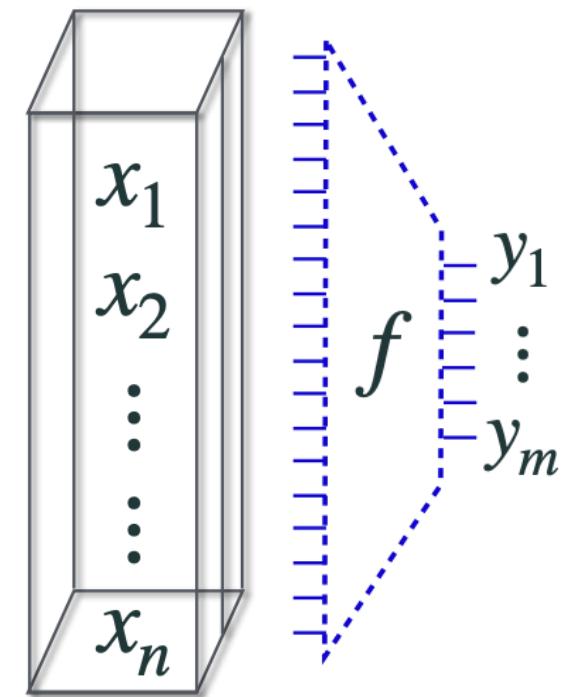


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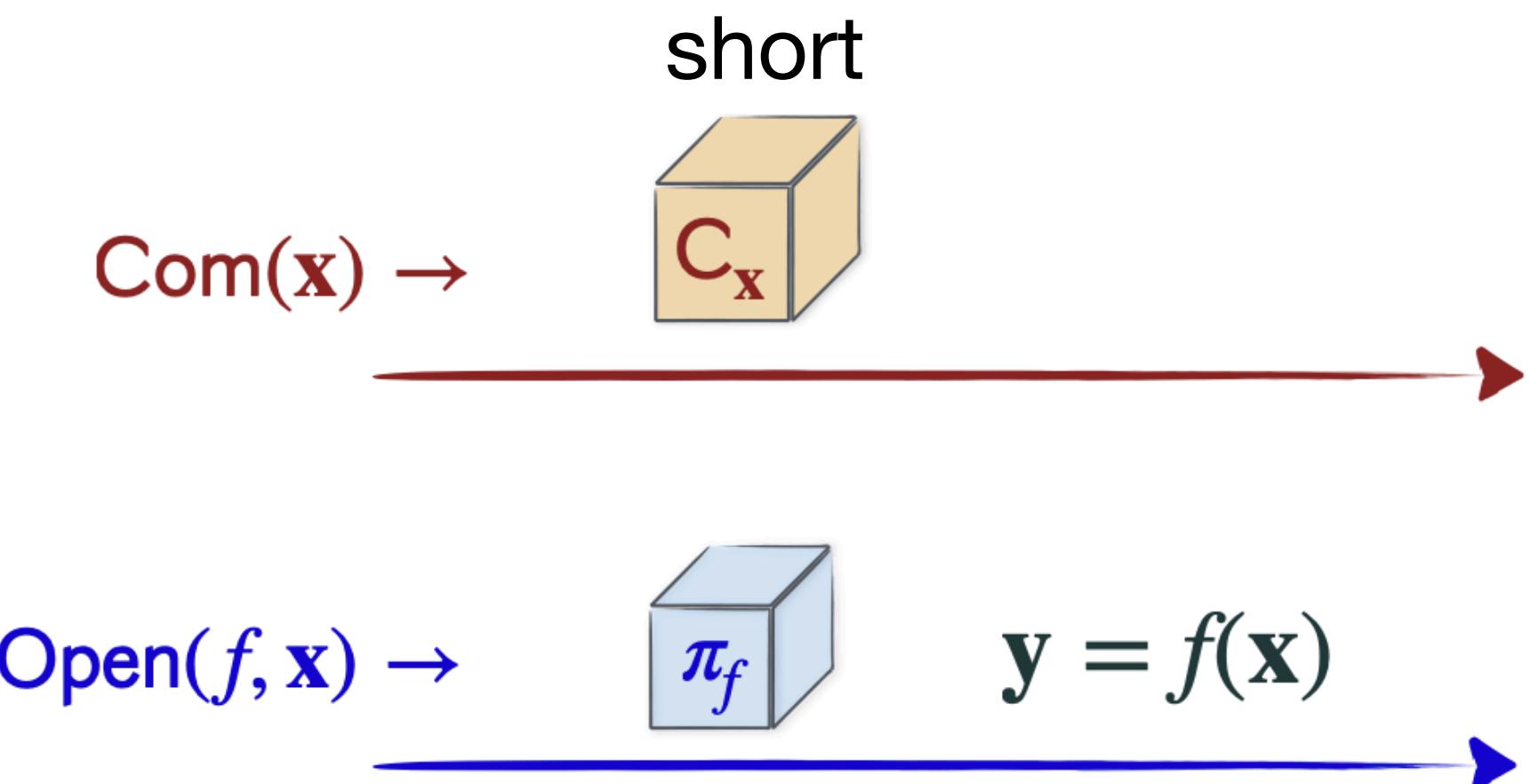
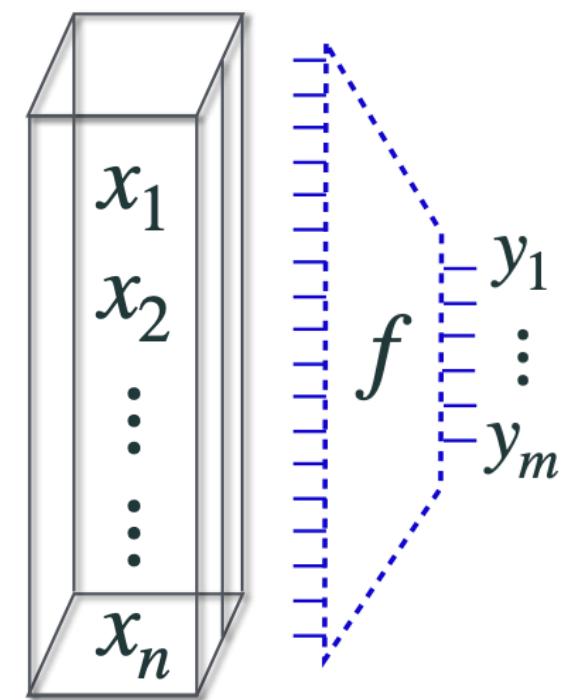
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$$\text{Ver}(\mathbf{C}_x, f, \mathbf{y}, \pi_f) \stackrel{?}{=} 1$$

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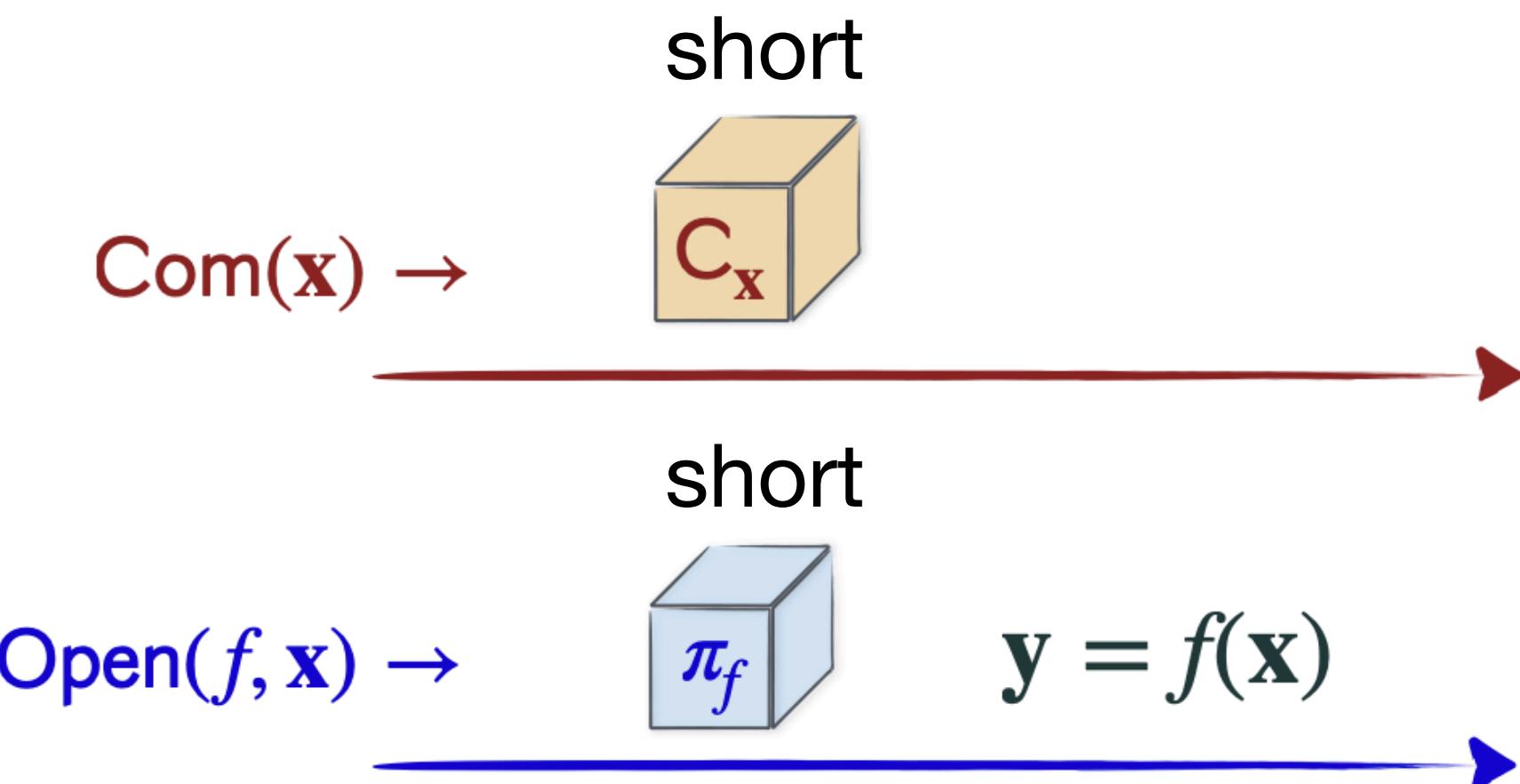
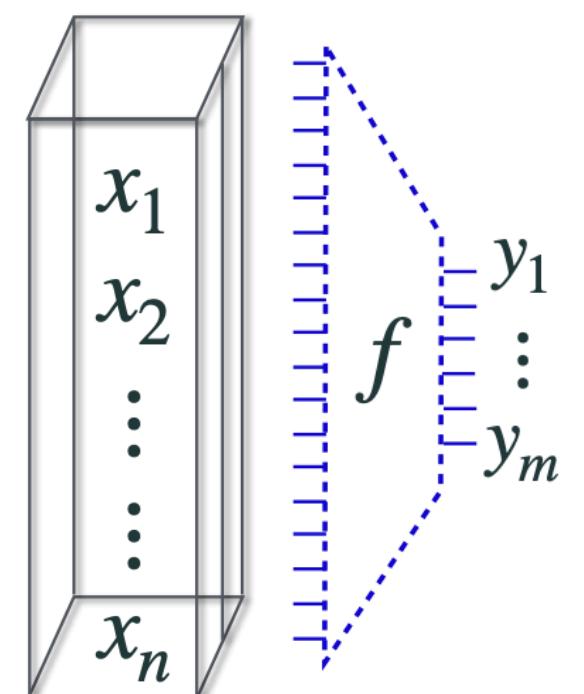
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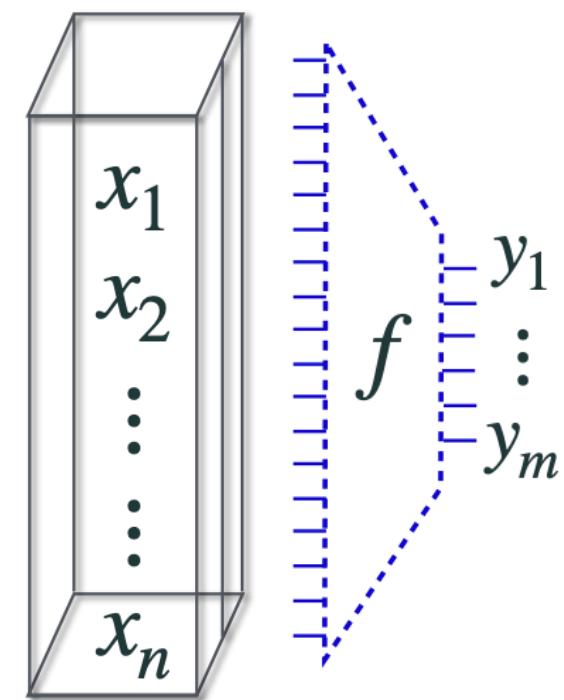
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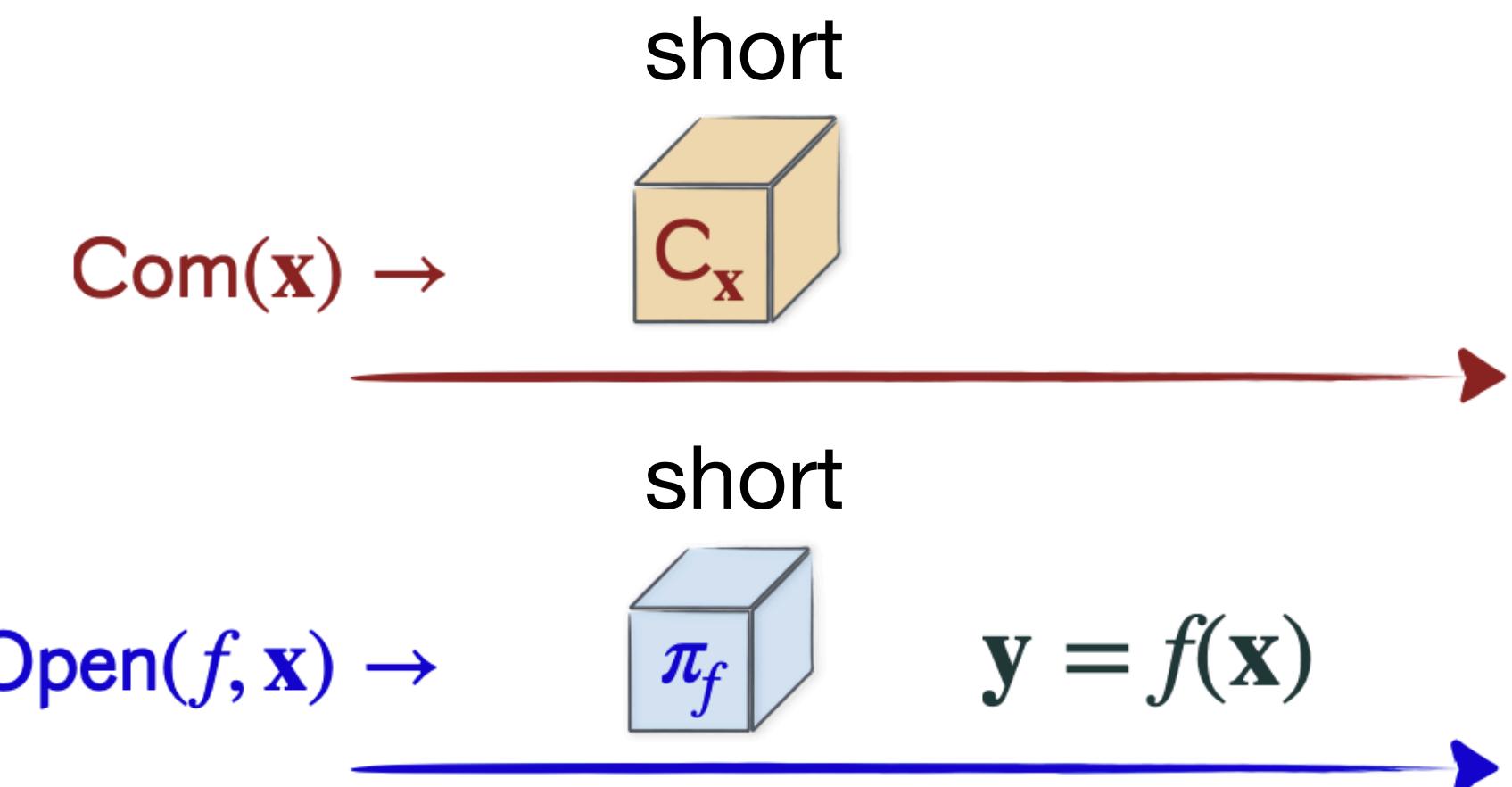
# What are Functional Commitments? (FC)



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Functional commitments generalize polynomial and vector commitments.



Client (Verifier)

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# Security of Functional Commitments

## Evaluation Binding



Malicious Prover



Client (Verifier)

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**Malicious Prover**

No adversary can succeed in providing inconsistent valid-looking outputs.



**Client (Verifier)**

# Security of Functional Commitments

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Malicious Prover

$$C_x \xrightarrow{f, \pi_f, y, \pi'_f, y'} y \neq y'$$



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# Security of Functional Commitments

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Client (Verifier)

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# Security of Functional Commitments

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**NB:** intuitively stronger than deterministic and non-deterministic soundness  
(but weaker than extractability).

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# **Incrementality in Functional Commitments**

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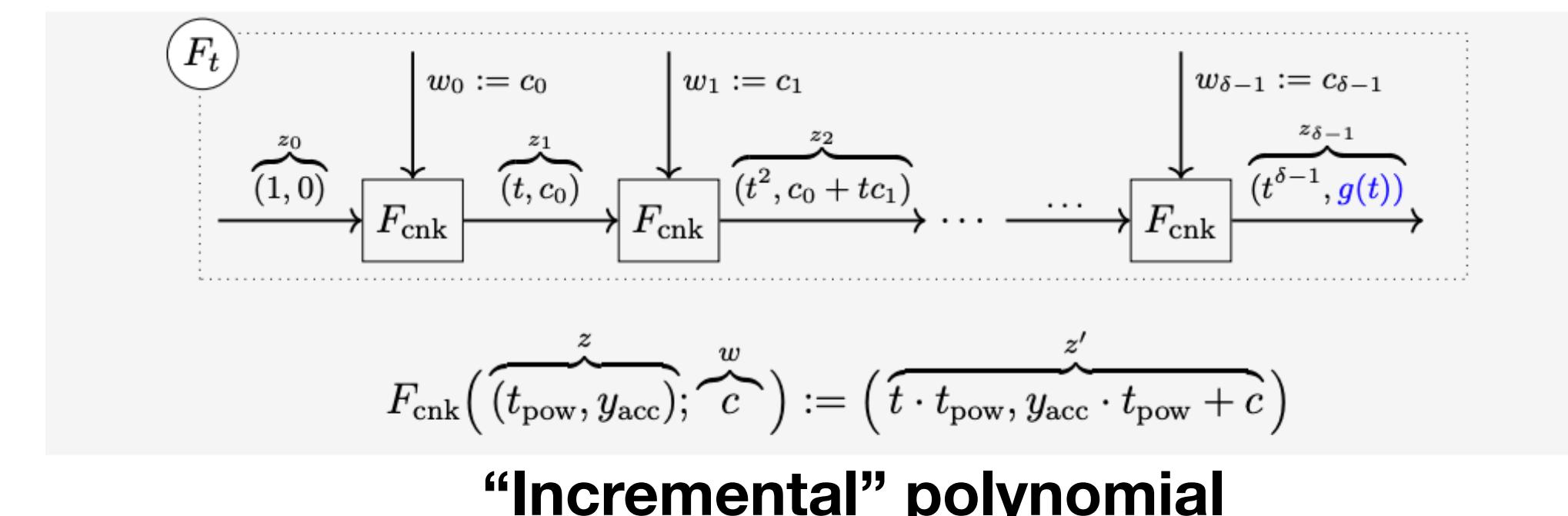
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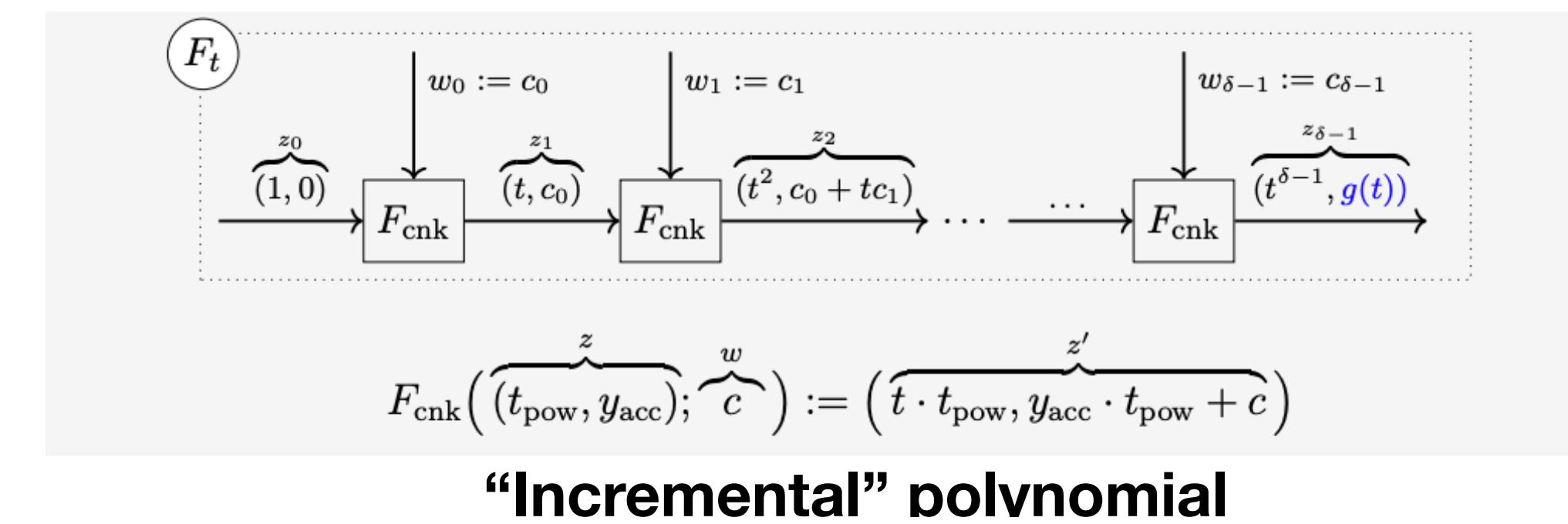
committing and (especially) proving should be doable in an incremental manner.

**Example applications:**

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**Our contributions for IFC:**

modeling, canonical construction,  
security proofs, connections to other results.



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**Summary and, where could one go from here?**

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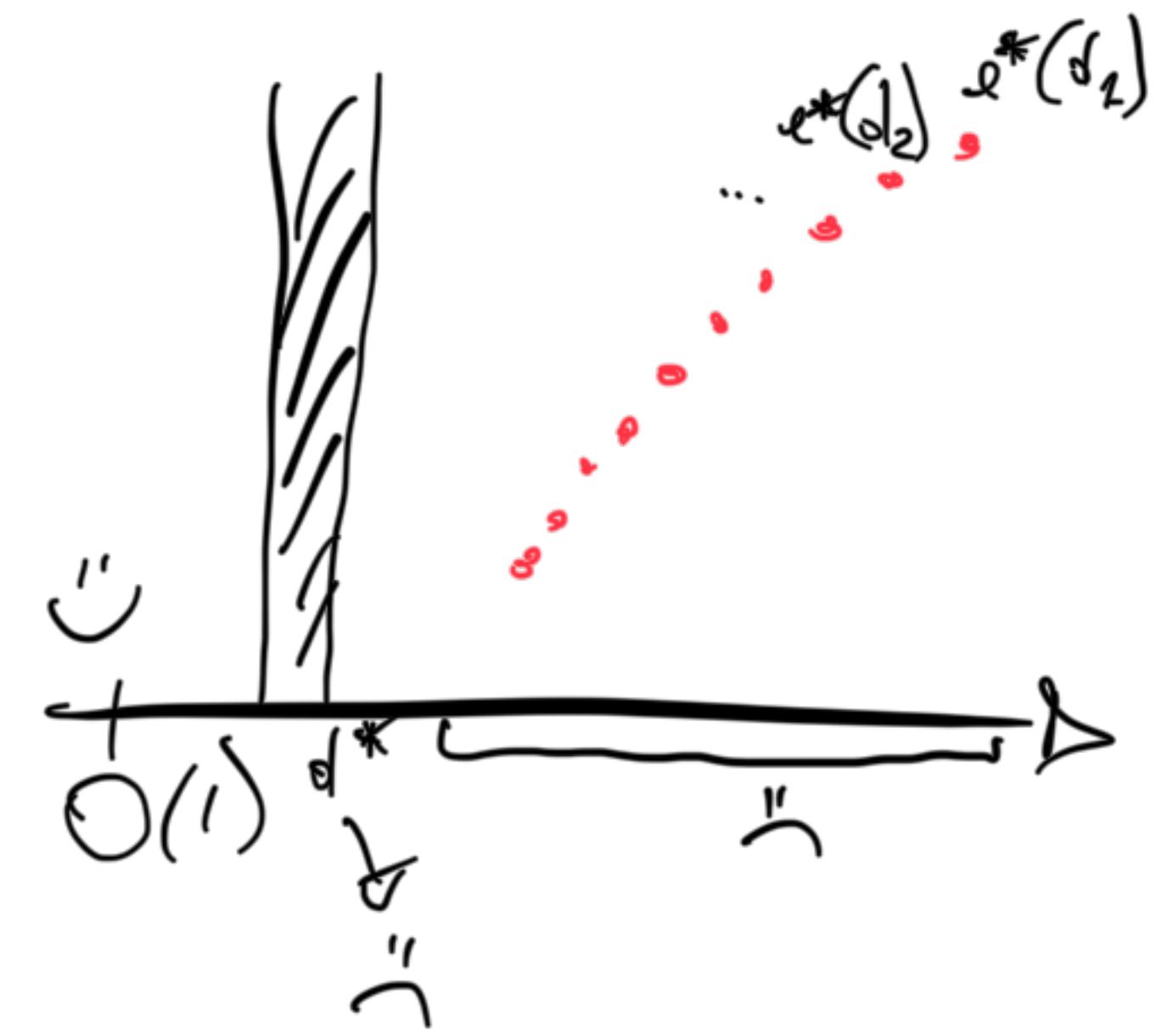
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# Extra slide on graceful sec. degradation



**Theorem 6.** Given any sequence of superconstant—i.e.,  $\omega(1)$ —depth bounds  $R_0, R_1, R_2, \dots$ , there always exists a superconstant depth bound  $L$  such that for all  $i$ ,  $R_i \succeq L$ .

$$\exists d_1, d_2, \dots : e^*(d_i) < e^*(d_{i-1})$$

$e^* := \approx \log$  of Adv. of  
"BEST"  $\mathcal{A}$

# Extra slide on io-soundness

**Theorem (Informal statement of Corollary 1).** Let  $\Pi$  be an IVC scheme and  $D = \omega(1)$  be a depth bound. Let  $E \subseteq \mathbb{N}$  be an infinite and “exponentially sparse” set of security parameters where  $\Pi$  achieves negligible soundness at depth bound  $D$ . Then there exists a depth bound  $d = \omega(1)$  where  $\Pi$  achieves (standard) negligible soundness.

**Theorem (Informal statement of Corollary 2).** Let  $\Pi, D = \omega(1)$ , and  $E$  as in the previous theorem. Then:

- $E$  exponentially sparse  $\implies d = O(\log D)$ .
- $E$  sub-exponentially sparse  $\implies d = O(\text{polylog} D)$ .

**Theorem 3.** Let  $E = \{\lambda_1 < \lambda_2 < \dots\} \subseteq \mathbb{N}$  be a constructible  $(2^{\kappa^T})$ -sparse set for some  $T$  with  $0 < T \leq 1$ . Let  $\Pi$  be an IVC that is i.o-sound with respect to  $E$  for depth bound  $D(\cdot)$ . Let  $d'(\cdot)$  be a depth bound. If for all  $i \in \mathbb{N}$ ,

$$d'(\lambda_{i+1} - 1) \leq D(\lambda_i), \tag{\blacktriangle}$$

then  $\Pi$  is (almost-everywhere) sound for depth bound  $d'$  if appropriately parameterized (Definition 3). The resulting proving time, verification time and proof size are like those originally in  $\Pi$  (up to constant factors).