

When Can We Incrementally Prove Computations of Arbitrary Depth?

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[eprint:2025/1413](https://eprint.2025/1413)

A joint work with
Dario Fiore and Mahak Pancholi
(IMDEA Software Institute)

This Talk in a Nutshell

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**A study of the security of
Incrementally Verifiable Computation (IVC) under
the lens of the depth of the proven computation.**

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the lens of the depth of the proven computation.**

**Our main motivation:
how can we prove security (or insecurity)
when we move beyond constant depth?**

Succinct Cryptographic Proofs (SNARKs)



Server (Prover)



Client (Verifier)

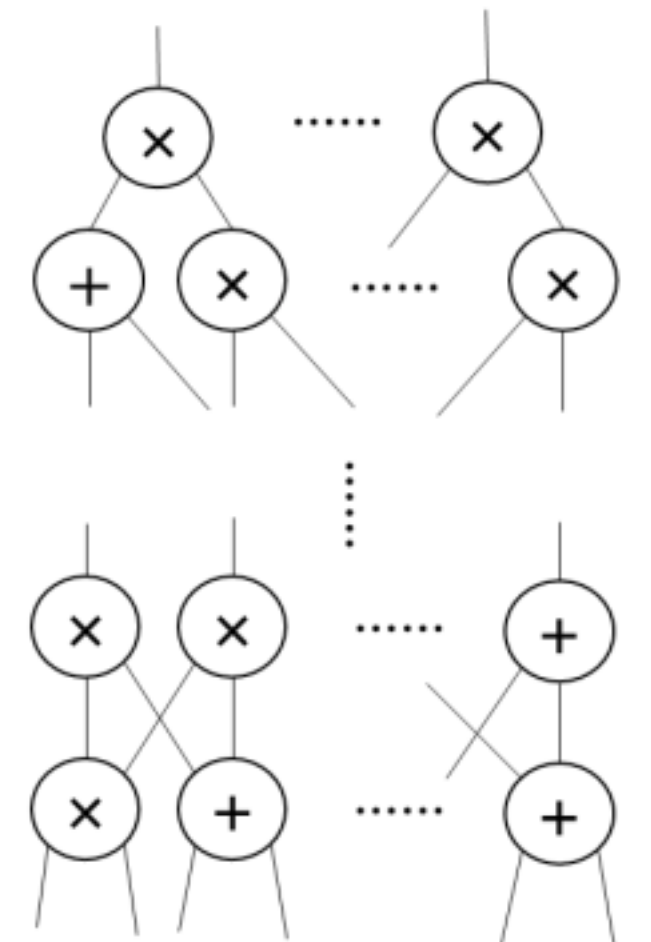
Succinct Cryptographic Proofs (SNARKs)



Server (Prover)



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Some program F

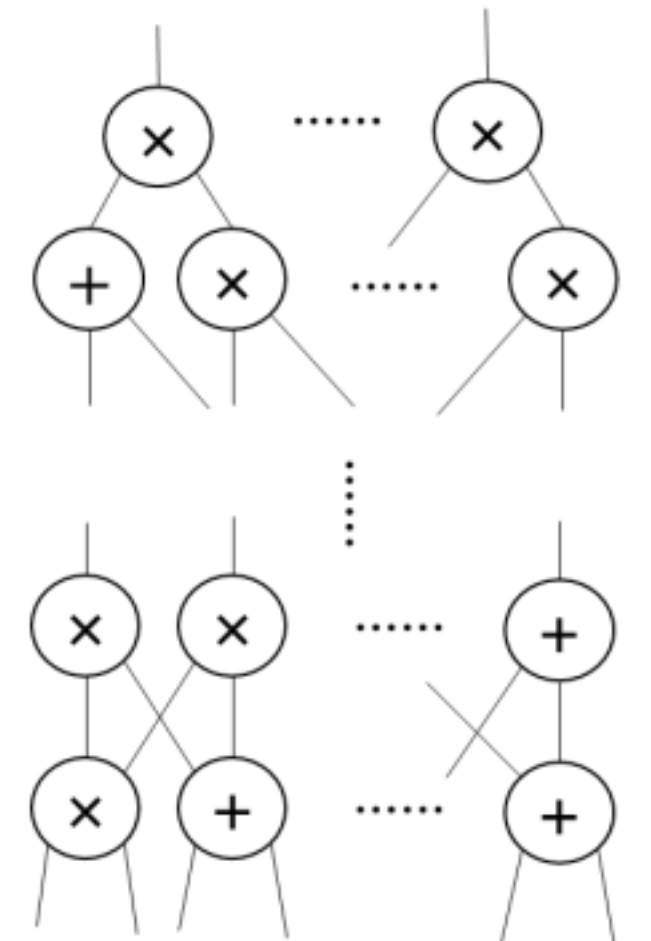
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Some program F

Client would like to know whether $\exists w : F(x, w) = 1$

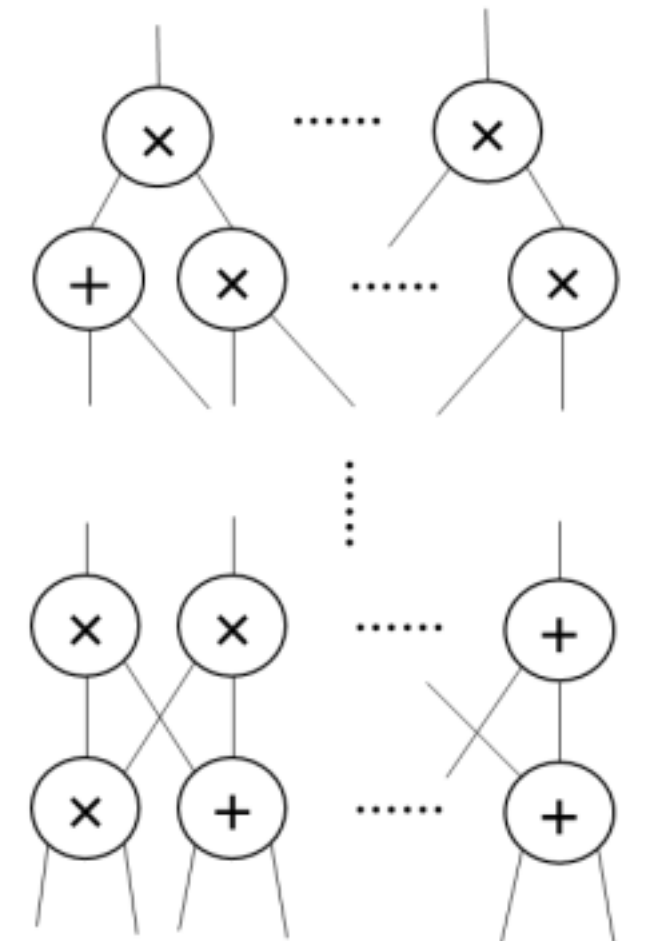
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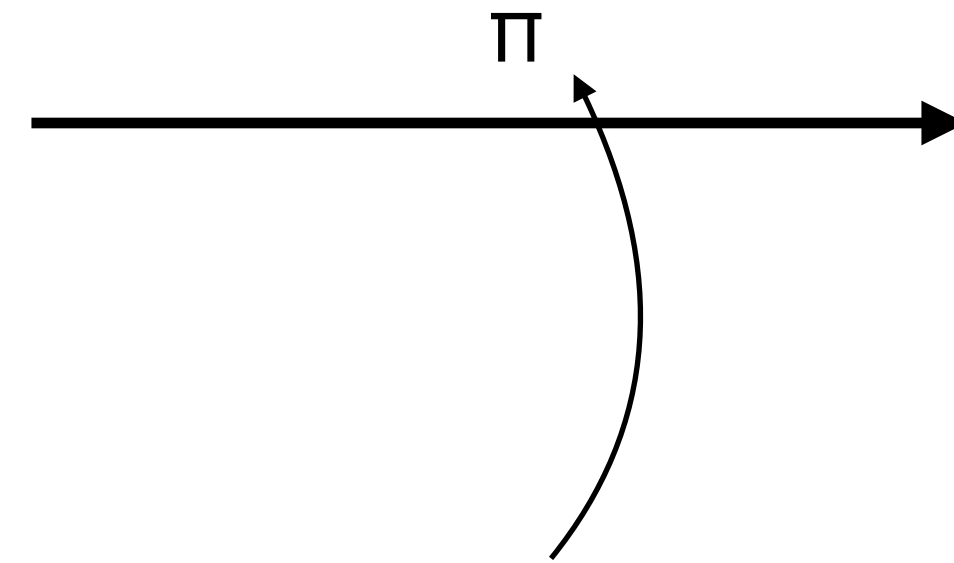
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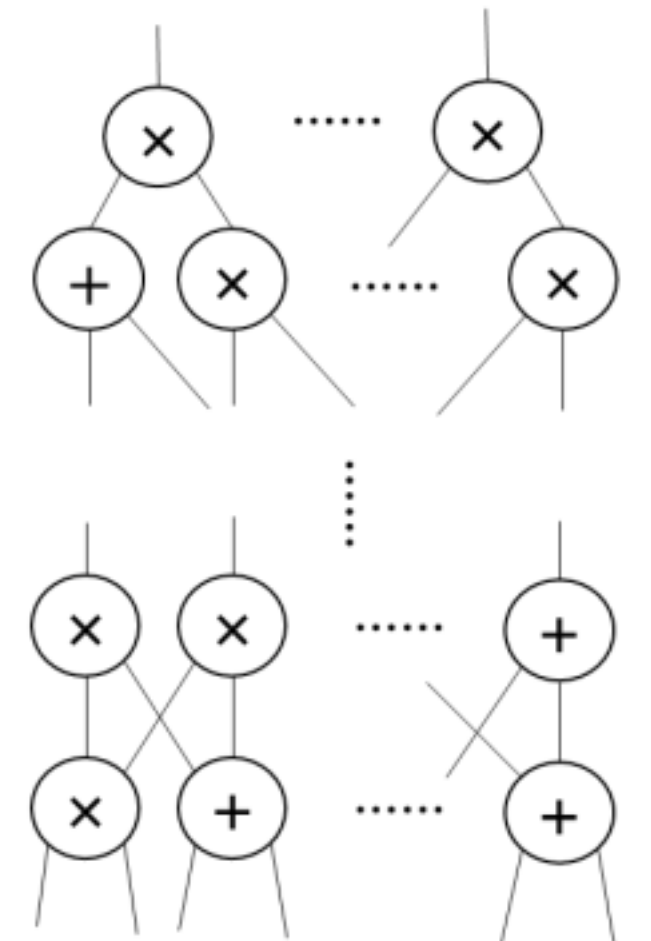
Server (Prover)



Proof that statement is true



Client (Verifier)



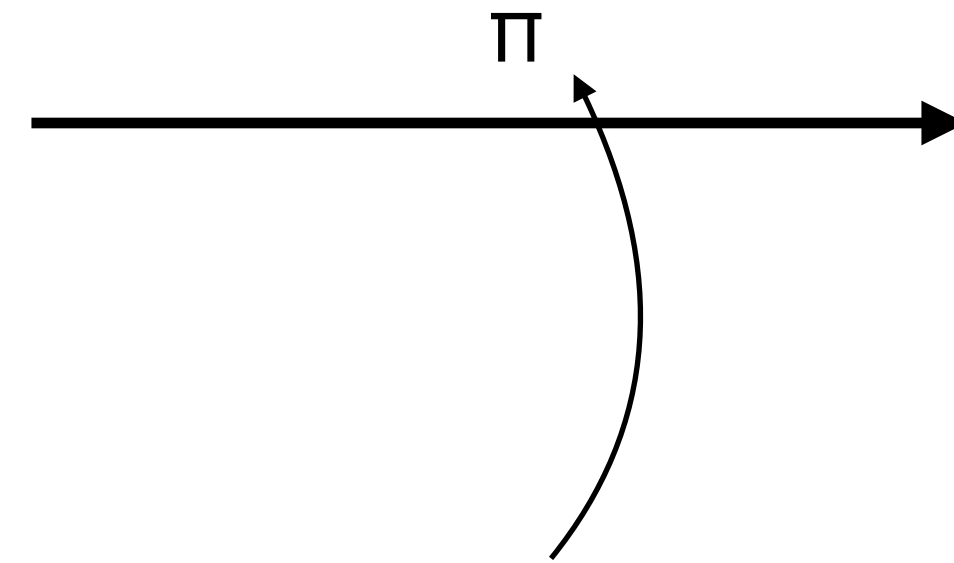
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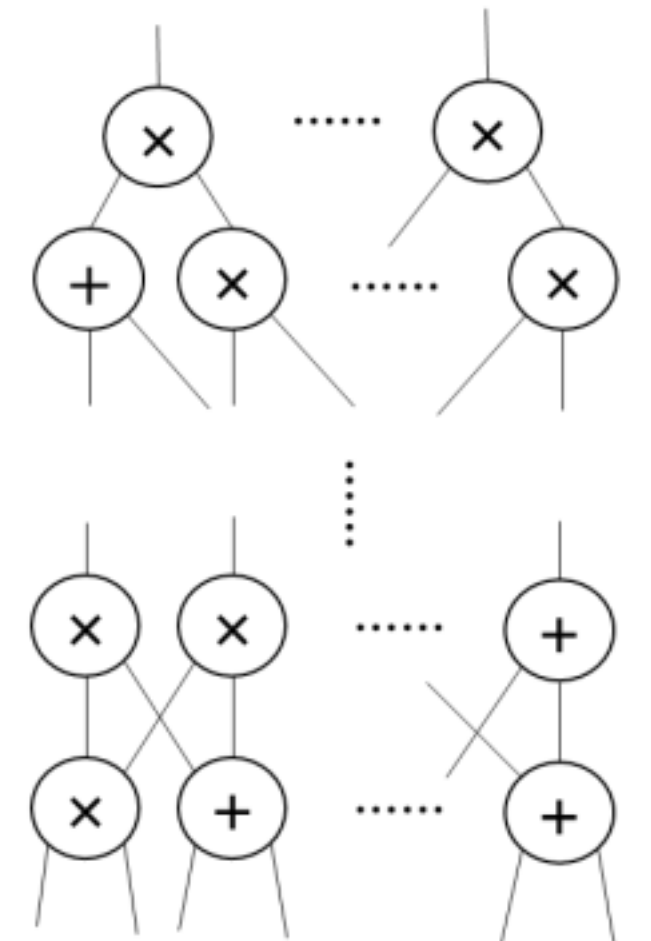


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Verify(x, π)



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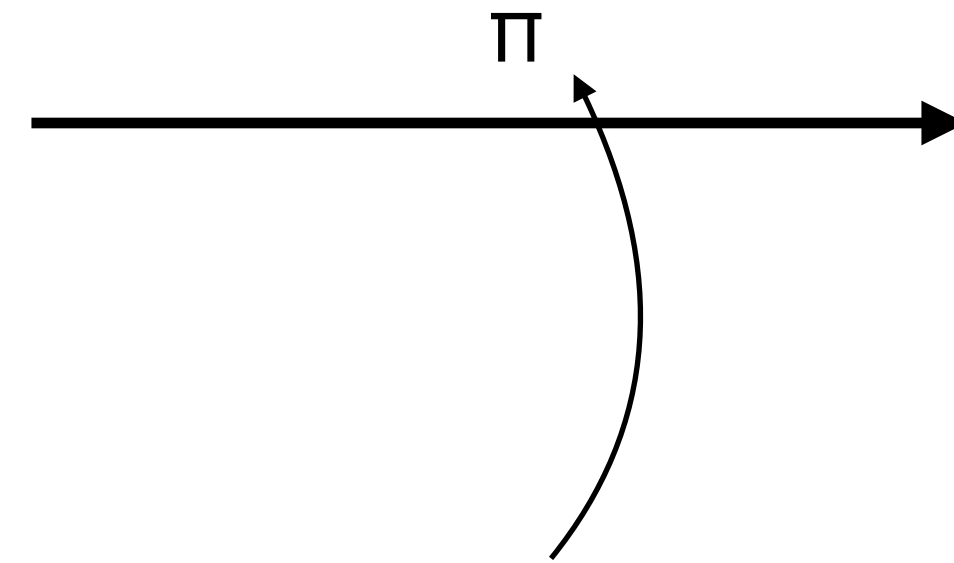
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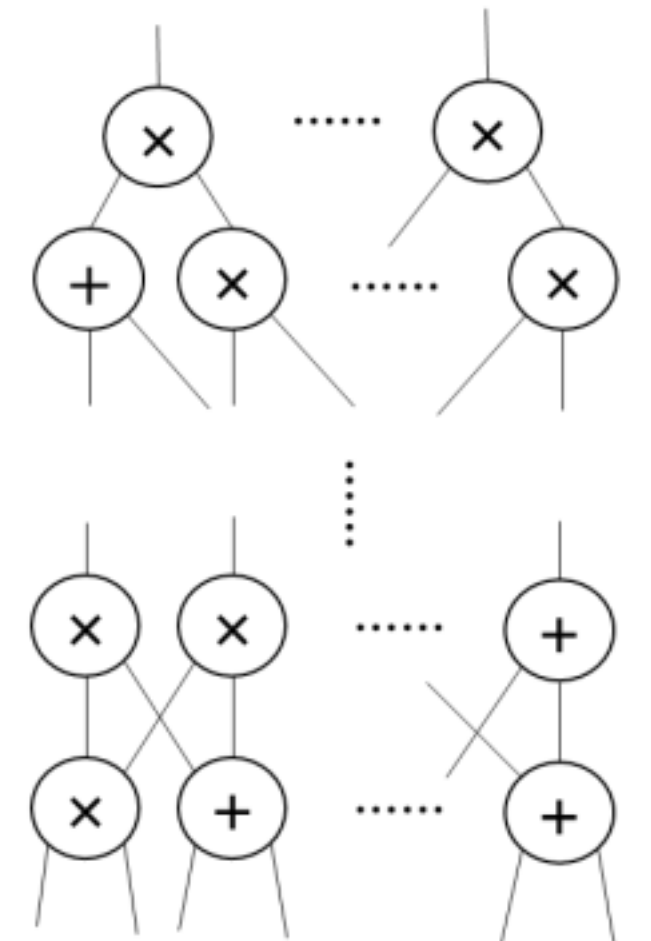


Proof that statement is true

Verify(x, π)



Client (Verifier)



Some program F

Common requirement: **Succinctness**
(π is very small; Verify is very fast)

Client would like to know whether $\exists w : F(x, w) = 1$

Limitations of Traditional “Monolithic” Proofs

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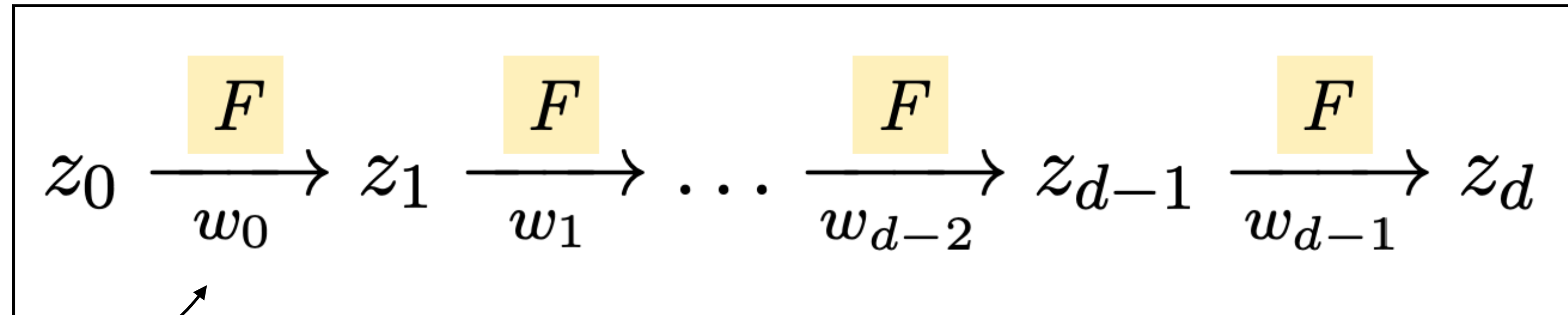
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- Large memory requirements
 - whole “trace” of the computation should in principle be kept all in memory at the same time
- No pipelining
 - Must finish the computation before starting proving
 - Cannot take advantage of incremental computations (next slide)

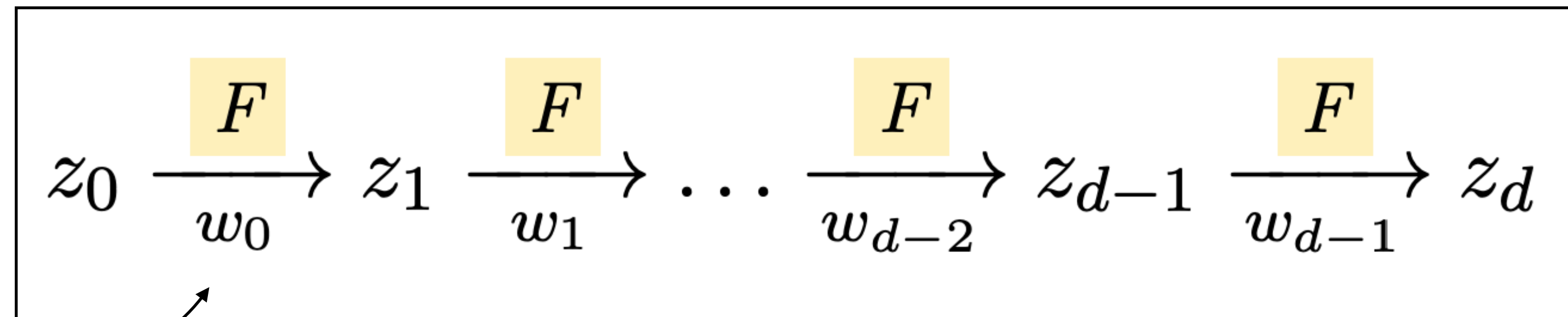
Incremental Computations



$$z_i = F(z_{i-1}, w_{i-1})$$

An arrow points from the equation $z_i = F(z_{i-1}, w_{i-1})$ to the first transition in the sequence, specifically to the F and w_0 components of the arrow from z_0 to z_1 .

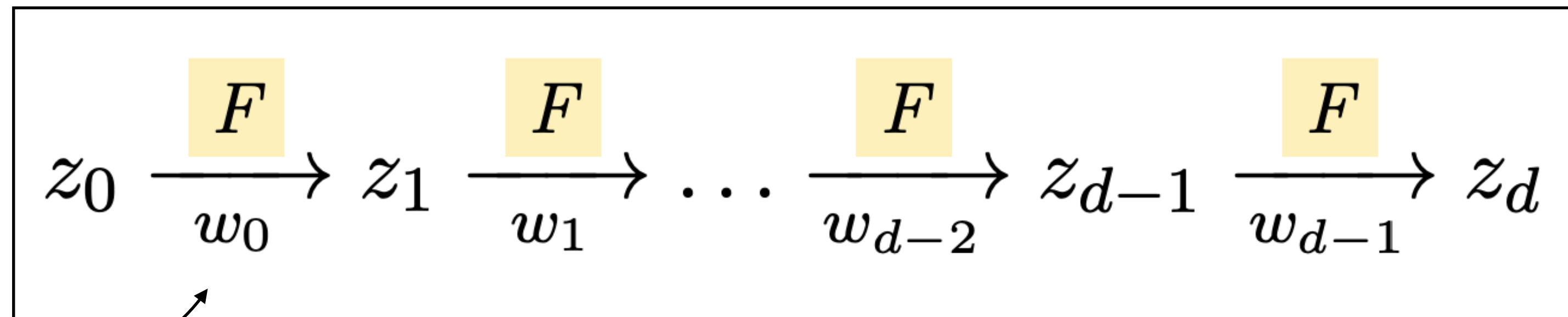
Incremental Computations



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Examples of natural applications:

Incremental Computations



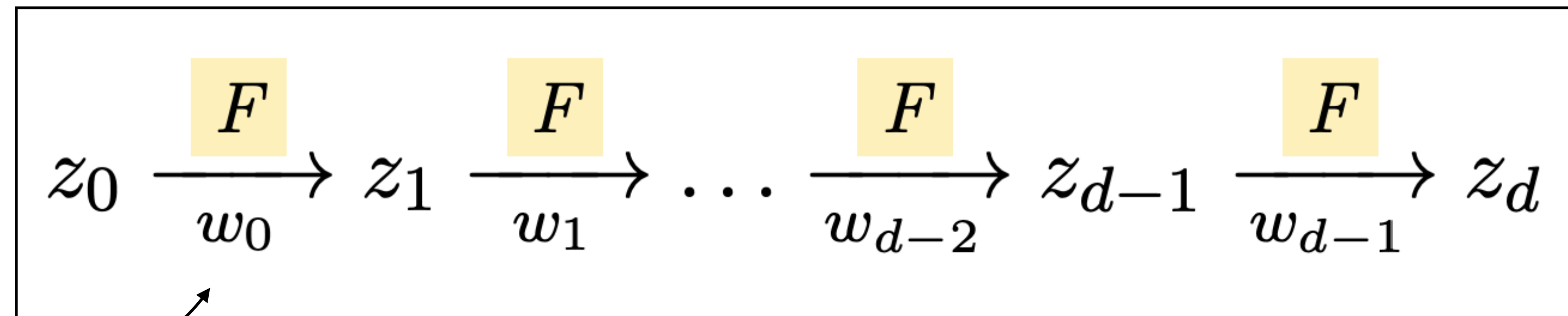
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Examples of natural applications:

- Streaming algorithms

Incremental Computations



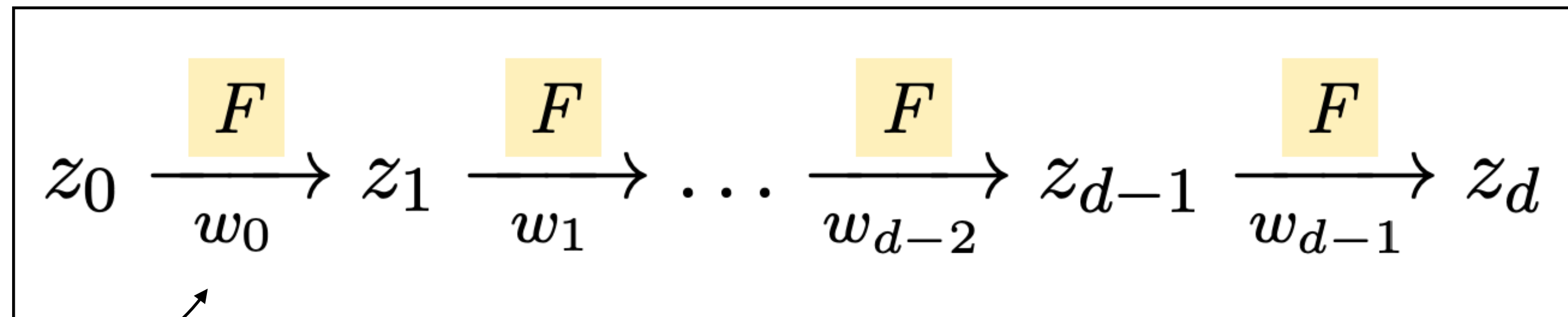
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Examples of natural applications:

- Streaming algorithms
- RAM computations

Incremental Computations



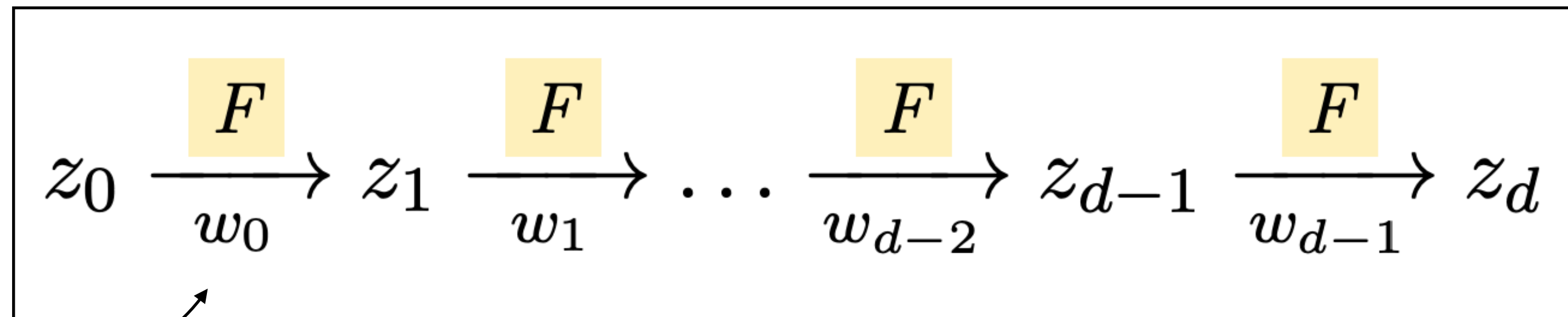
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Examples of natural applications:

- Streaming algorithms
- RAM computations
- Verifiable Delay Functions (VDF)

Incremental Computations



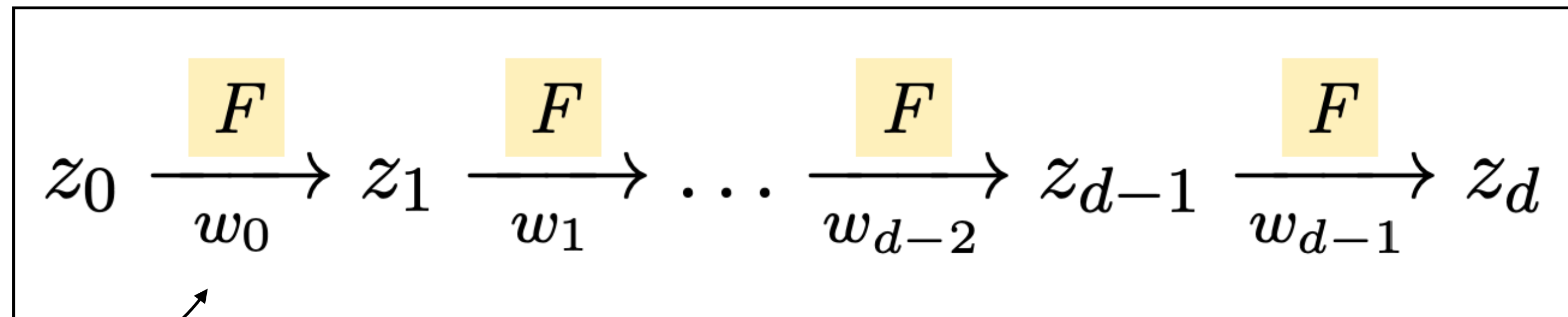
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- Streaming algorithms
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- Round functions in symmetric primitives

Incremental Computations



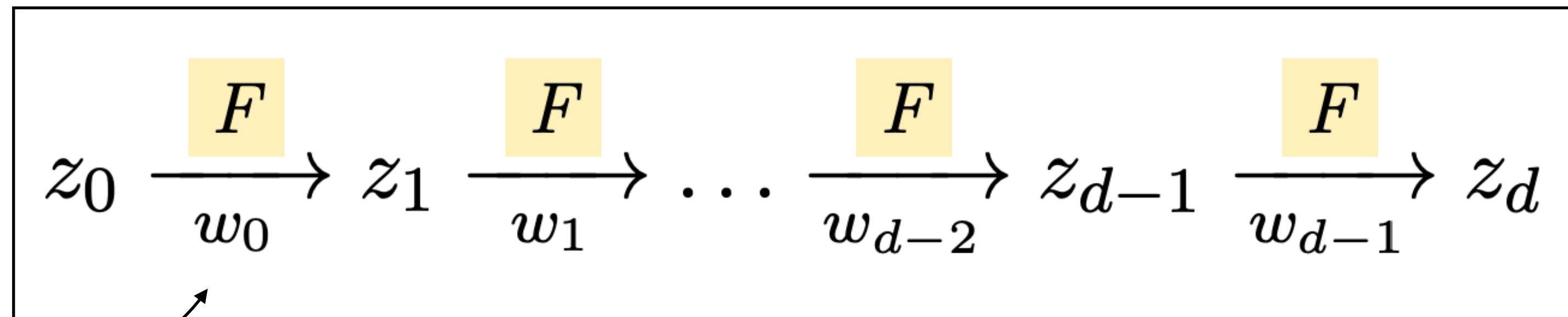
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Examples of natural applications:

- Streaming algorithms
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- Verifiable Delay Functions (VDF)
- Round functions in symmetric primitives
- Recurrent neural networks

Incremental Computations



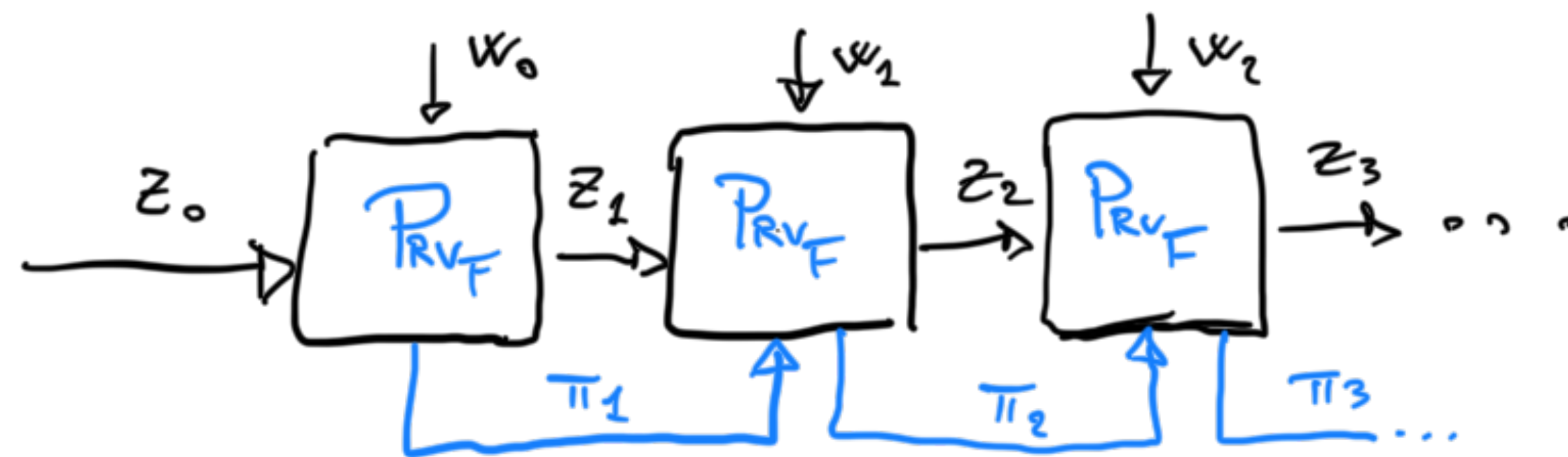
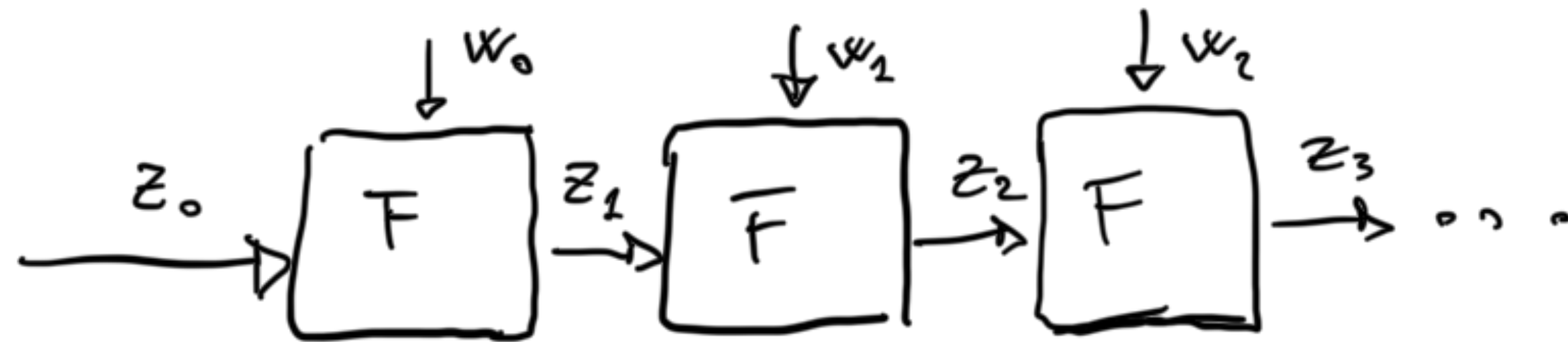
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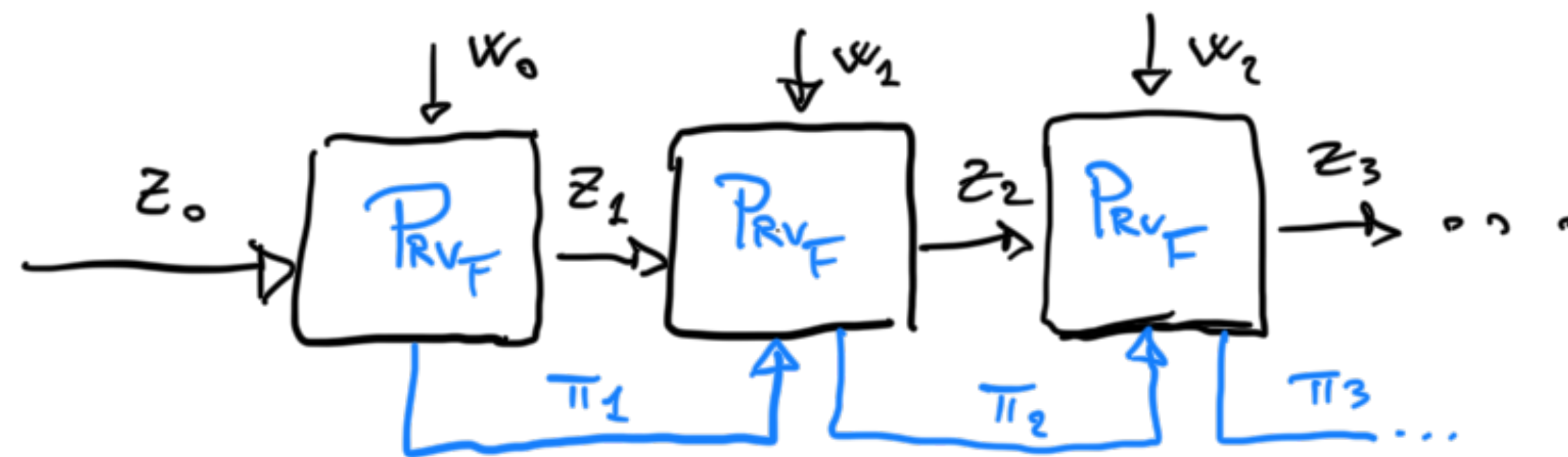
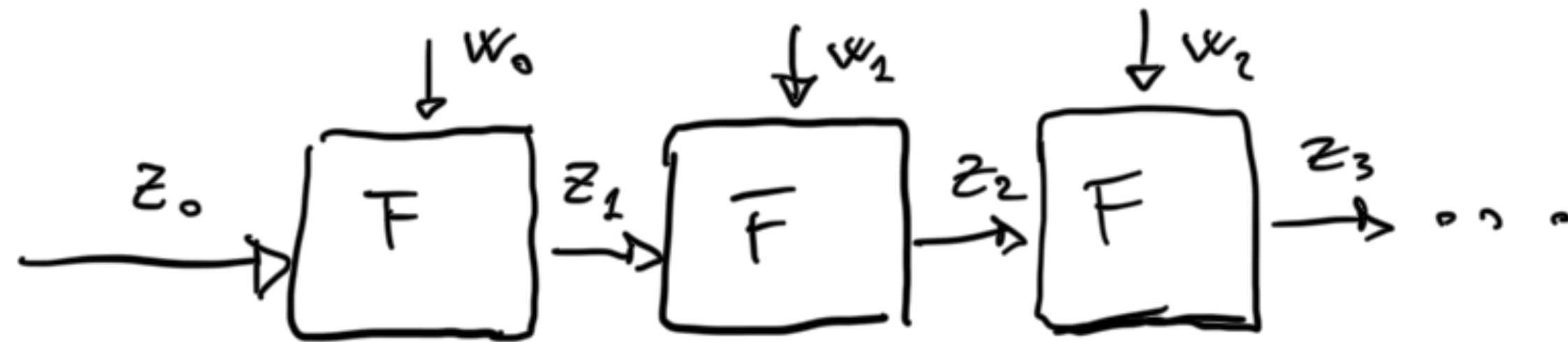
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- ...

Incrementally Verifiable Computations (IVC)



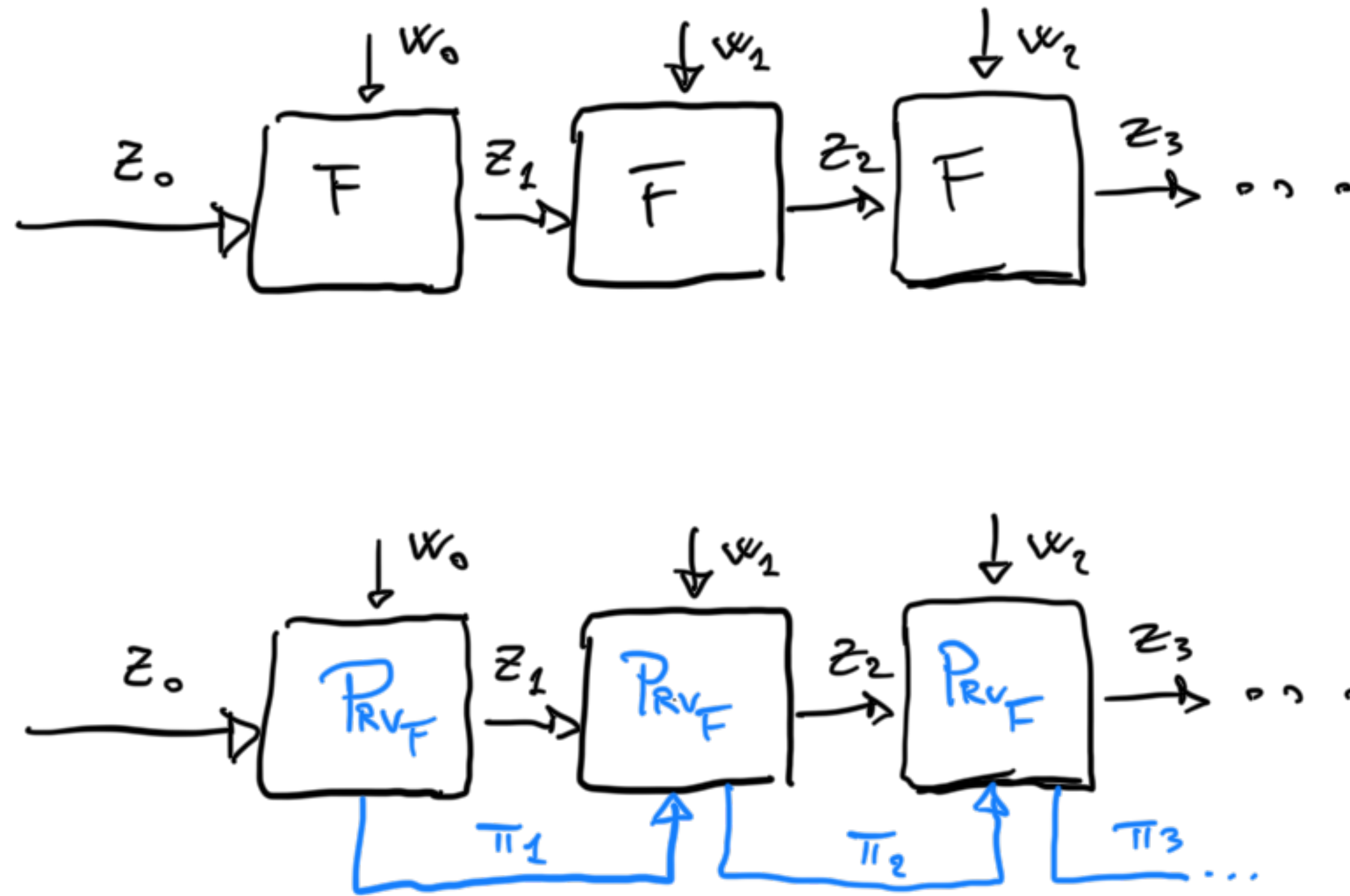
Incrementally Verifiable Computations (IVC)



Proof size should be sublinear in the # of steps (the *depth* of the computation)

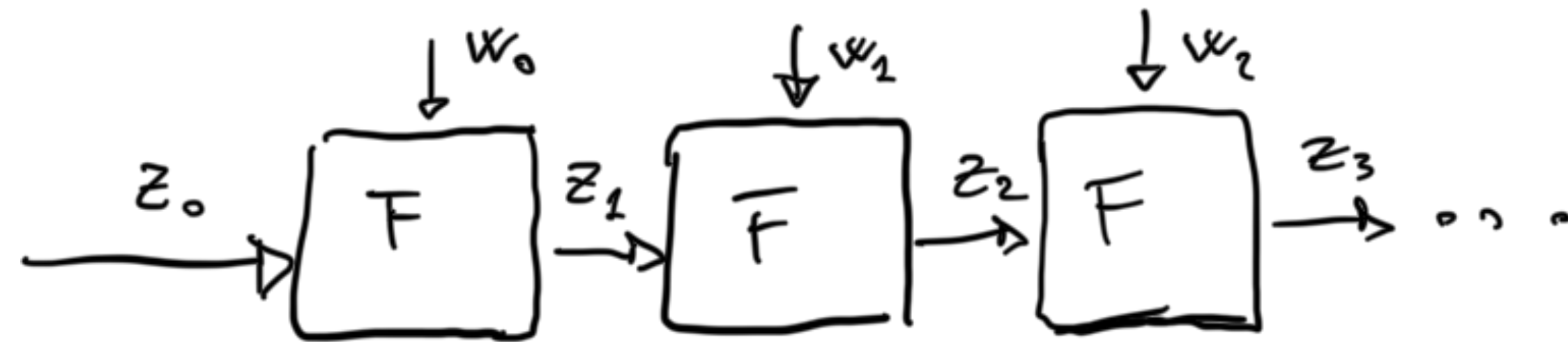
Incrementally Verifiable Computations (IVC)

Advantages



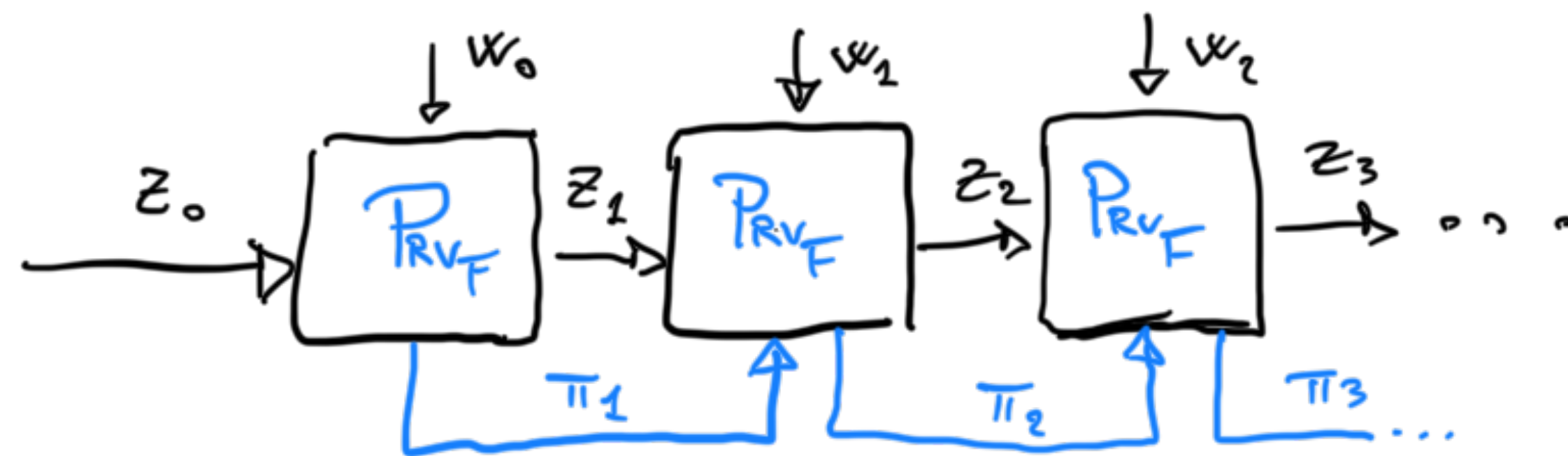
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Incrementally Verifiable Computations (IVC)



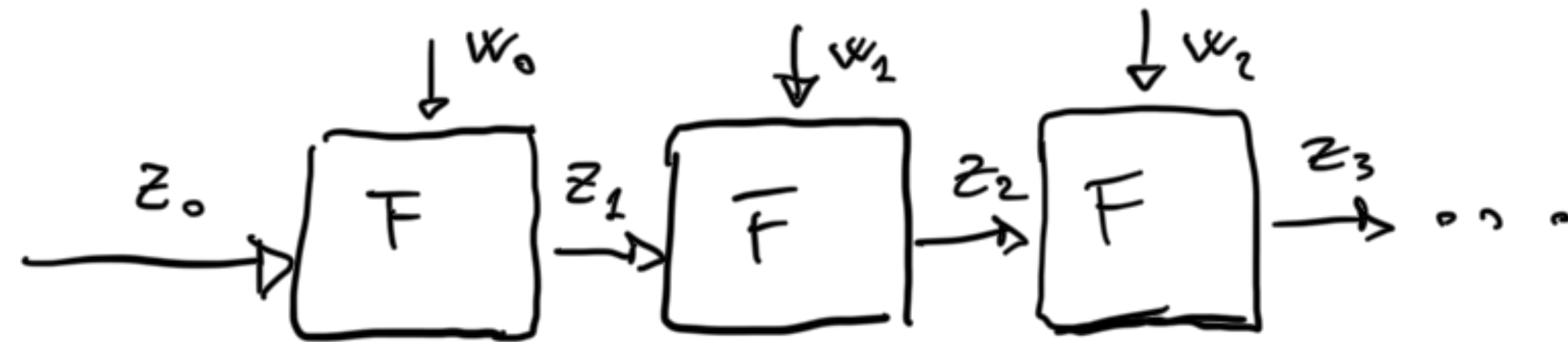
Advantages

- Low memory footprint



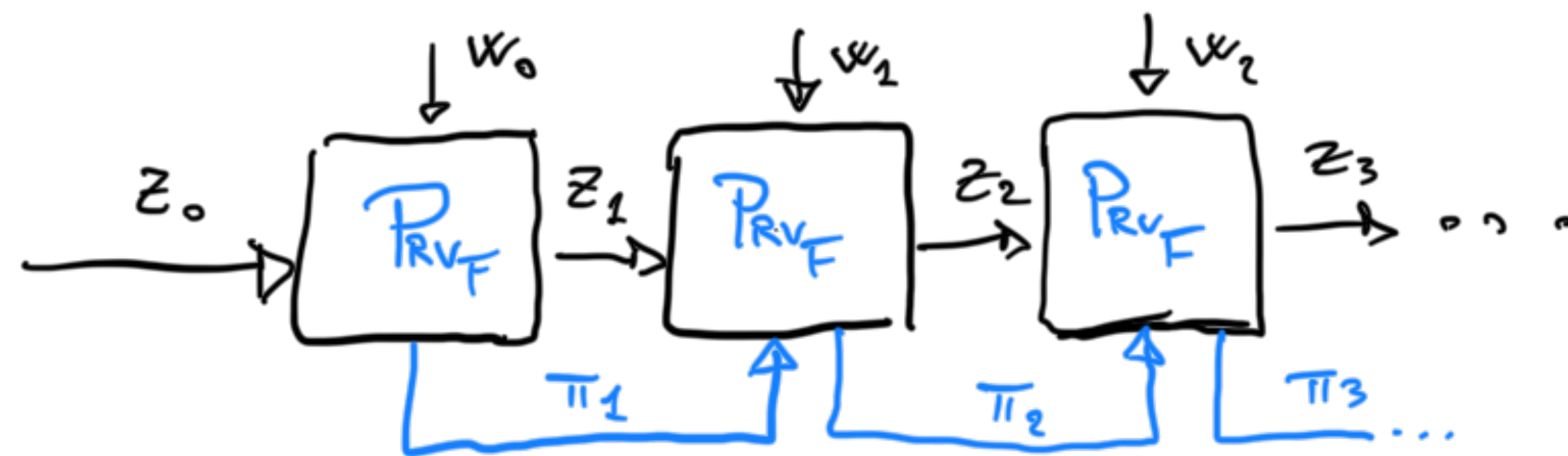
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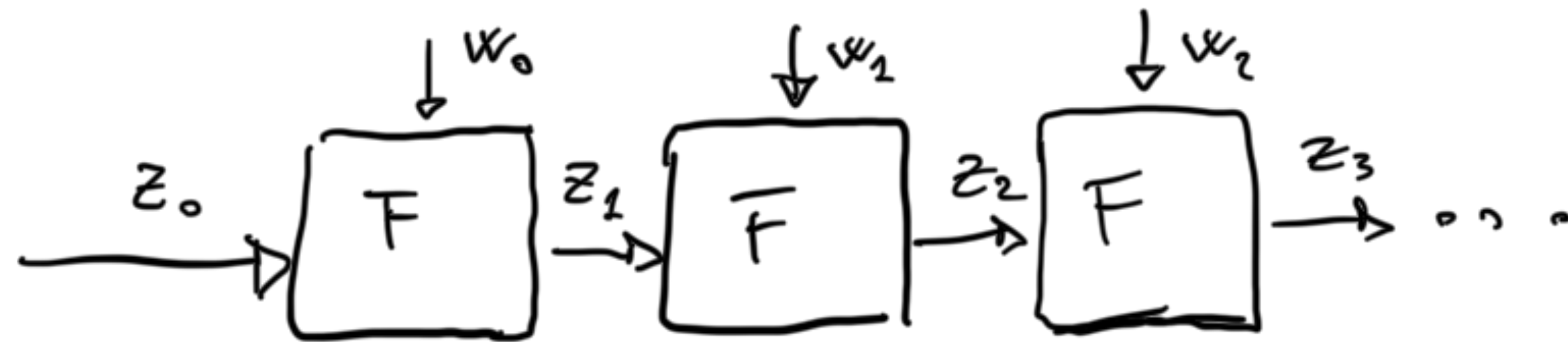
Advantages

- Low memory footprint
- Pipelining opportunities



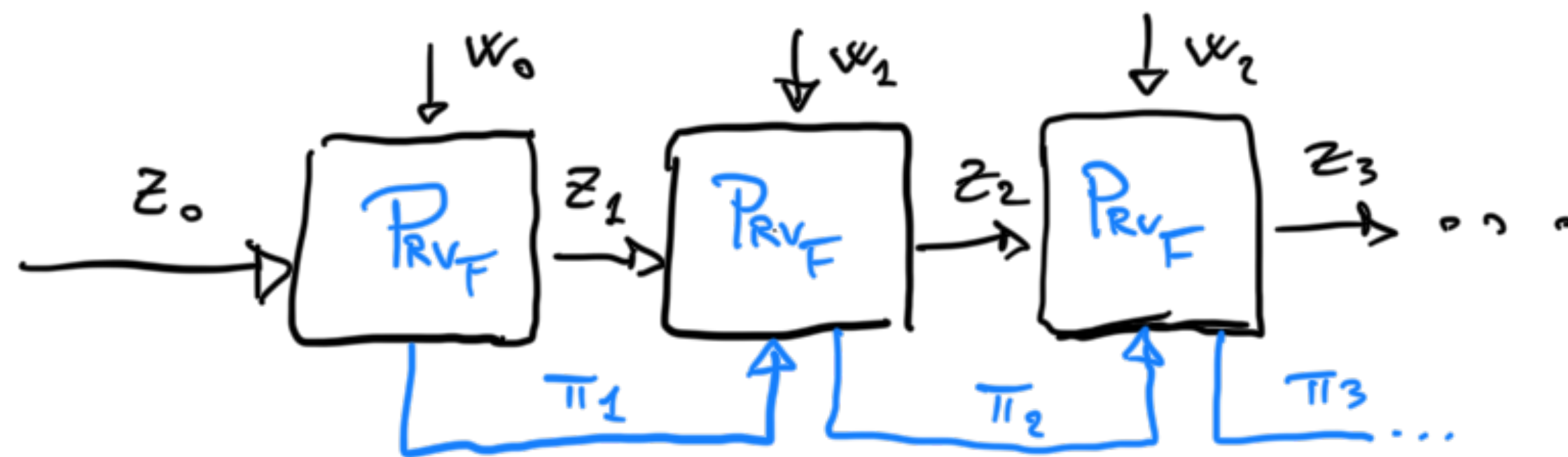
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Incrementally Verifiable Computations (IVC)



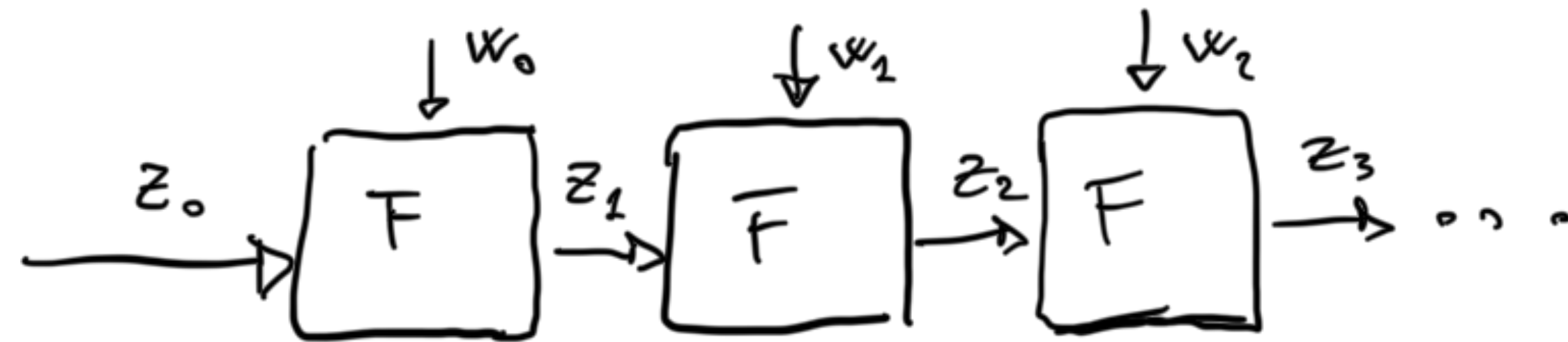
Advantages

- Low memory footprint
- Pipelining opportunities
- Natural model for incremental computations



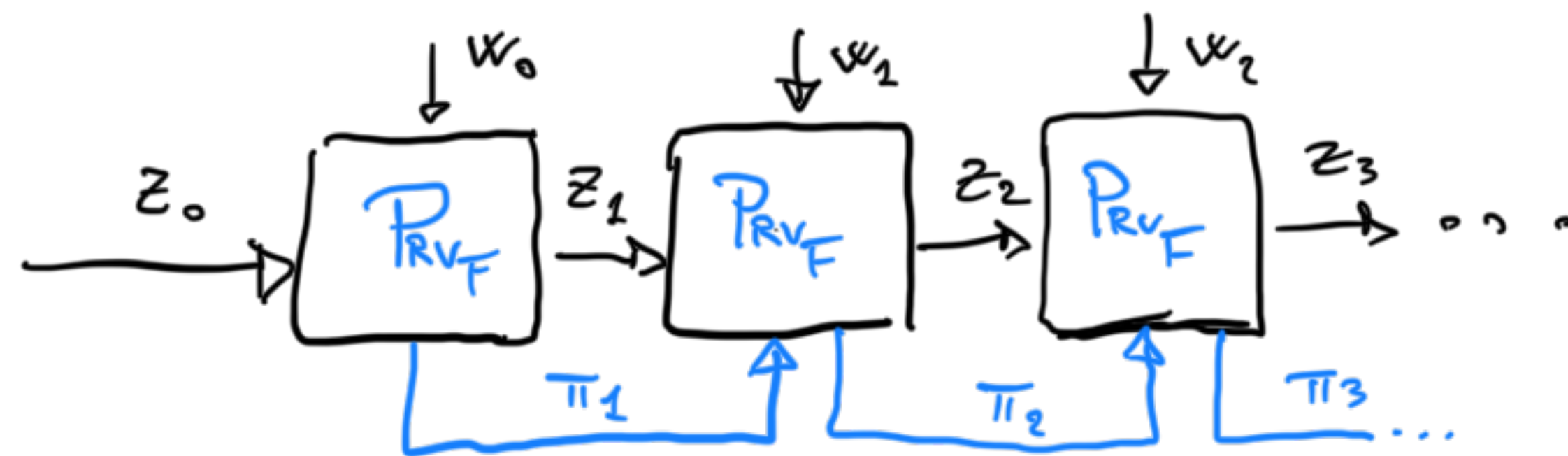
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Incrementally Verifiable Computations (IVC)



Advantages

- Low memory footprint
- Pipelining opportunities
- Natural model for incremental computations
- Proofs can be distributed (e.g., in settings with zero-knowledge)



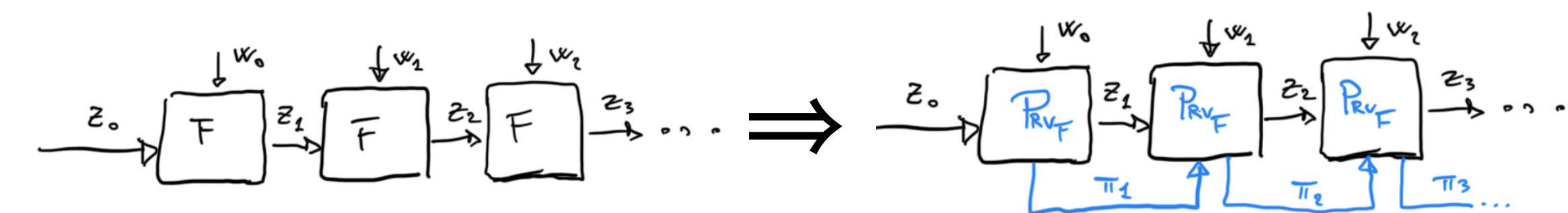
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Constructions of IVC

(Practical or nearly-practical)

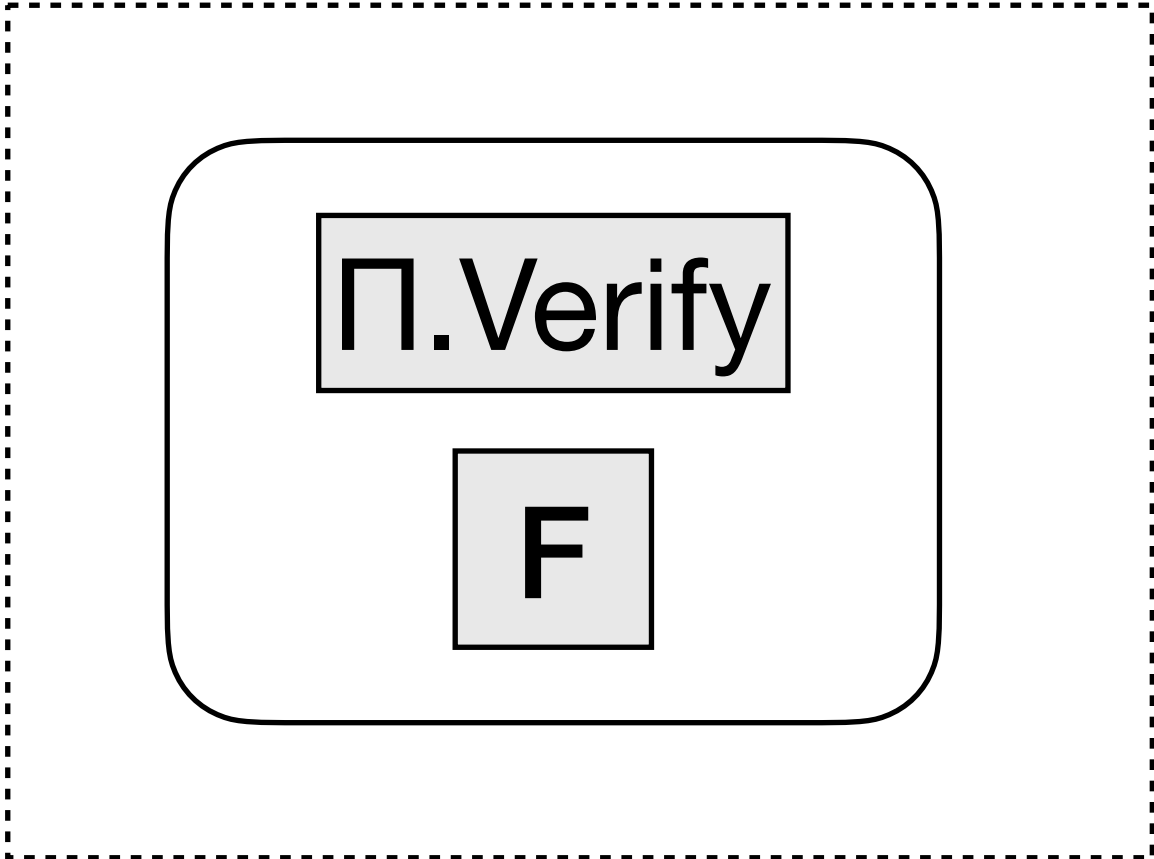
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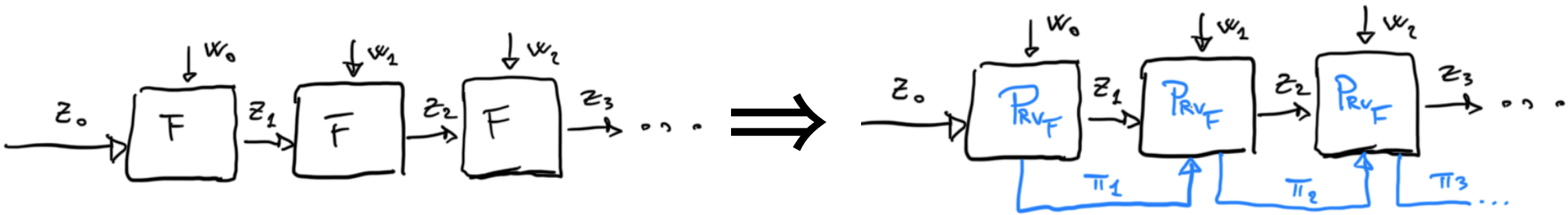
Constructions of IVC

(Practical or nearly-practical)



SNARK Prover's statement (Π is a SNARK)

Canonical construction (SNARK recursion)



Constructions of IVC

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Nova: Recursive Zero-Knowledge Arguments
from Folding Schemes

Abhiram Kothapalli[†]

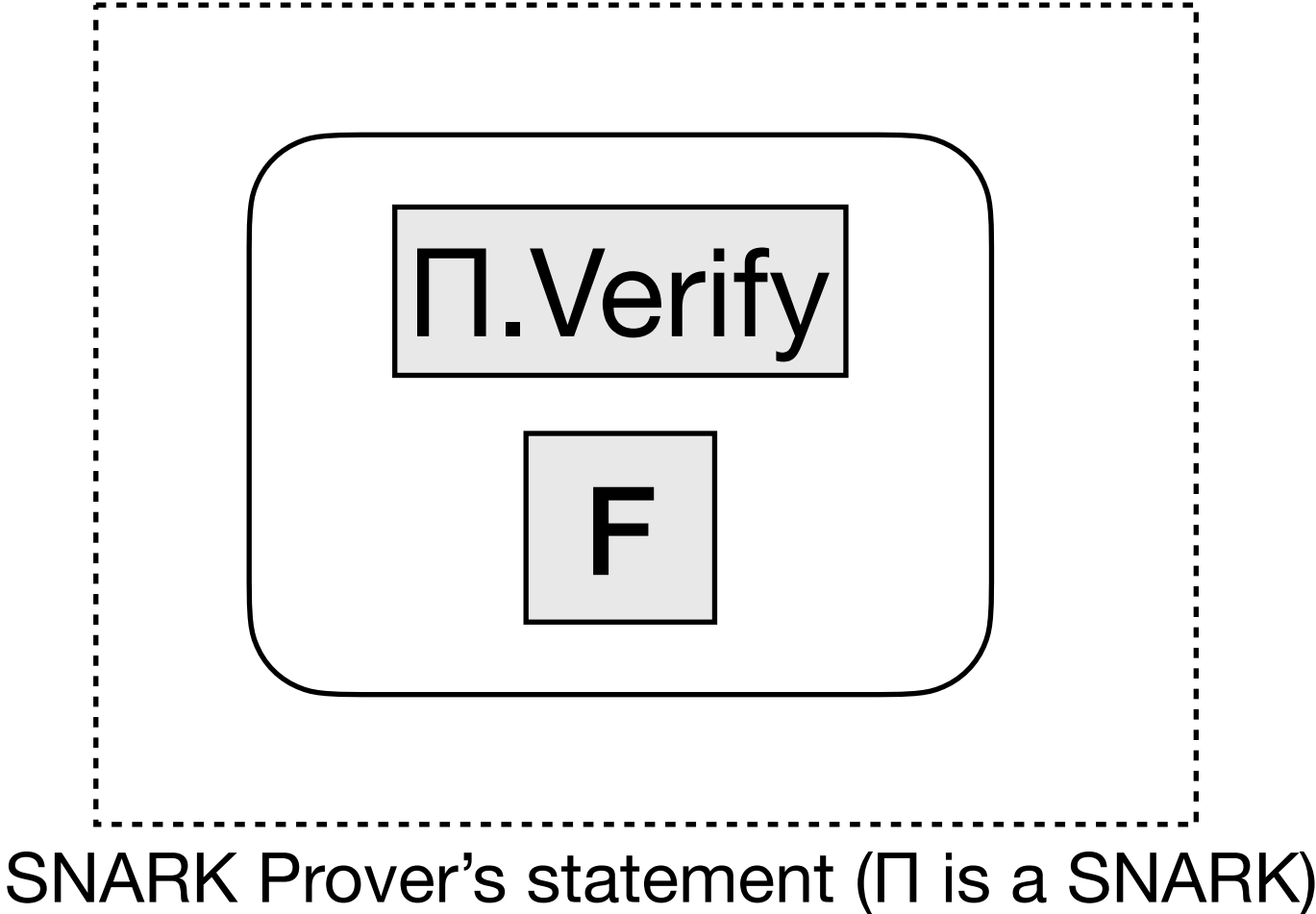
Srinath Setty^{*}

Ioanna Tzialla[‡]

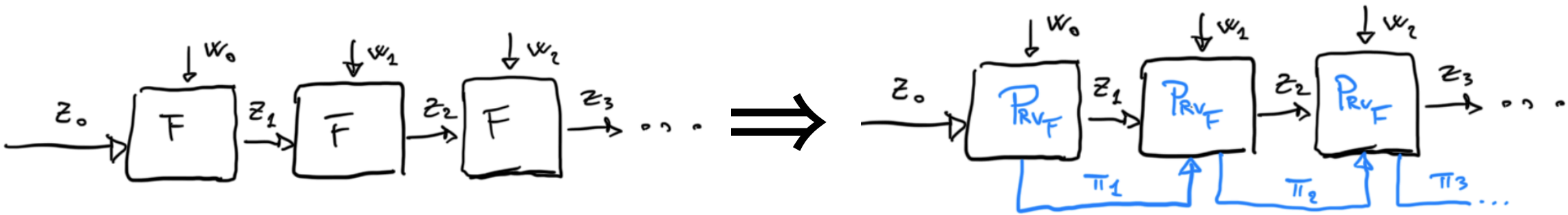
[†]Carnegie Mellon University

^{*}Microsoft Research

[‡]New York University



Canonical construction (SNARK recursion)



Constructions of IVC

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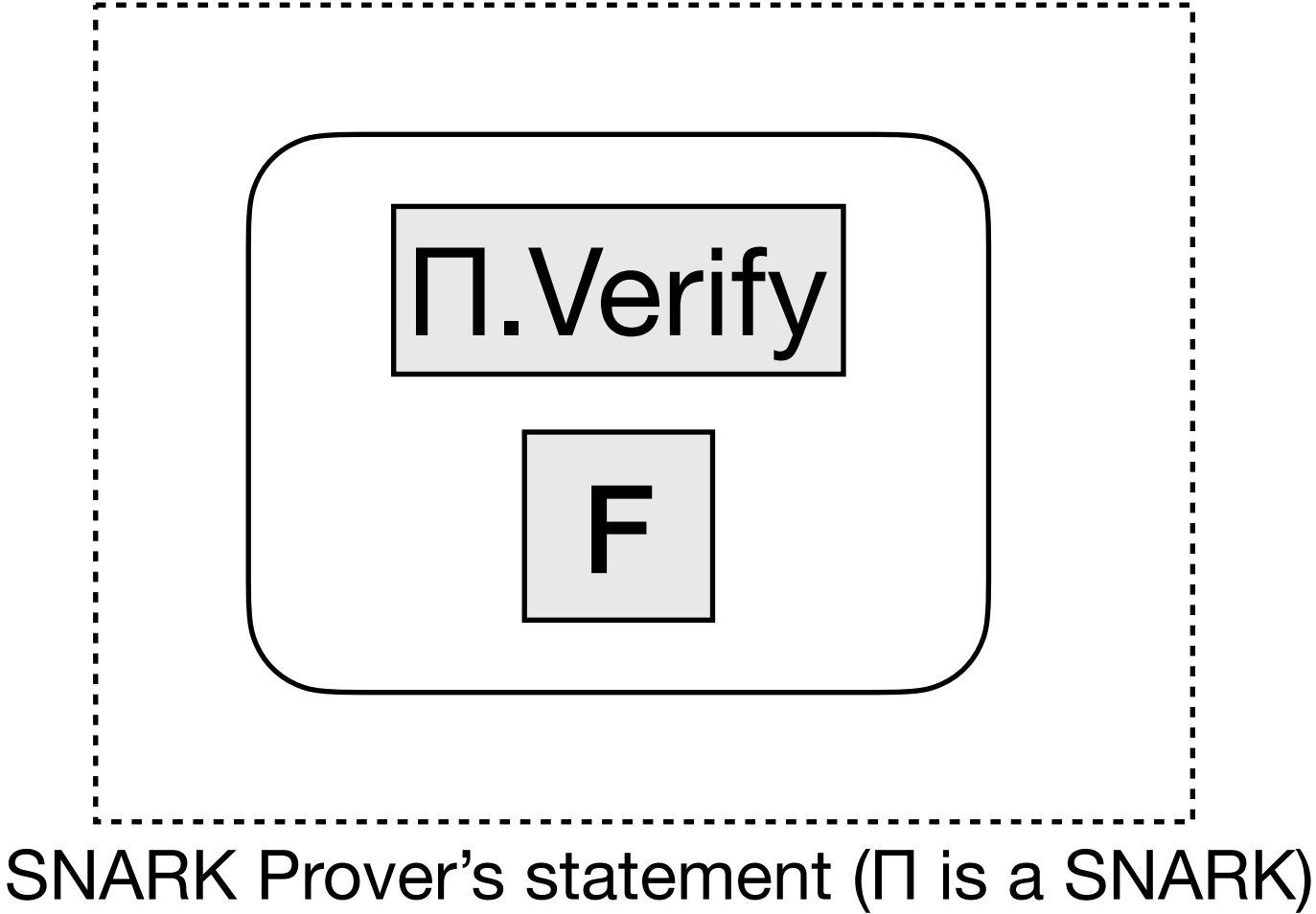
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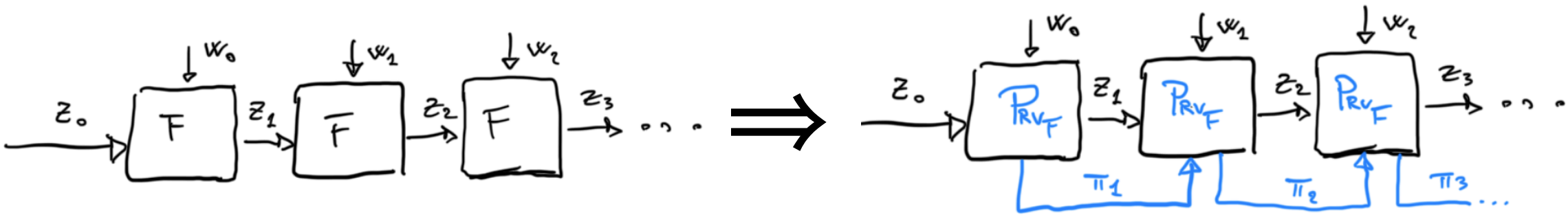
Proof-Carrying Data without Succinct Arguments

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Stanford University UC Berkeley

William Lin Pratyush Mishra Nicholas Spooner
will.lin@berkeley.edu pratyush@berkeley.edu nspooner@bu.edu
UC Berkeley UC Berkeley Boston University



Canonical construction
(SNARK recursion)



Constructions of IVC

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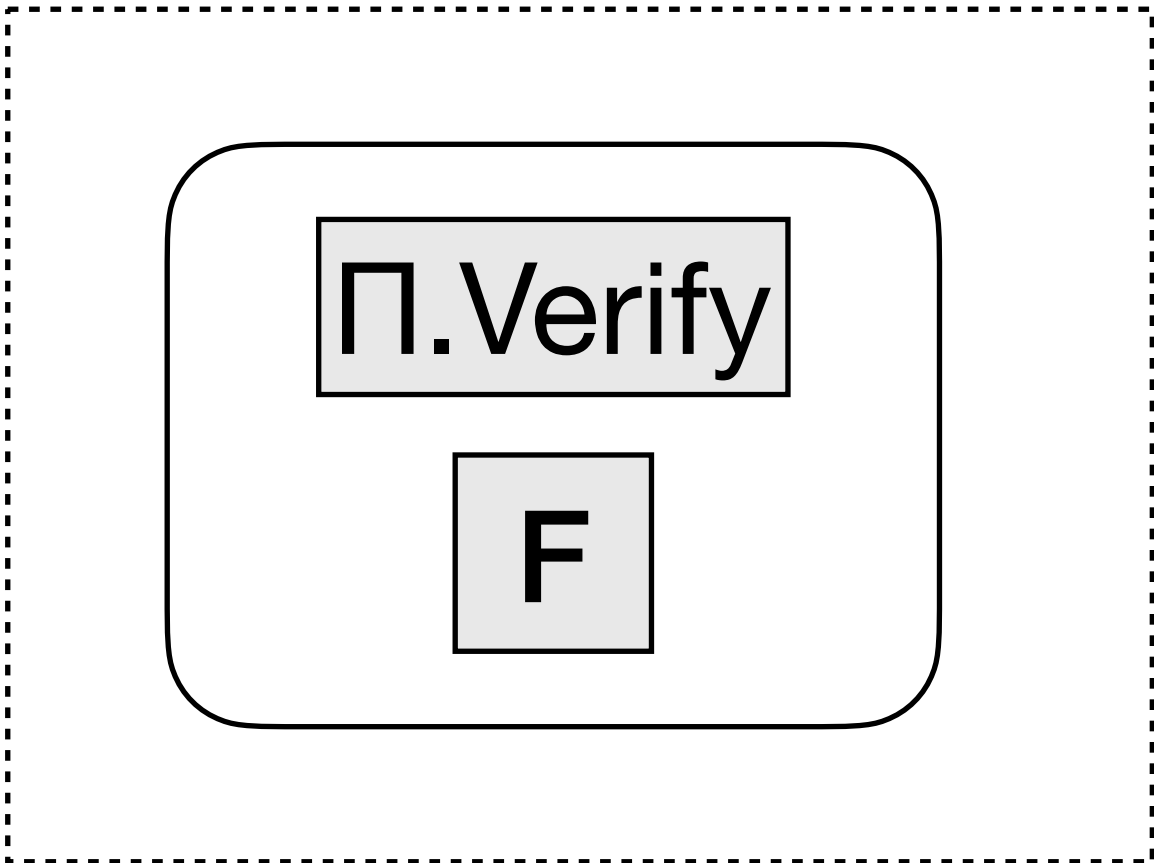
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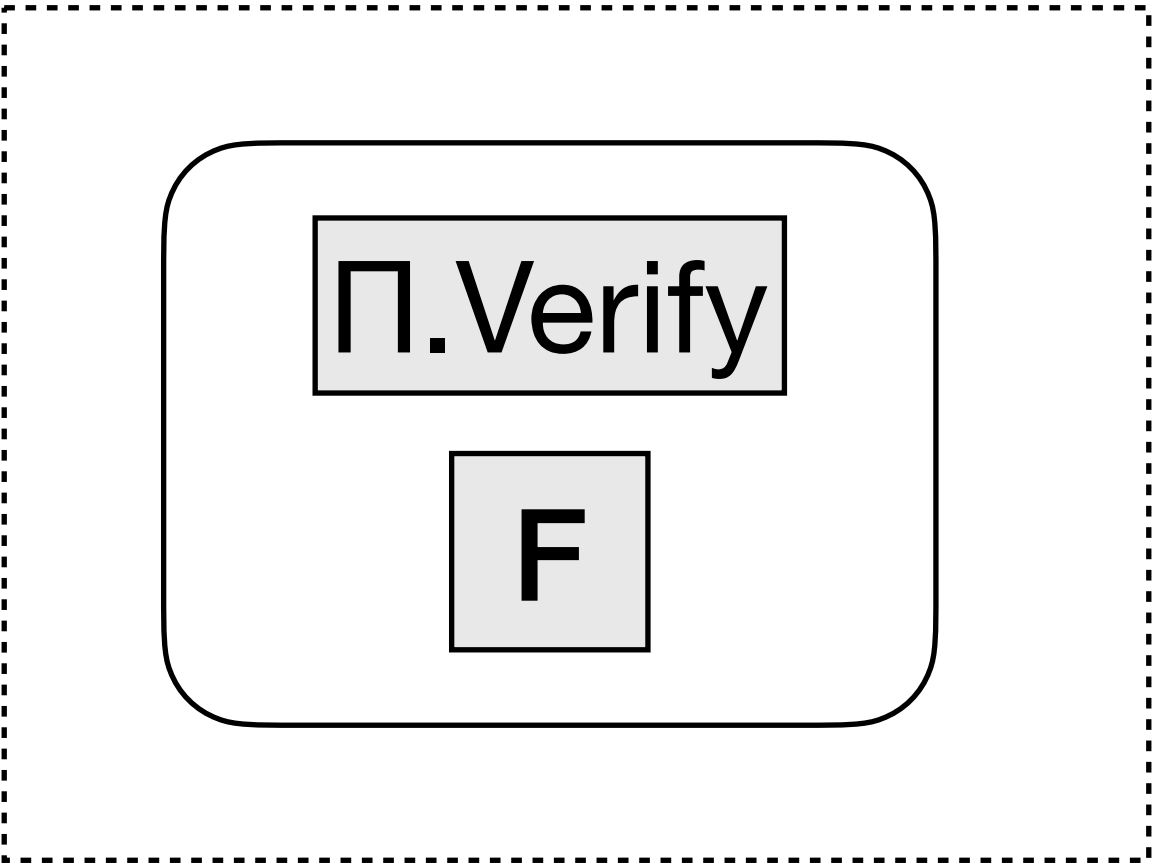
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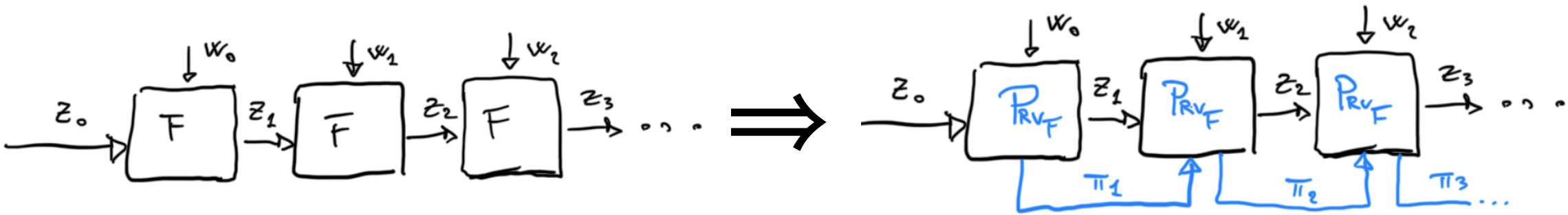
SNARK Prover's statement (Π is a SNARK)

Canonical construction
(SNARK recursion)



Folding/acc. Prover's statement (Π is a folding/acc. scheme)

Lightweight version
(folding/accumulation recursion)



* very approximate rendition (there are more details)

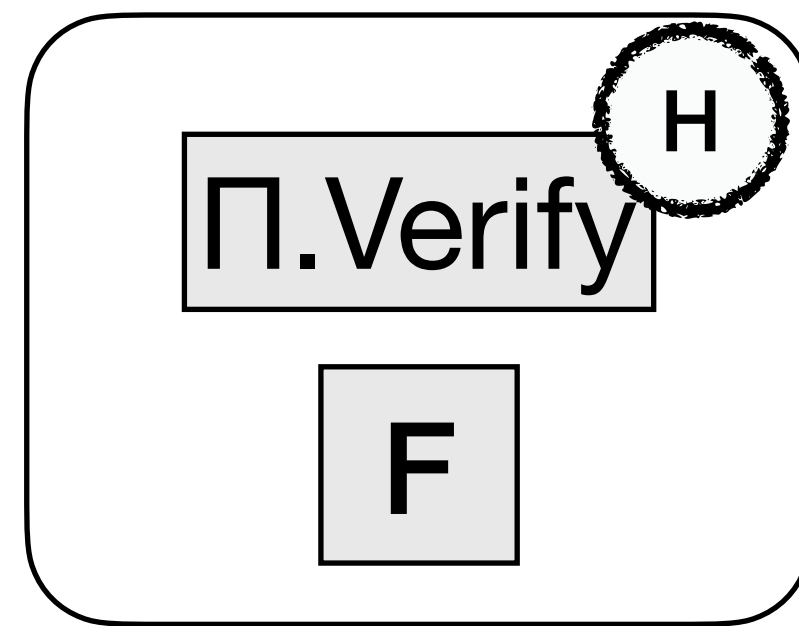
Challenges in Proving the Security of IVC

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- **First challenge:** idealized models and “theoretical hygiene”

Challenges in Proving the Security of IVC

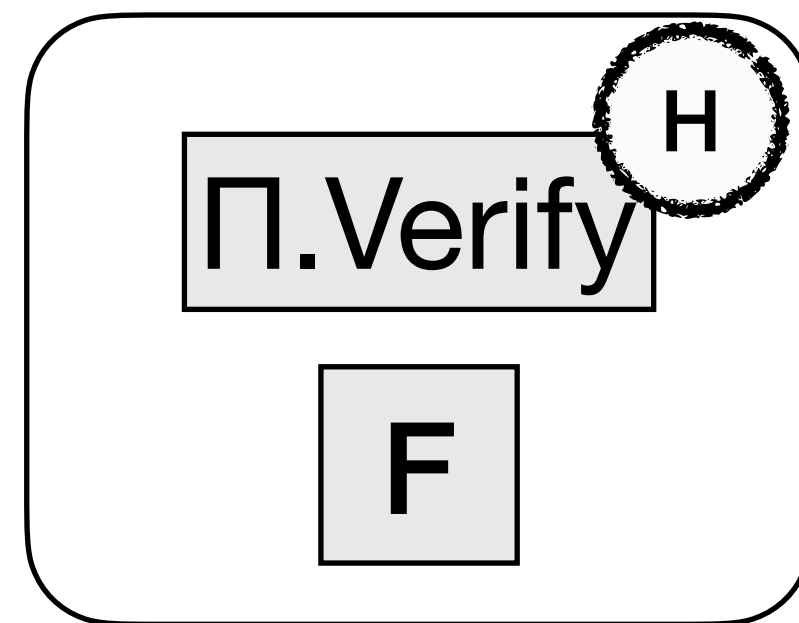
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Random Oracle

Challenges in Proving the Security of IVC

- **First challenge:** idealized models and “theoretical hygiene”



Random Oracle

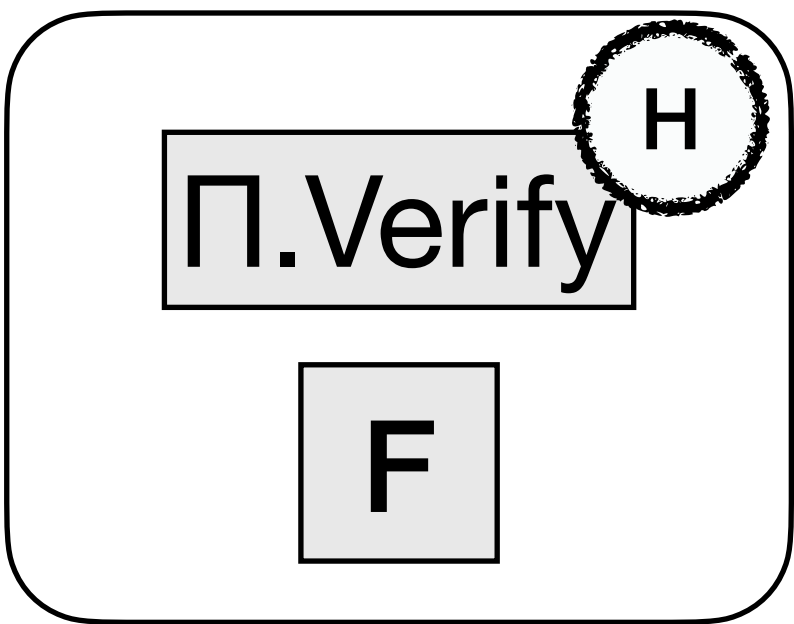
$$\mathcal{A}_{\text{AGM}}(g, g', \dots) \rightarrow \left(\overline{\Pi}, \text{EXPL.} \right)$$

Below the first argument g is the notation $\bigcap \mathbb{G}$. Below the second argument g' is the notation $\bigcap \mathbb{G}$. Below the output $\overline{\Pi}$ is the notation (h_1^2, h_2^2, \dots) . Below the output EXPL. is the notation $(\tilde{v}_1^2, \tilde{v}_2^2)$. Below (h_1^2, h_2^2, \dots) is $\bigcap \mathbb{G}$. Below $(\tilde{v}_1^2, \tilde{v}_2^2)$ is $\bigcap \mathbb{F}^k$.

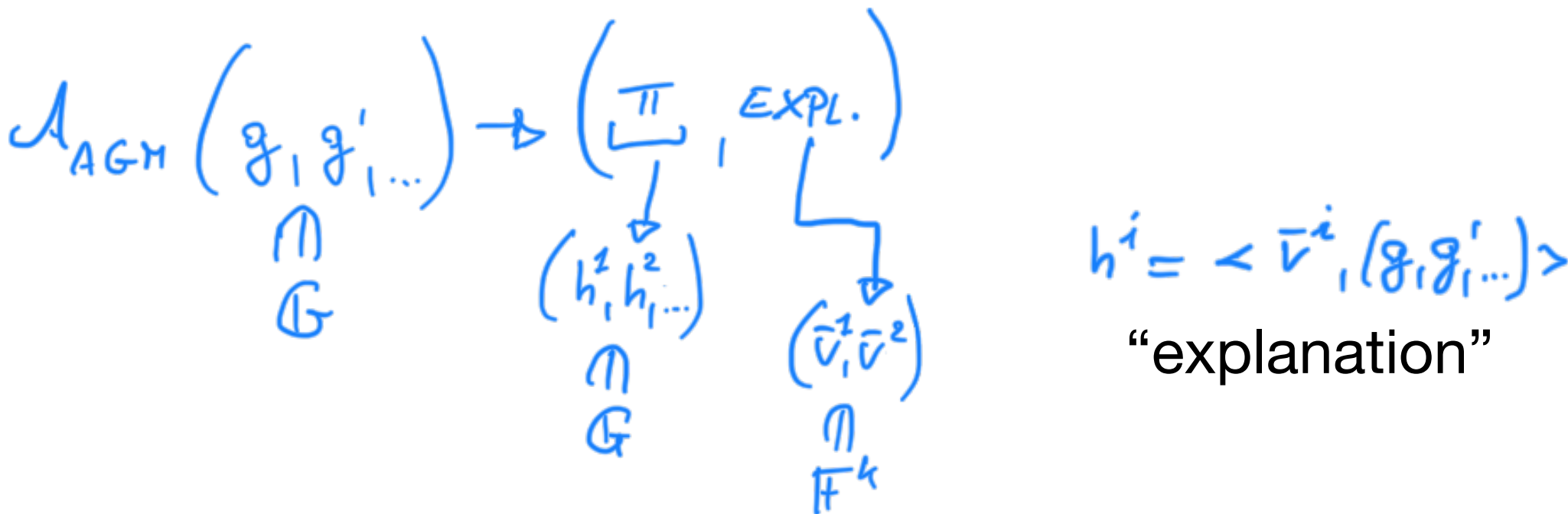
Algebraic Group Model (AGM)

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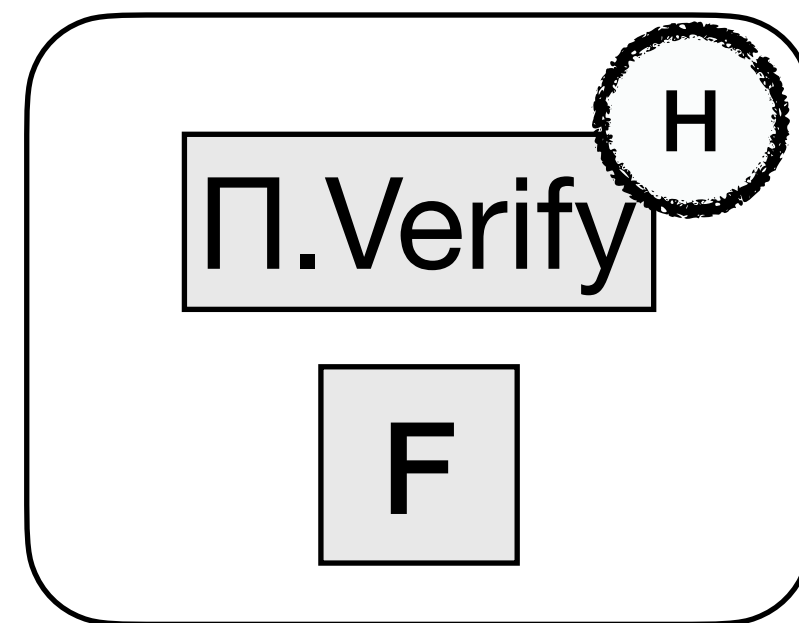
Algebraic Group Model (AGM)

$$h^i = \langle \bar{v}^i, (g, g', \dots) \rangle$$

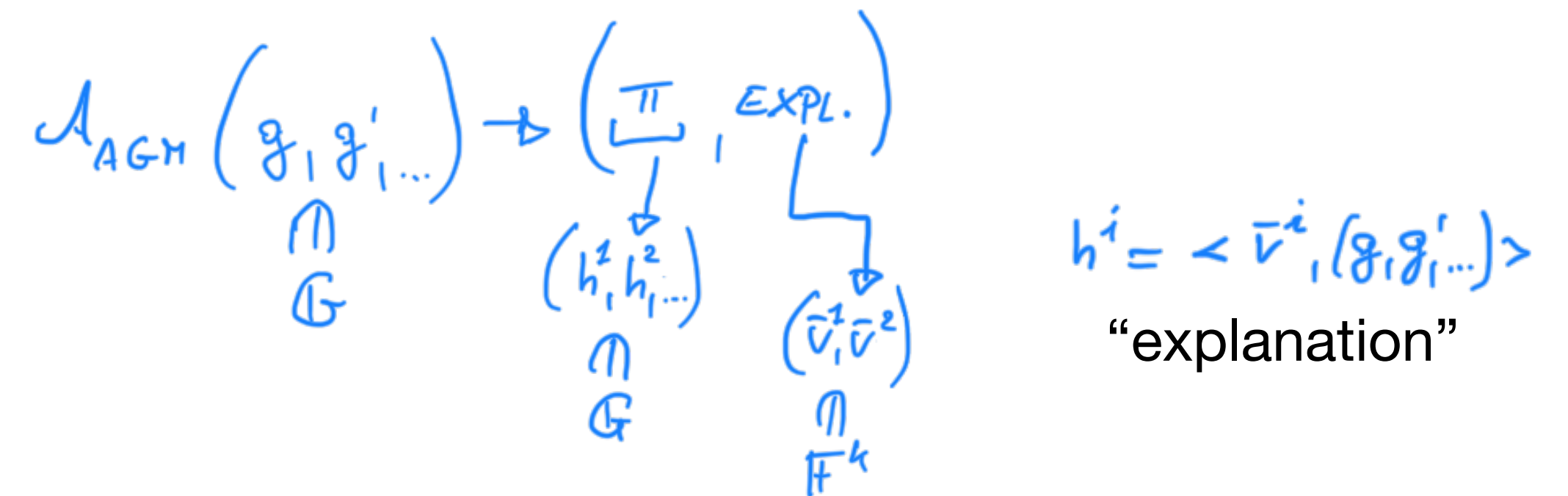
“explanation”

Challenges in Proving the Security of IVC

- **First challenge:** idealized models and “theoretical hygiene”



Random Oracle



Algebraic Group Model (AGM)

- **Second challenge (our focus):** depth of the computation

How Do We Usually Prove Security in IVC?

A glimpse of what can go wrong and what depth has to do with it

$$\mathcal{A}_{\text{ive}} \rightarrow (z_0, z_d, d, \pi_d)$$

$$z_0 \xrightarrow[w_0]{F} z_1 \xrightarrow[w_1]{F} \dots \xrightarrow[w_{d-2}]{F} z_{d-1} \xrightarrow[w_{d-1}]{F} z_d$$

How Do We Usually Prove Security in IVC?

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IVC extractability

$$\forall \text{PPT } \mathcal{A}_{\text{ive}} \exists \mathcal{E}_{\text{ive}}:$$

$$\left[\begin{array}{l} \mathcal{A}_{\text{ive}} \rightarrow (z_0, z_d, d, \pi_d) \\ \mathcal{E}_{\text{ive}} \rightarrow (w_0, \dots, w_{d-1}) \\ (z_0, z_d, d, \pi_d) \xrightarrow{\text{verify}} z_0 \xrightarrow[w_0, \dots, w_{d-1}]{F} z_d \end{array} \right]$$

$$\mathcal{A}_{\text{ive}} \rightarrow (z_0, z_d, d, \pi_d)$$

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How Do We Usually Prove Security in IVC?

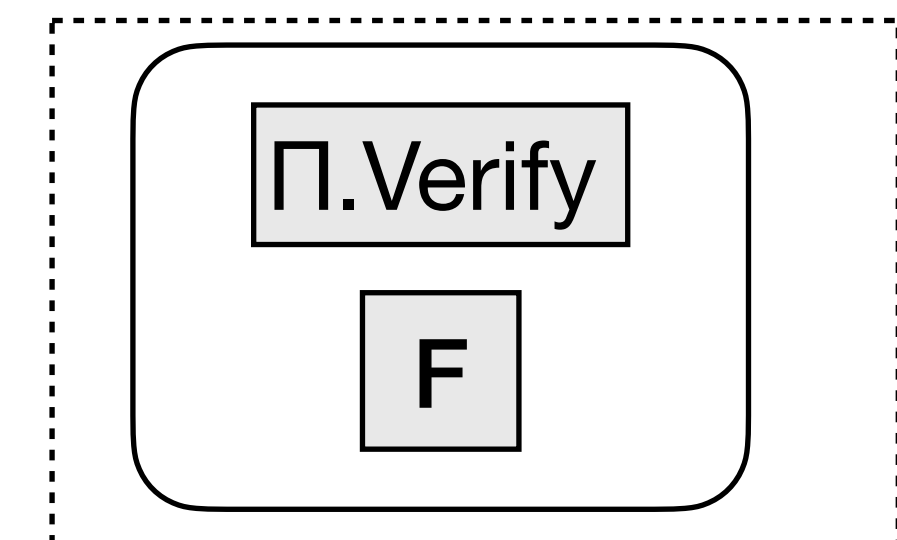
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SNARK Prover's statement (Π is a SNARK)

How Do We Usually Prove Security in IVC?

A glimpse of what can go wrong and what depth has to do with it

IVC extractability

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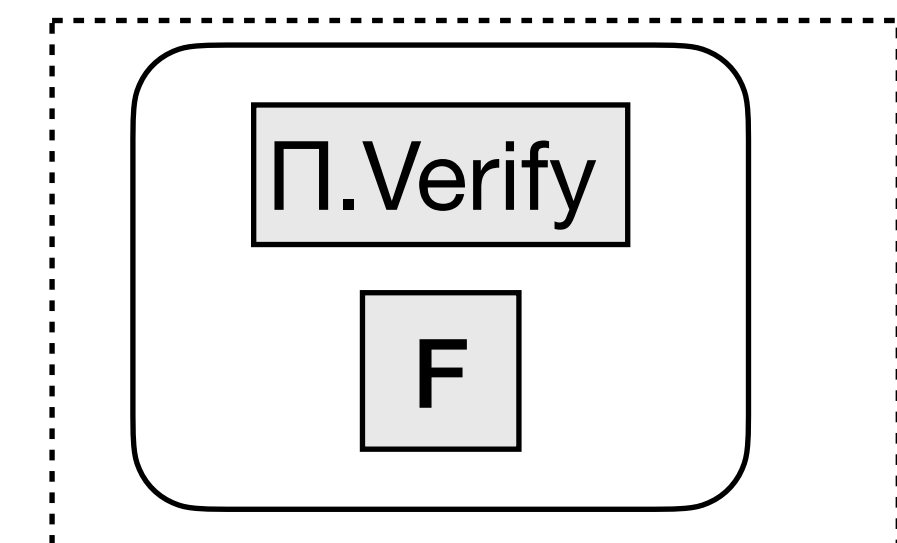
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SNARK extractability

$$\forall \text{PPT } \mathcal{A}_{\pi} \exists \mathcal{E}_{\pi}^{(\text{poly time})} :$$

$$\left[\begin{array}{l} \mathcal{A}_{\pi} \rightarrow (x, \pi) \\ \mathcal{E}_{\pi} \rightarrow w \\ (x, \pi) \xrightarrow{\text{verify}} (x, w) \in R_F \end{array} \right.$$



SNARK Prover's statement (Π is a SNARK)

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A glimpse of what can go wrong and what depth has to do with it

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$$\mathcal{A}_{\text{IVE}} \rightarrow (z_0, z_d, d, \pi_d)$$

$$z_0 \xrightarrow[w_0]{F} z_1 \xrightarrow[w_1]{F} \dots \xrightarrow[w_{d-2}]{F} z_{d-1} \xrightarrow[w_{d-1}]{F} z_d$$

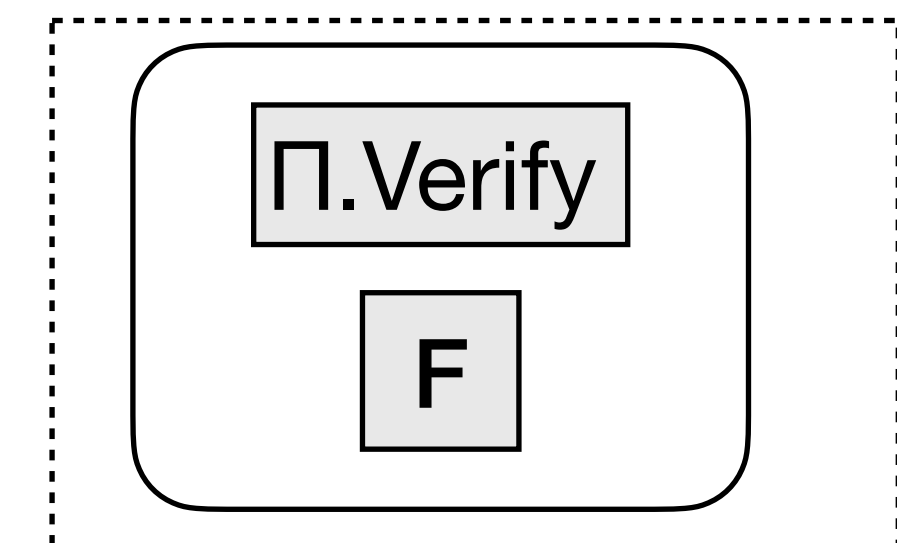
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$$|\mathcal{A}| = \tau \Rightarrow \mathcal{E}_{\mathcal{A}} = \tau^{-k}$$

w_{d-1}

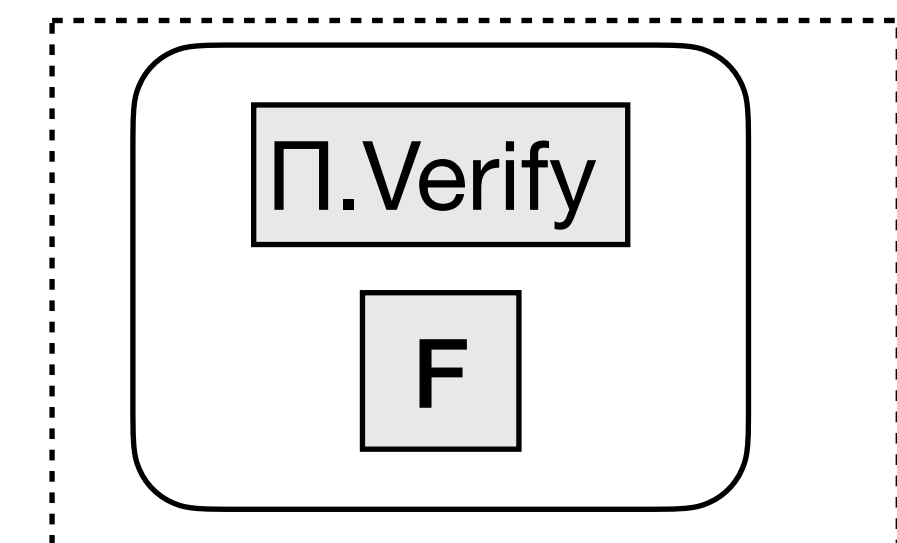
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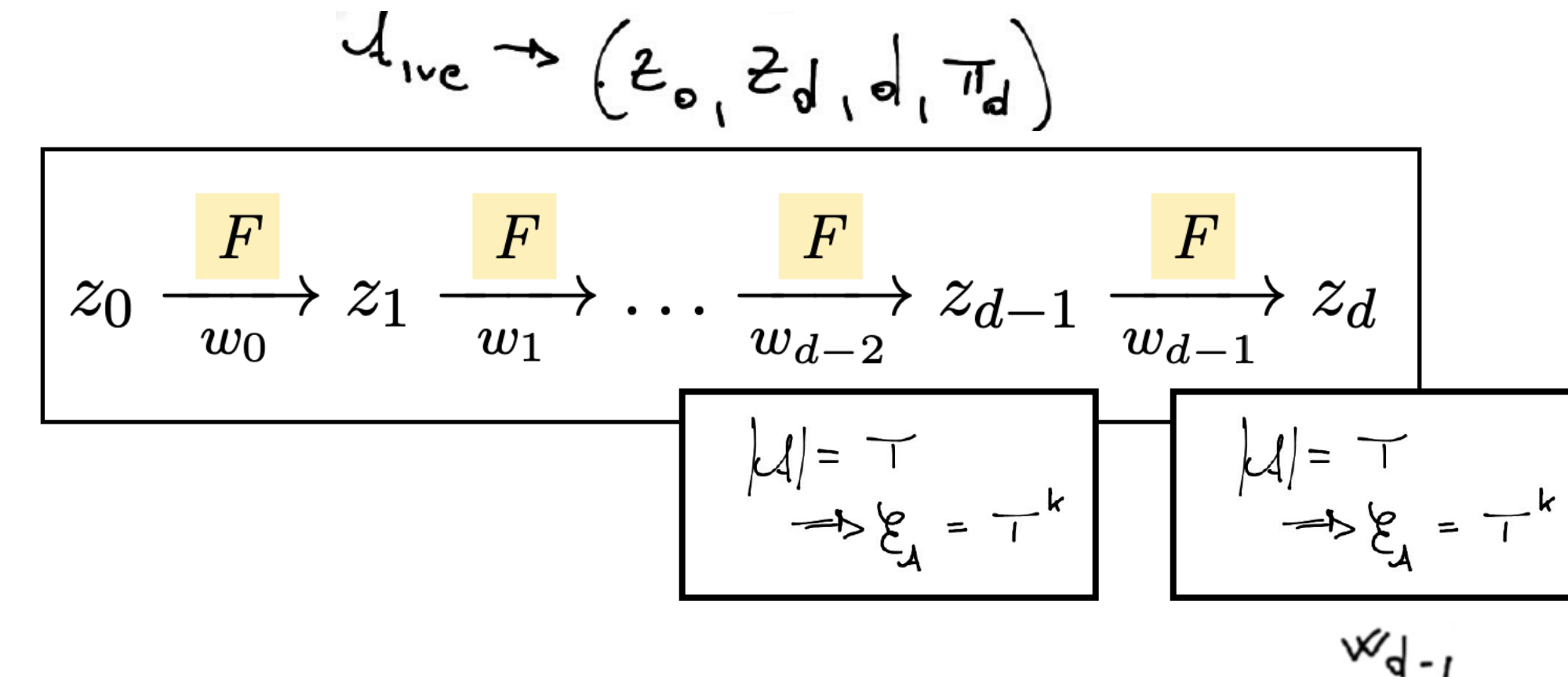
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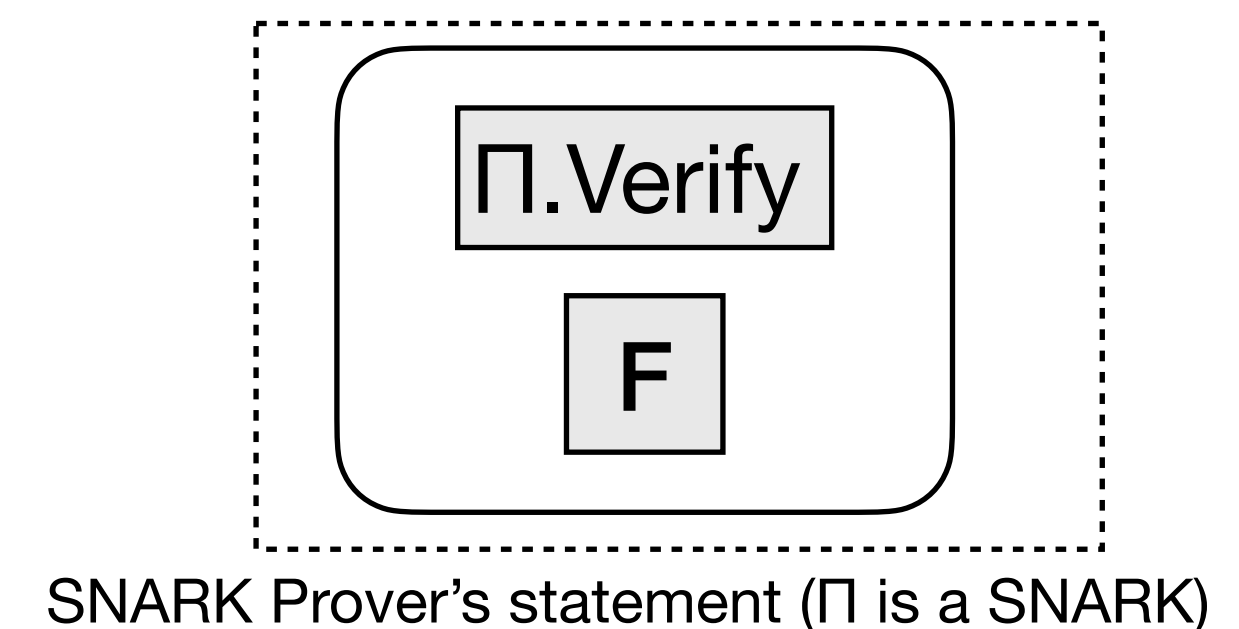
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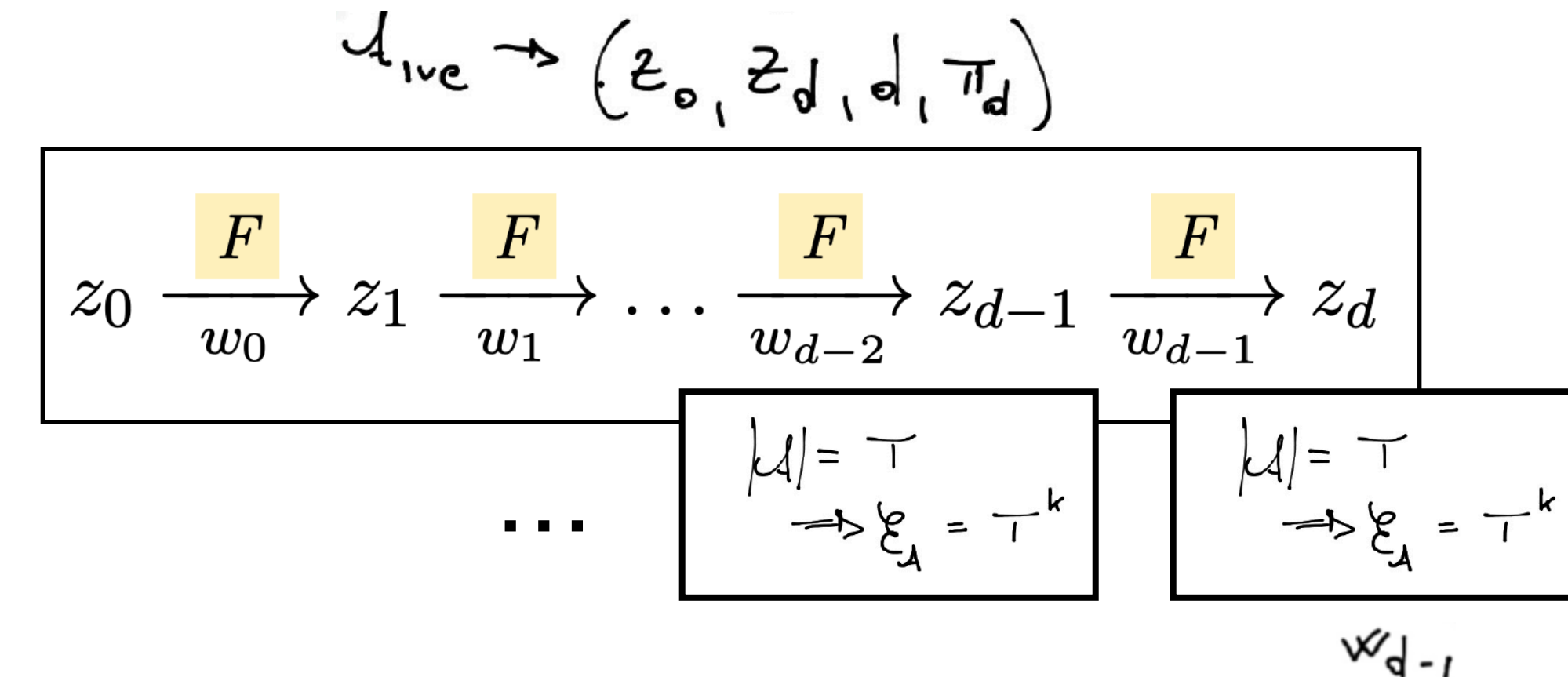
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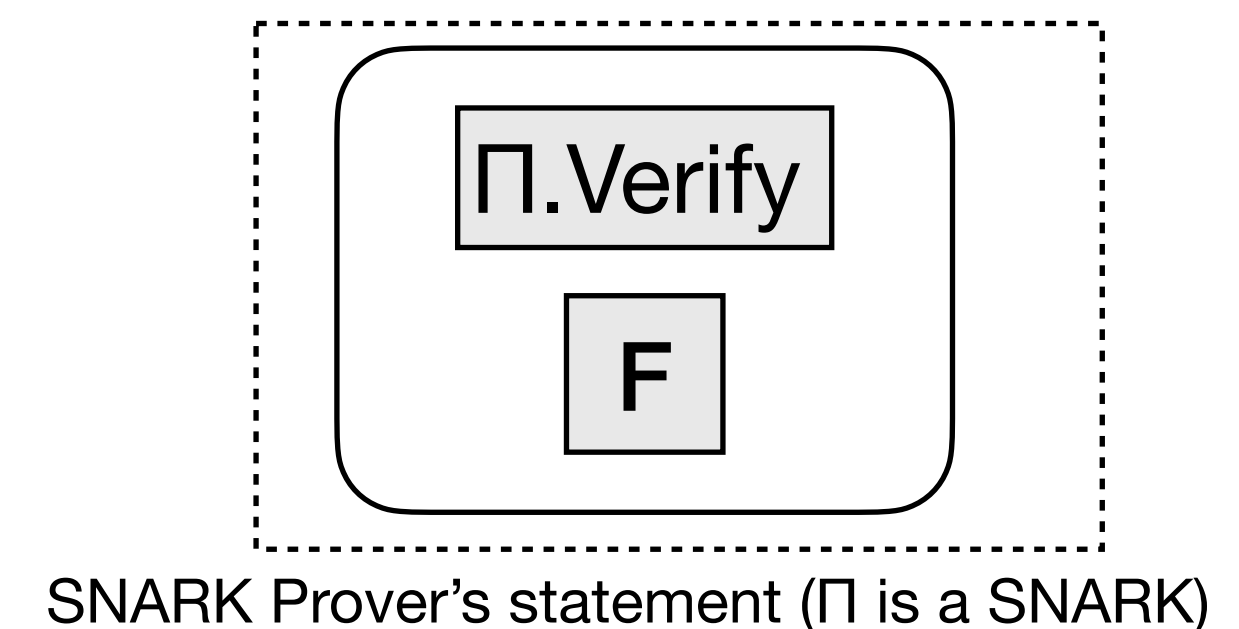
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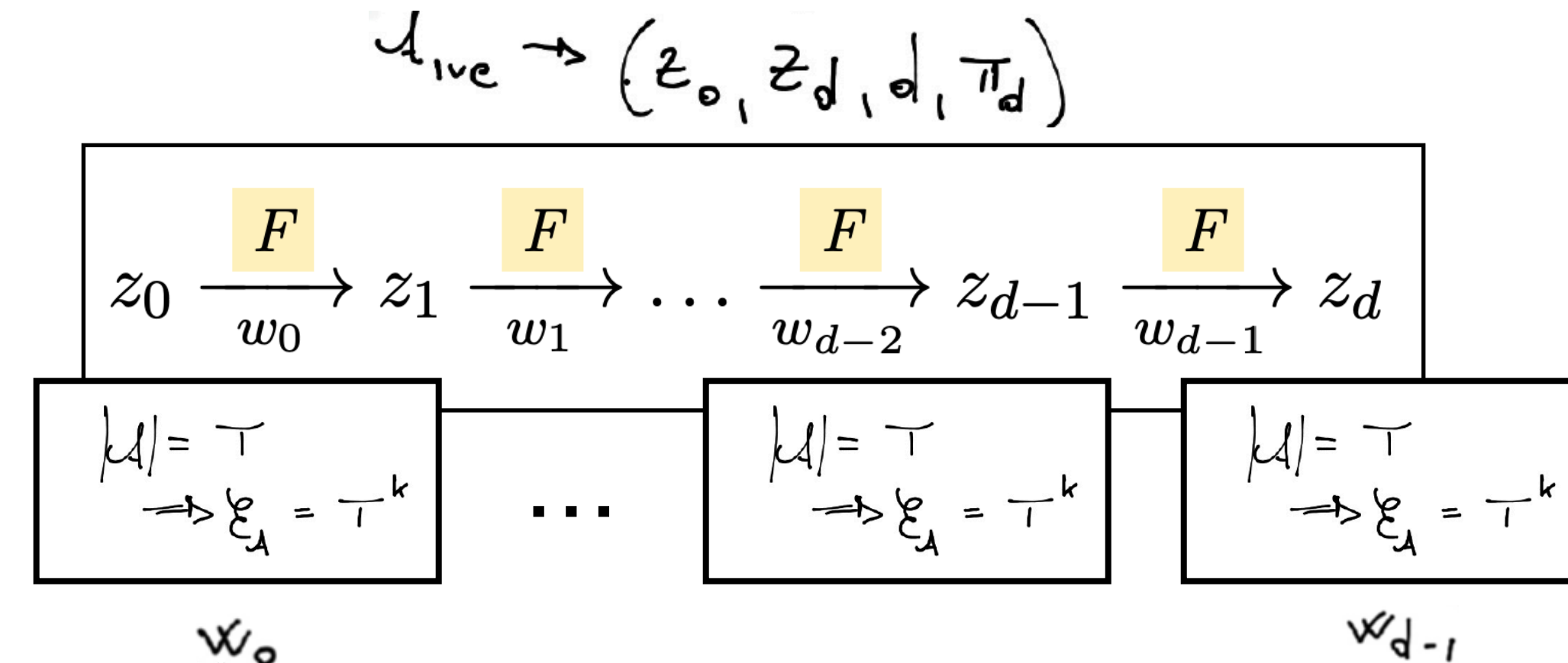
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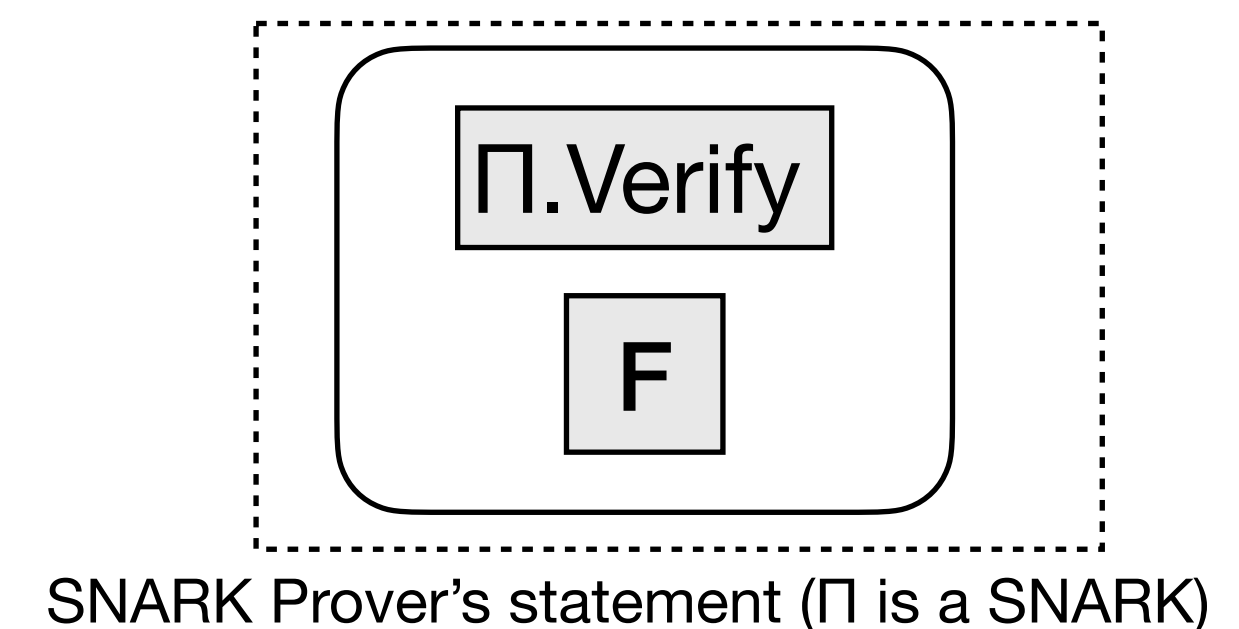
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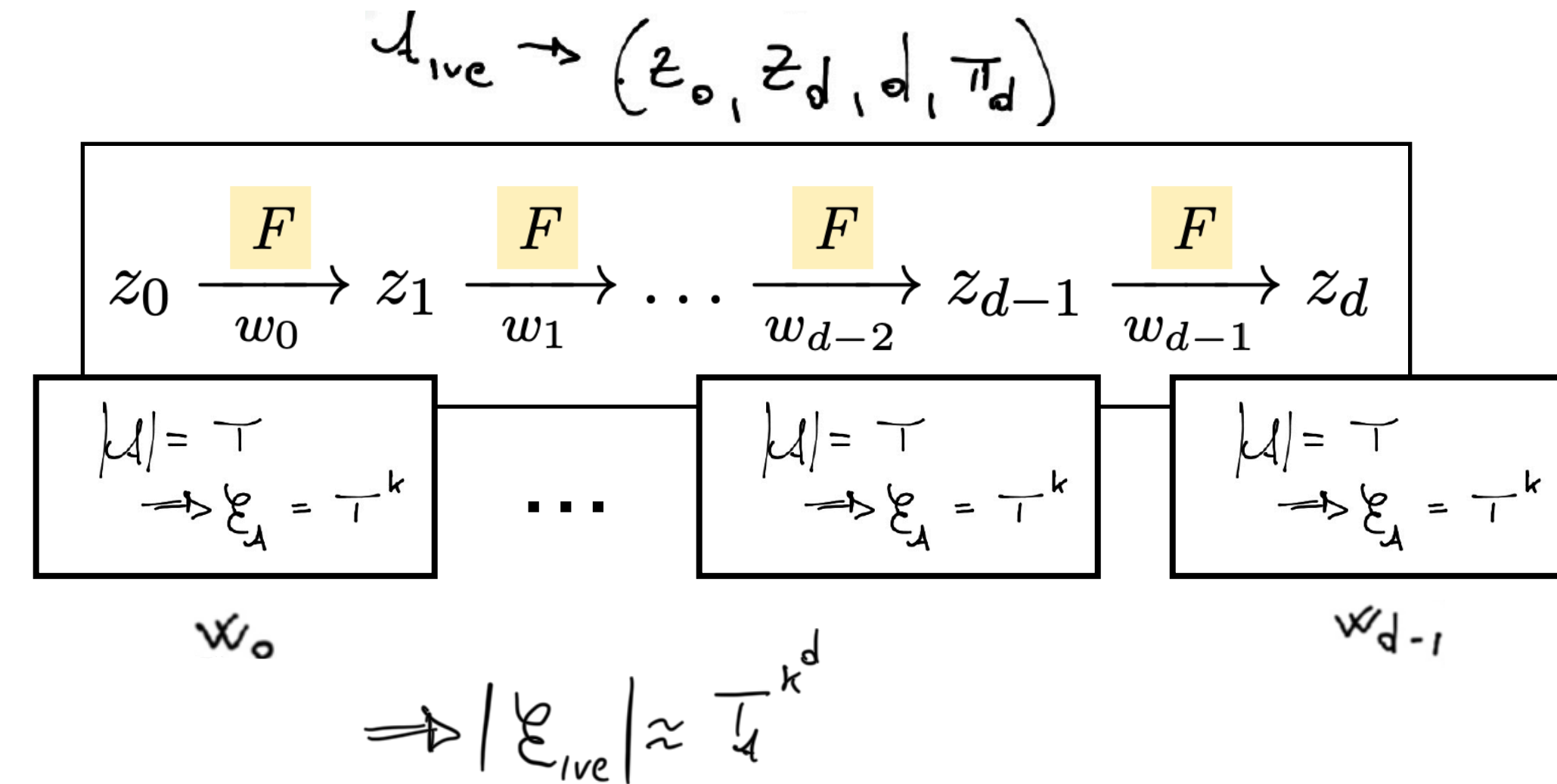
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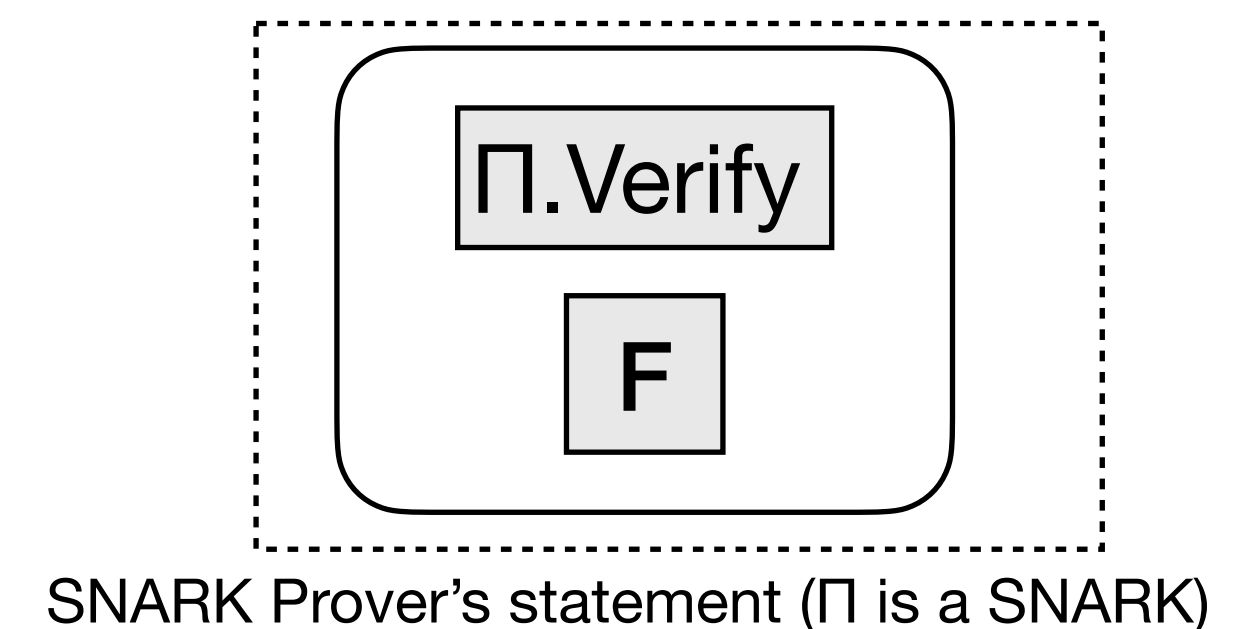
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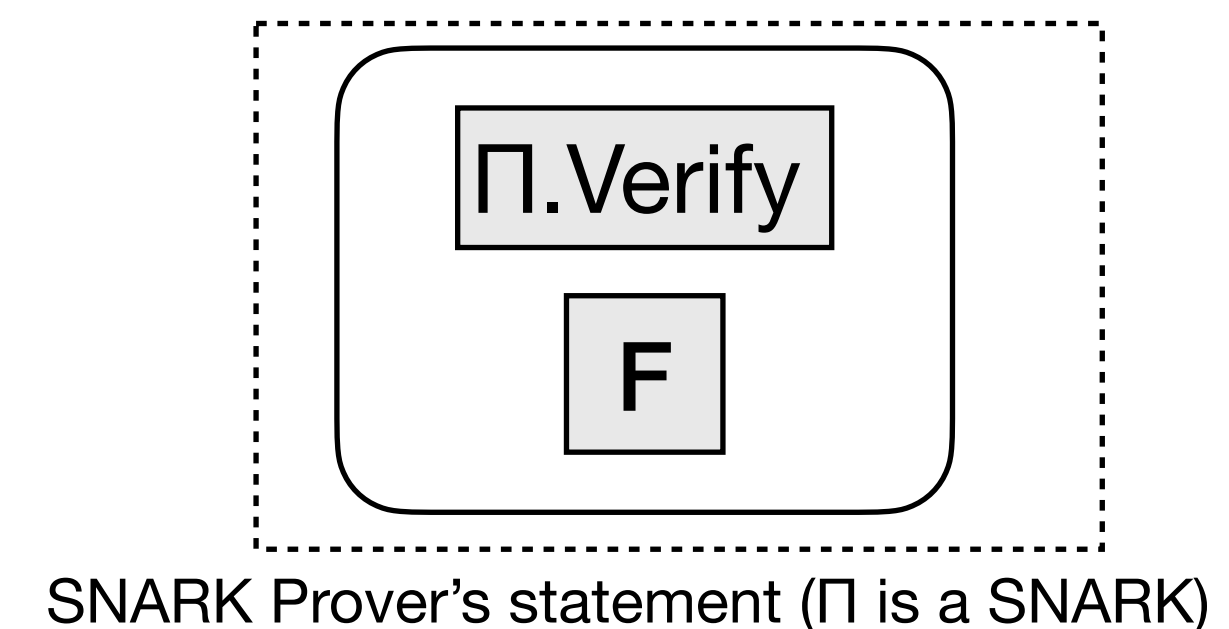
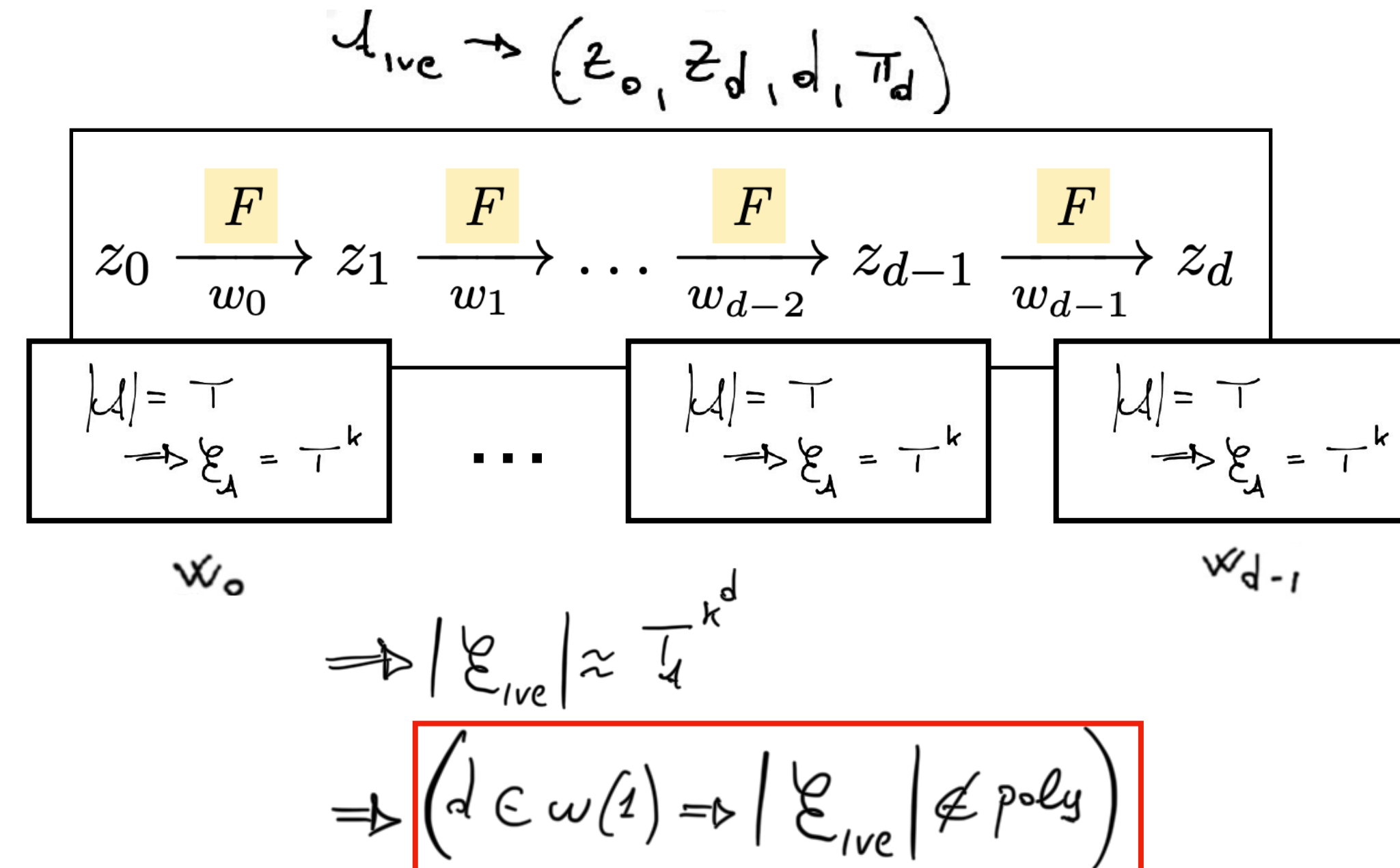
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The “Tree Approach”

A canonical way to go around the problem we just saw (via extractability)

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Recursive Composition and Bootstrapping
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Mangrove: A Scalable Framework for Folding-based SNARKs

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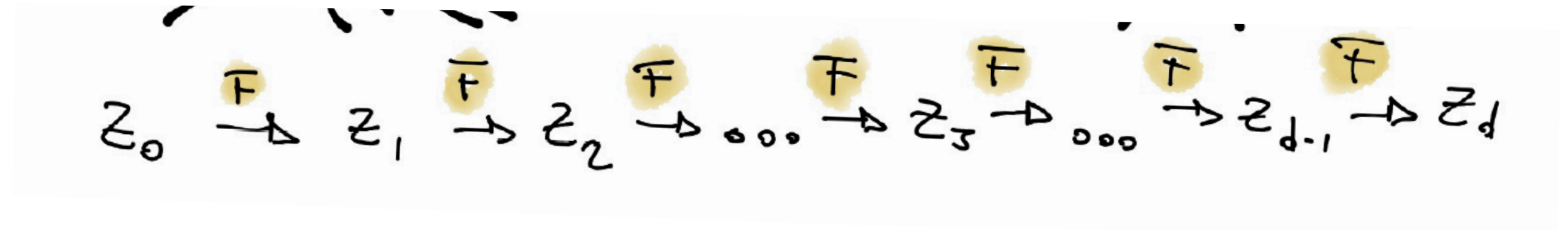
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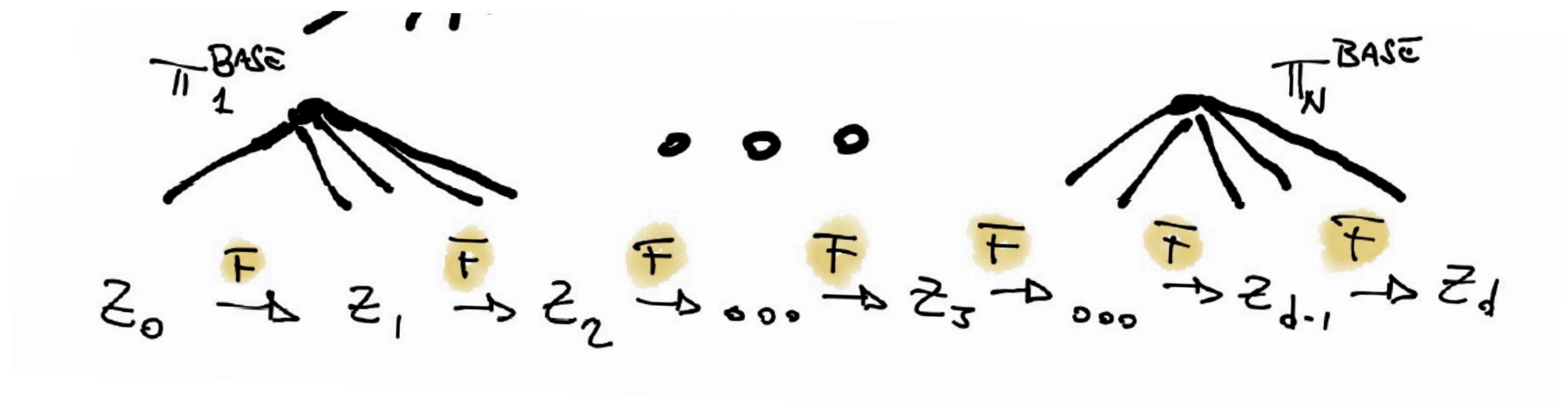
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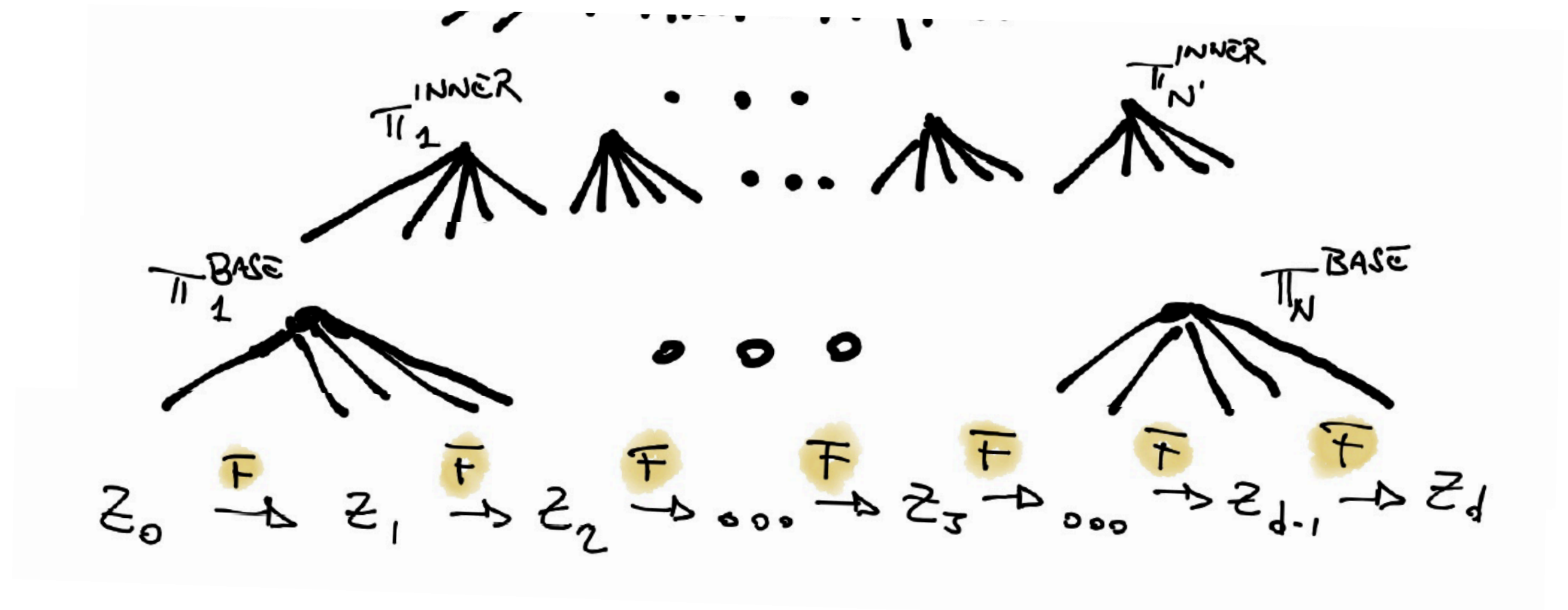
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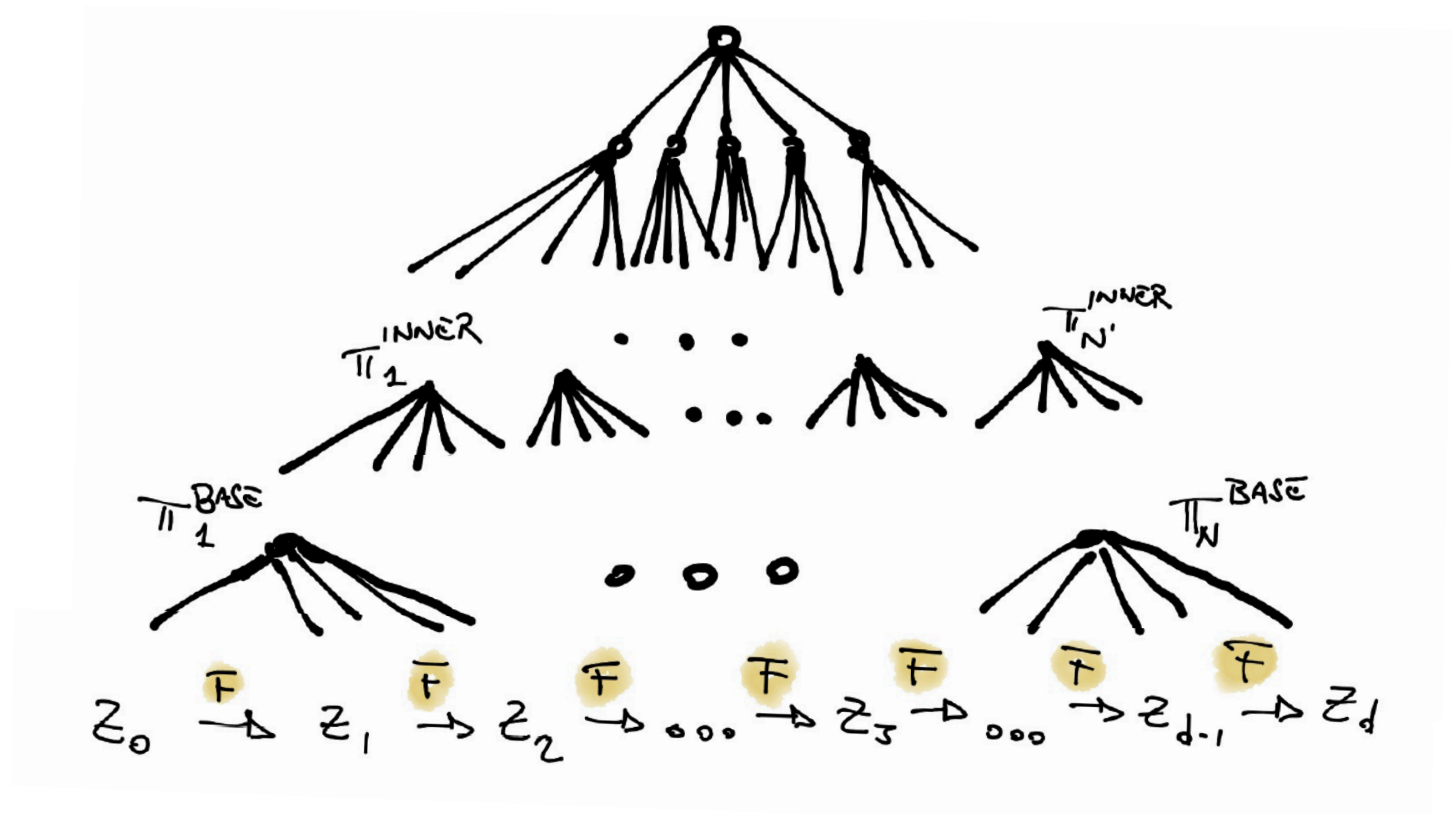
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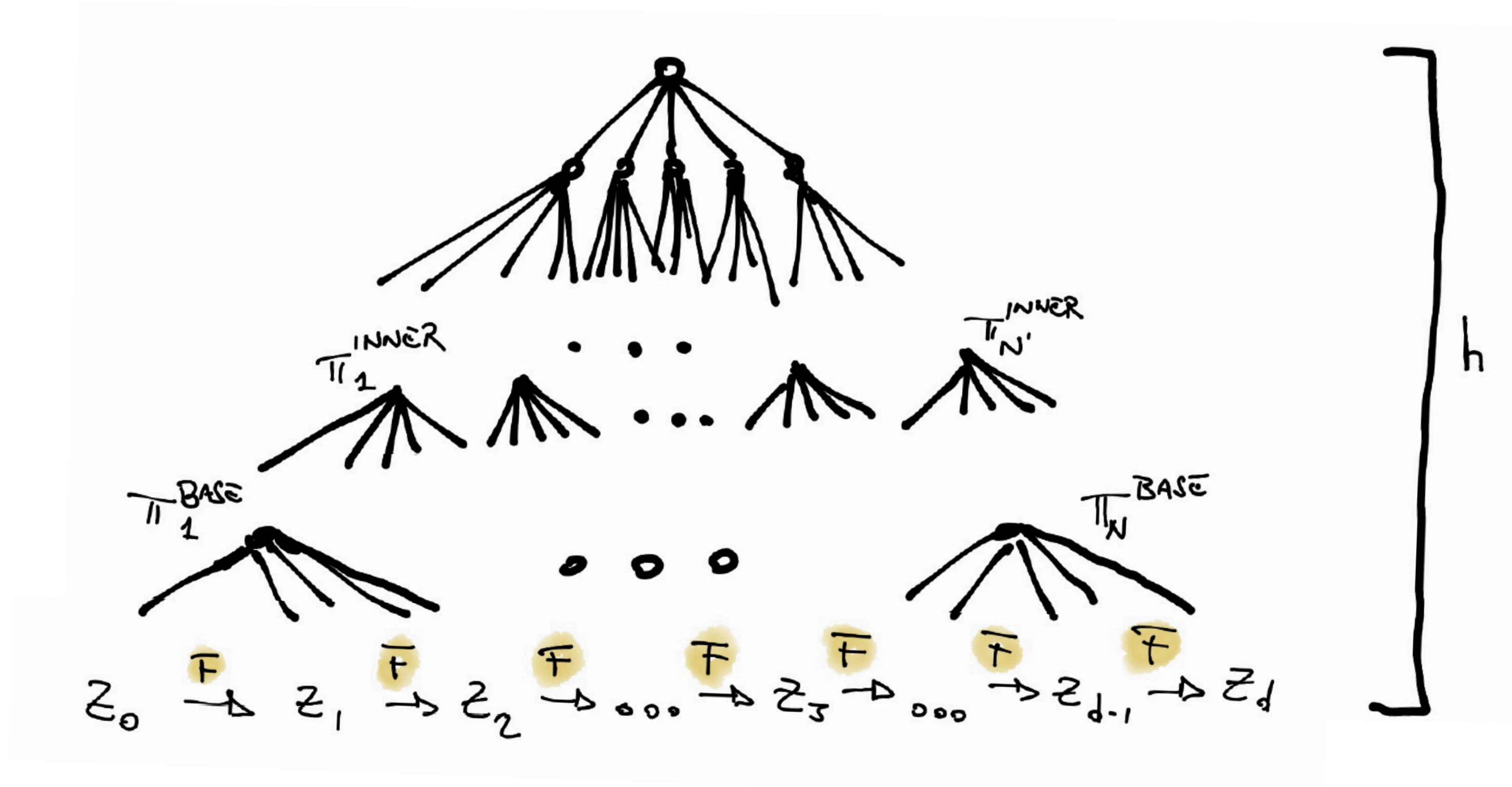
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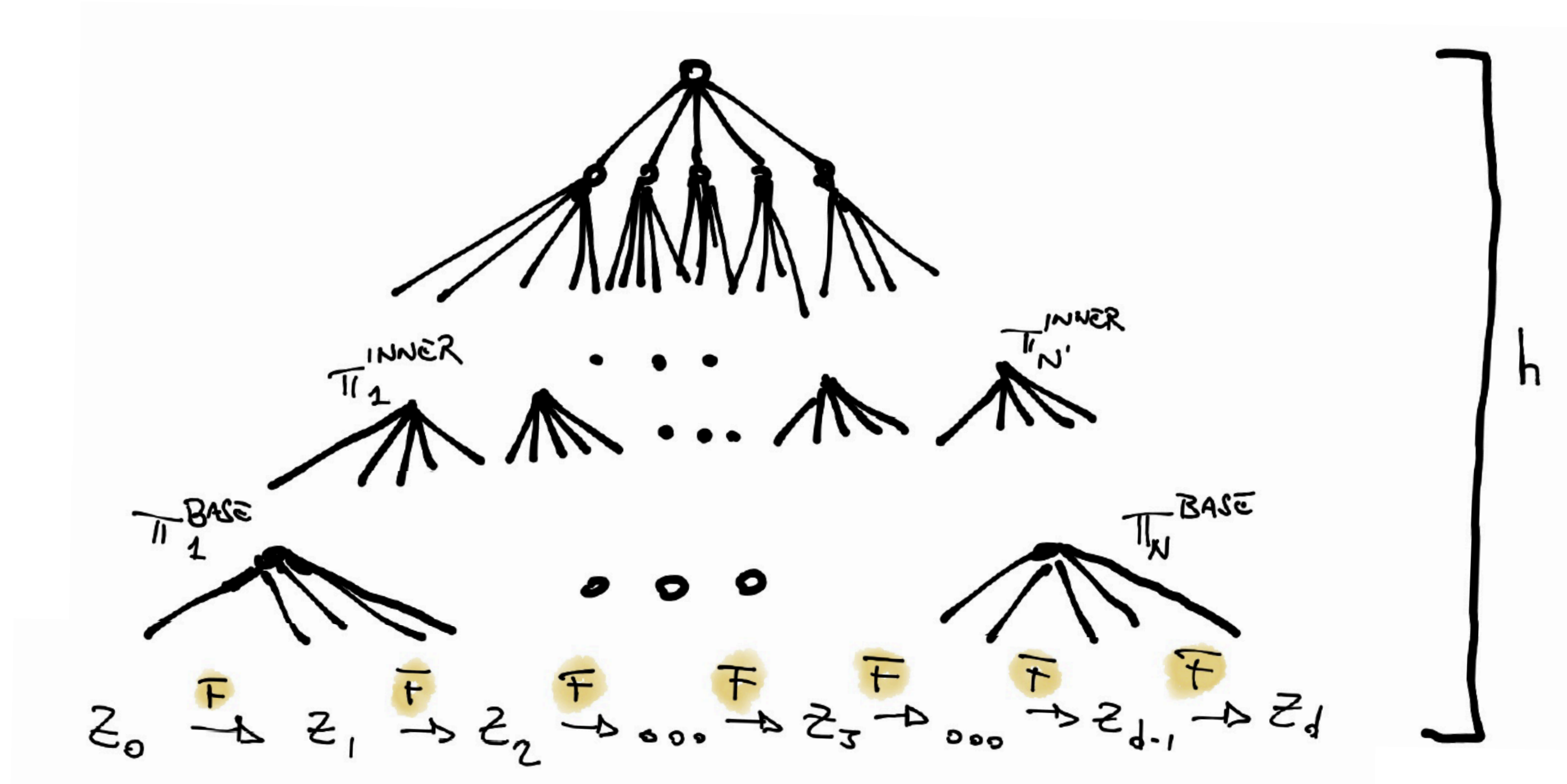
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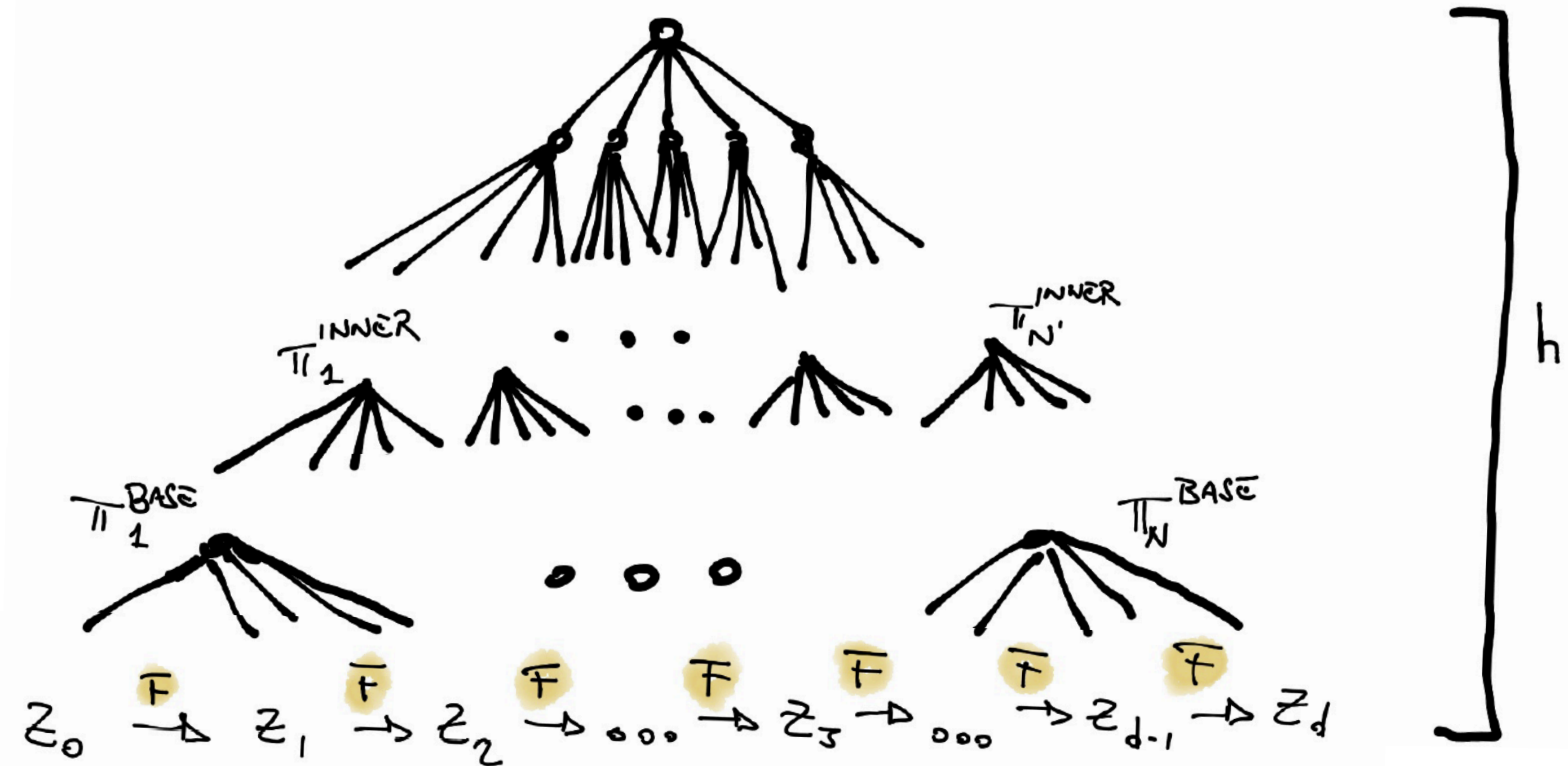
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This construction works but it is not the plain recursive construction from before anymore.

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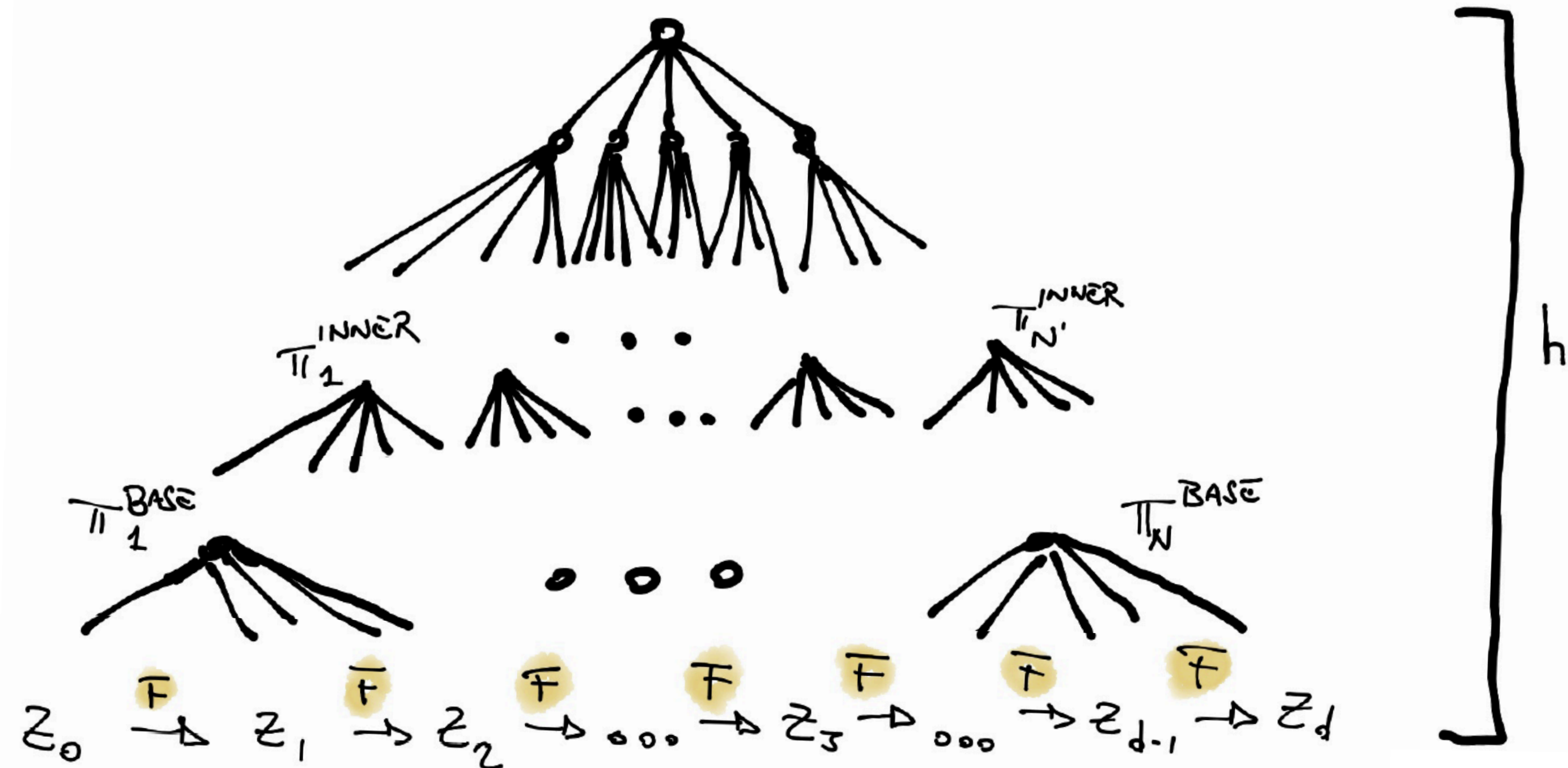
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A digression on motivation

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 - round functions require $\approx \lambda$ iterations
- General improved understanding of *where* we can use *which* constructions

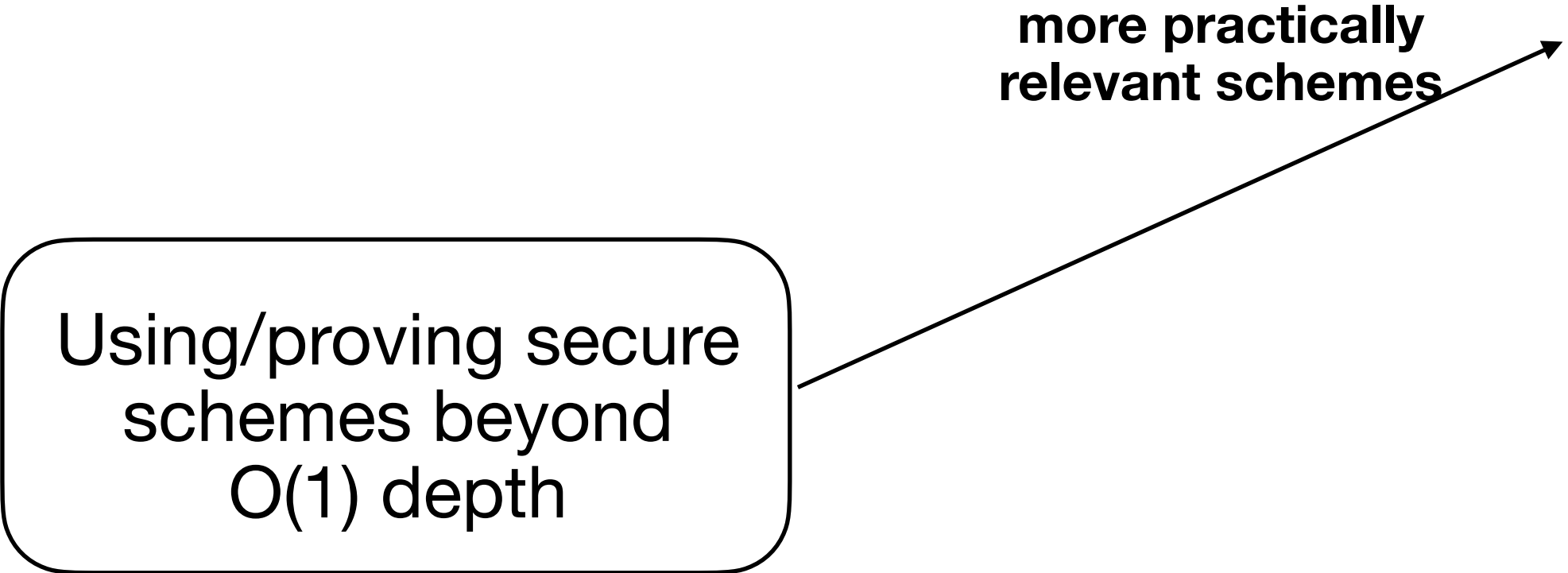
How the Community Has Addressed This—A Landscape

Using/proving secure
schemes beyond
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more practically
relevant schemes

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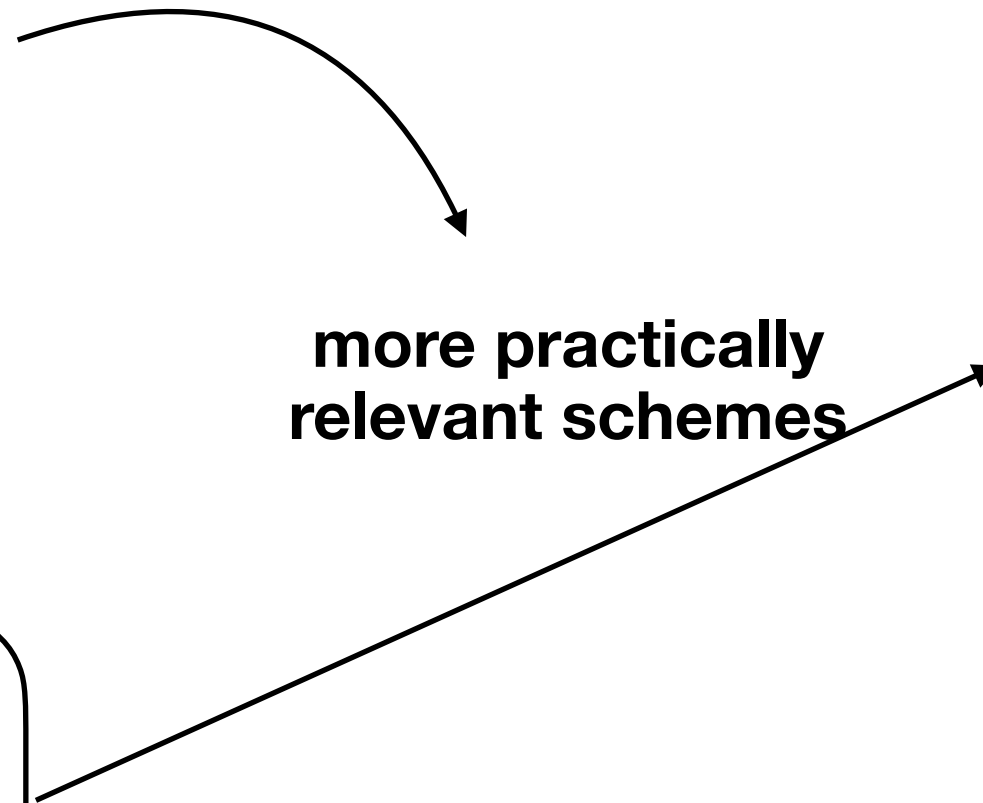
```
graph LR; A[Using/proving secure schemes beyond O(1) depth] --> B[more practically relevant schemes]
```

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includes “recursion-based”
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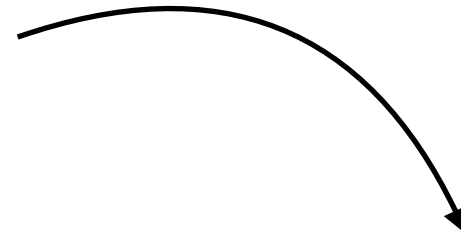
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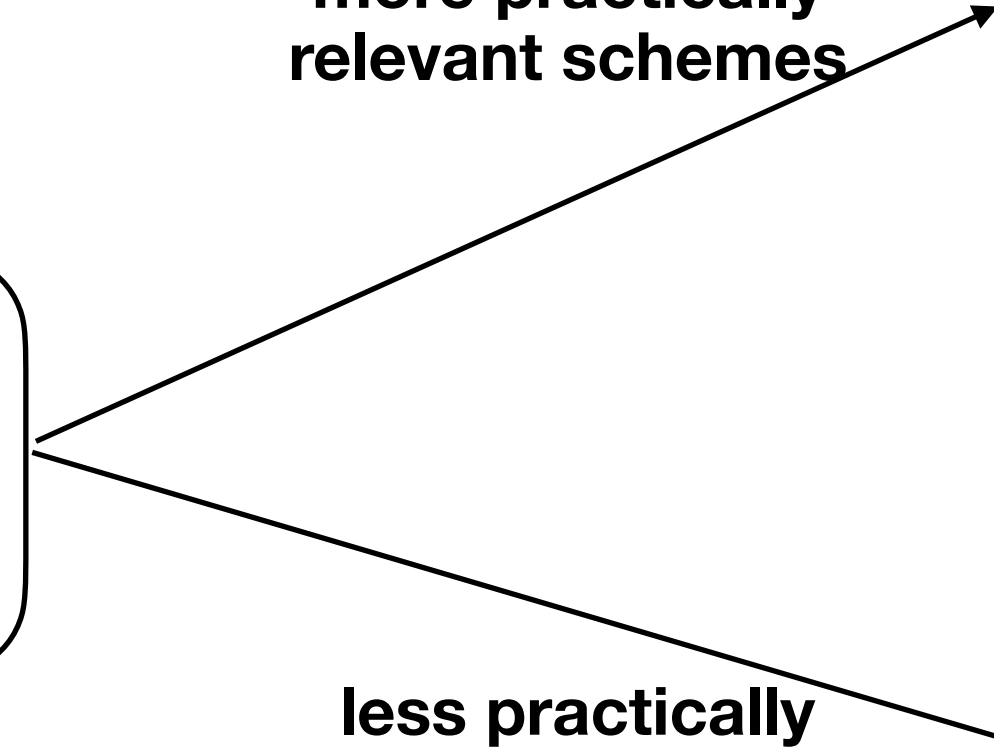
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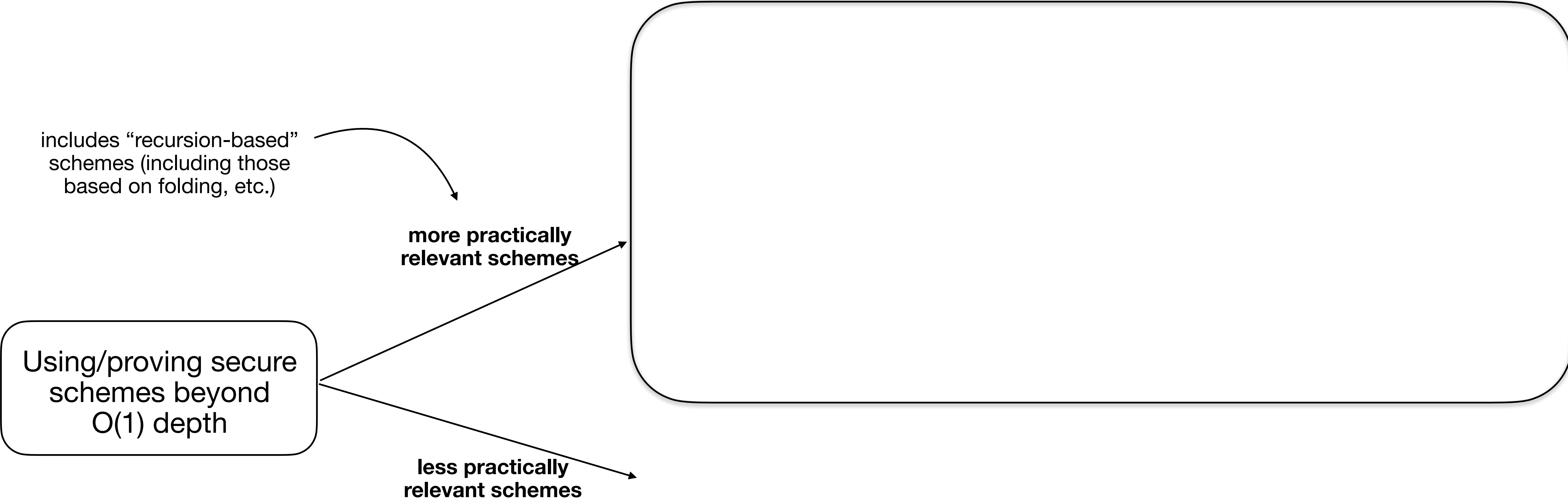
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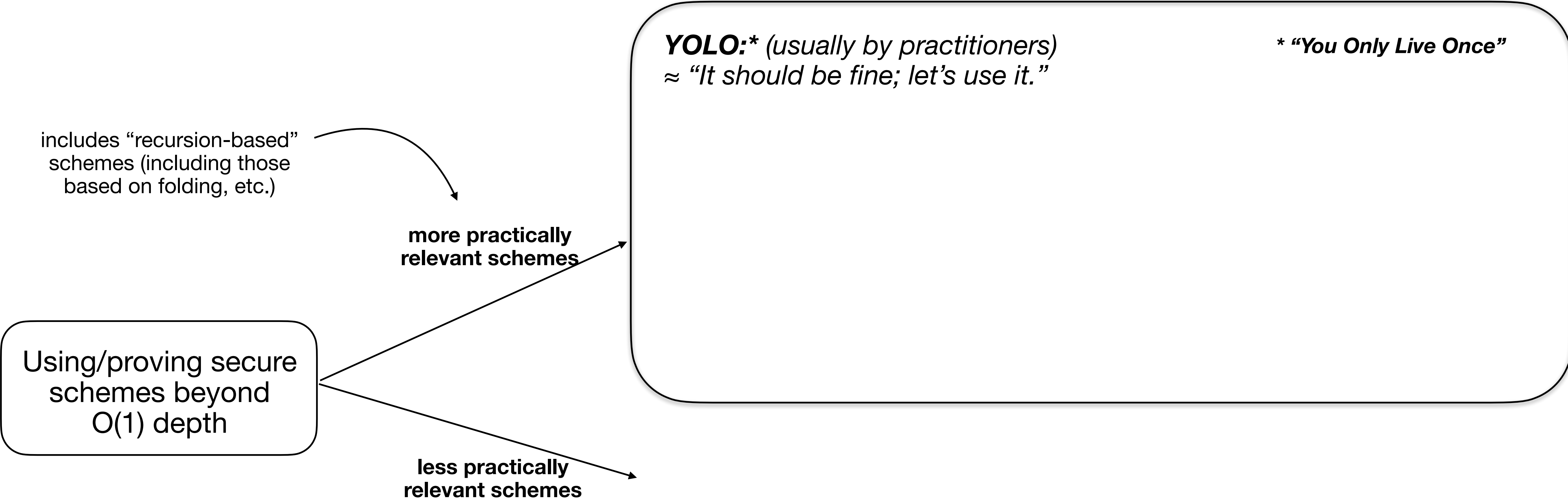


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How the Community Has Addressed This—A Landscape



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YOLO:* (*usually by practitioners*)
≈ “It should be fine; let’s use it.”

* “*You Only Live Once*”

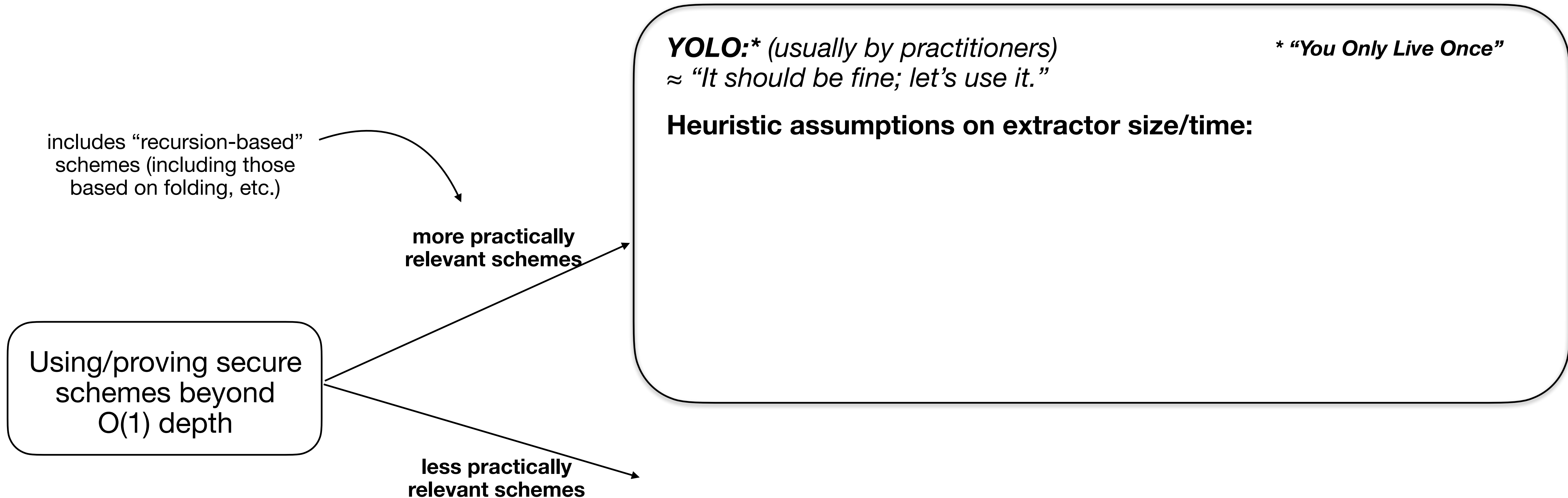
Heuristic assumptions on extractor size/time:

includes “recursion-based”
schemes (including those
based on folding, etc.)

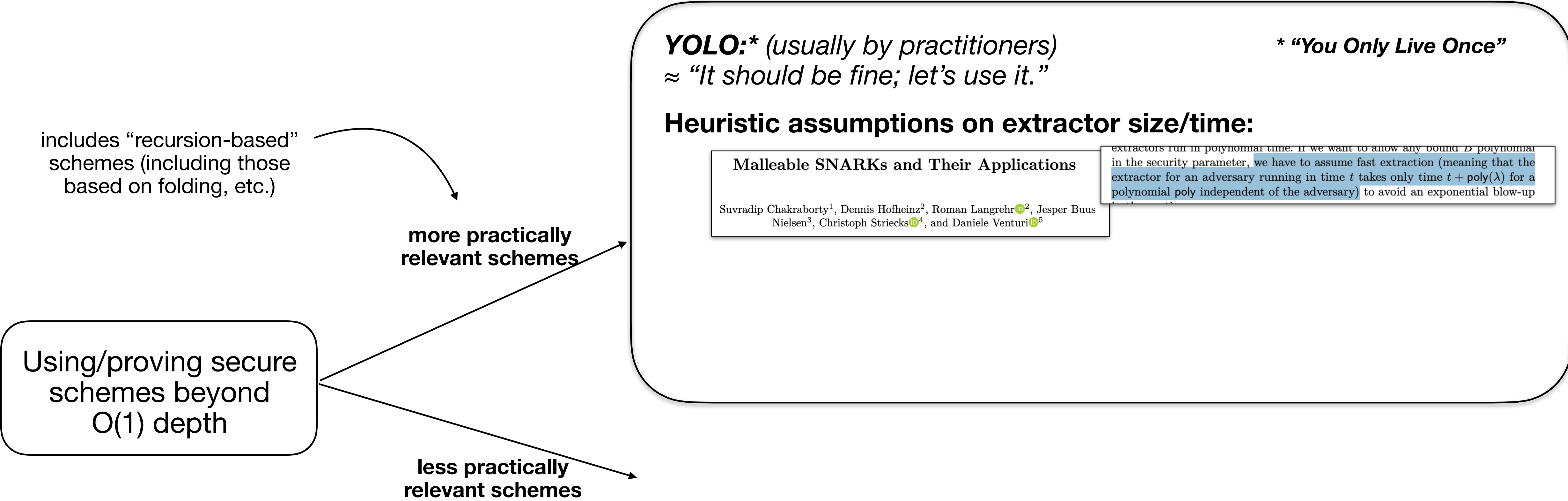
more practically
relevant schemes

Using/proving secure
schemes beyond
 $O(1)$ depth

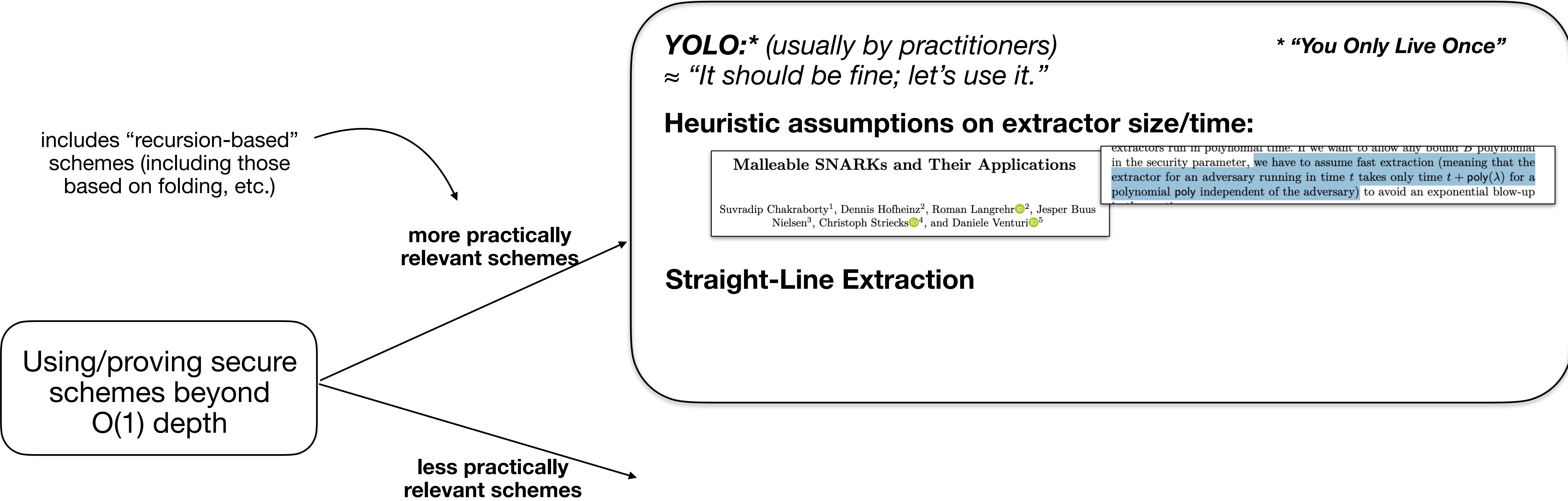
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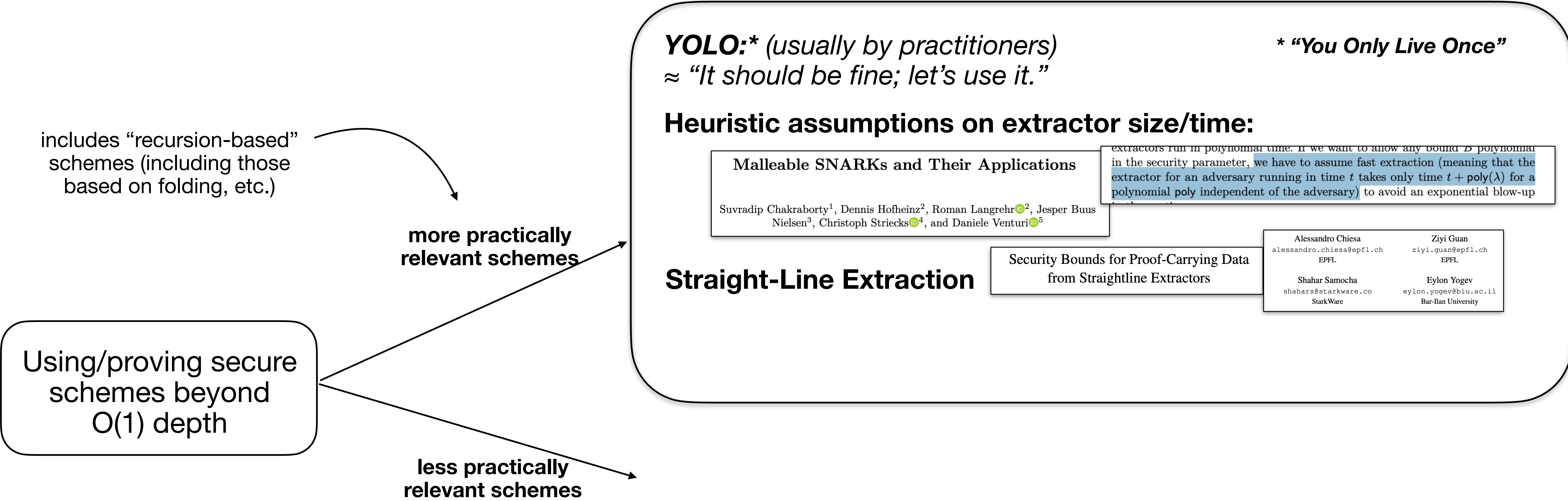
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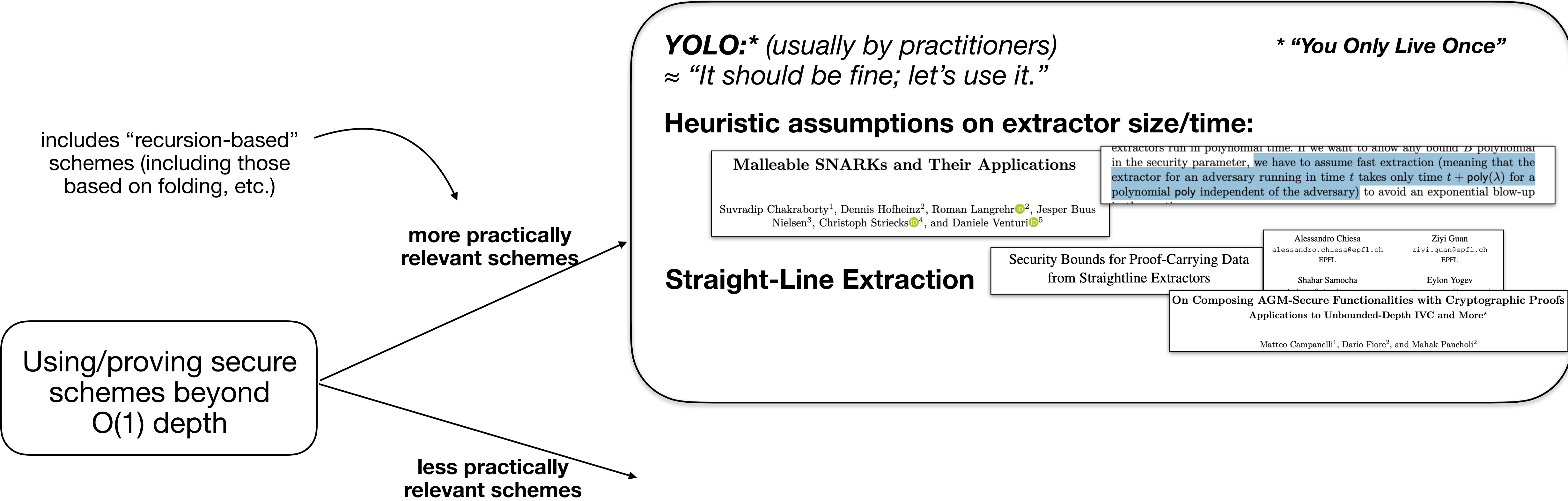
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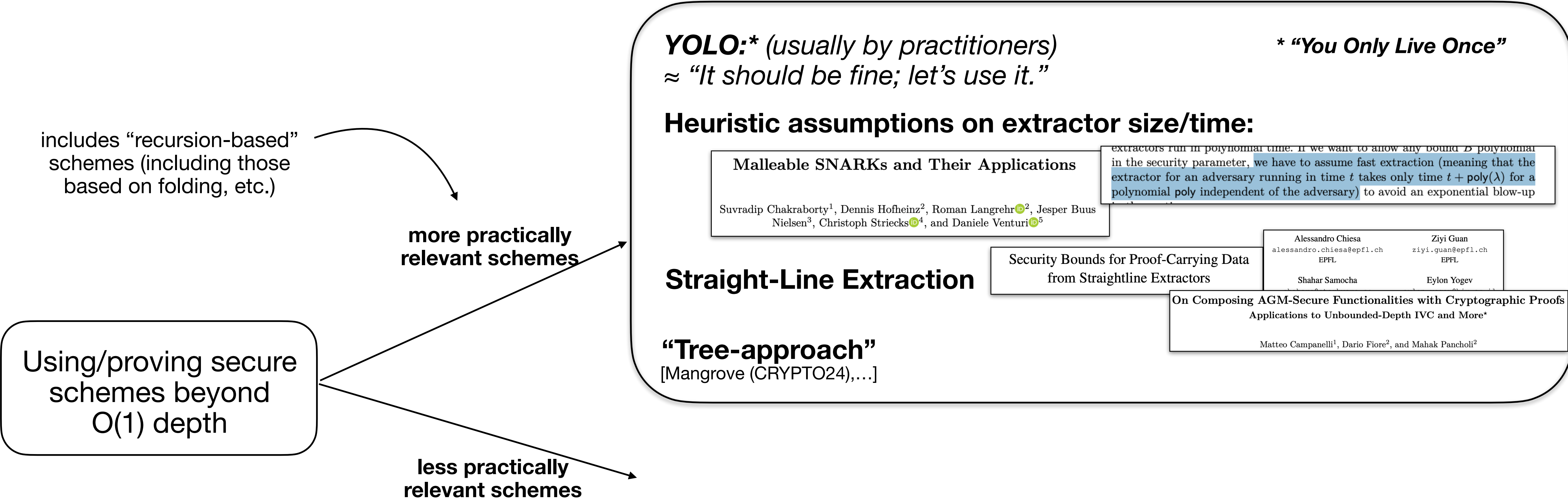
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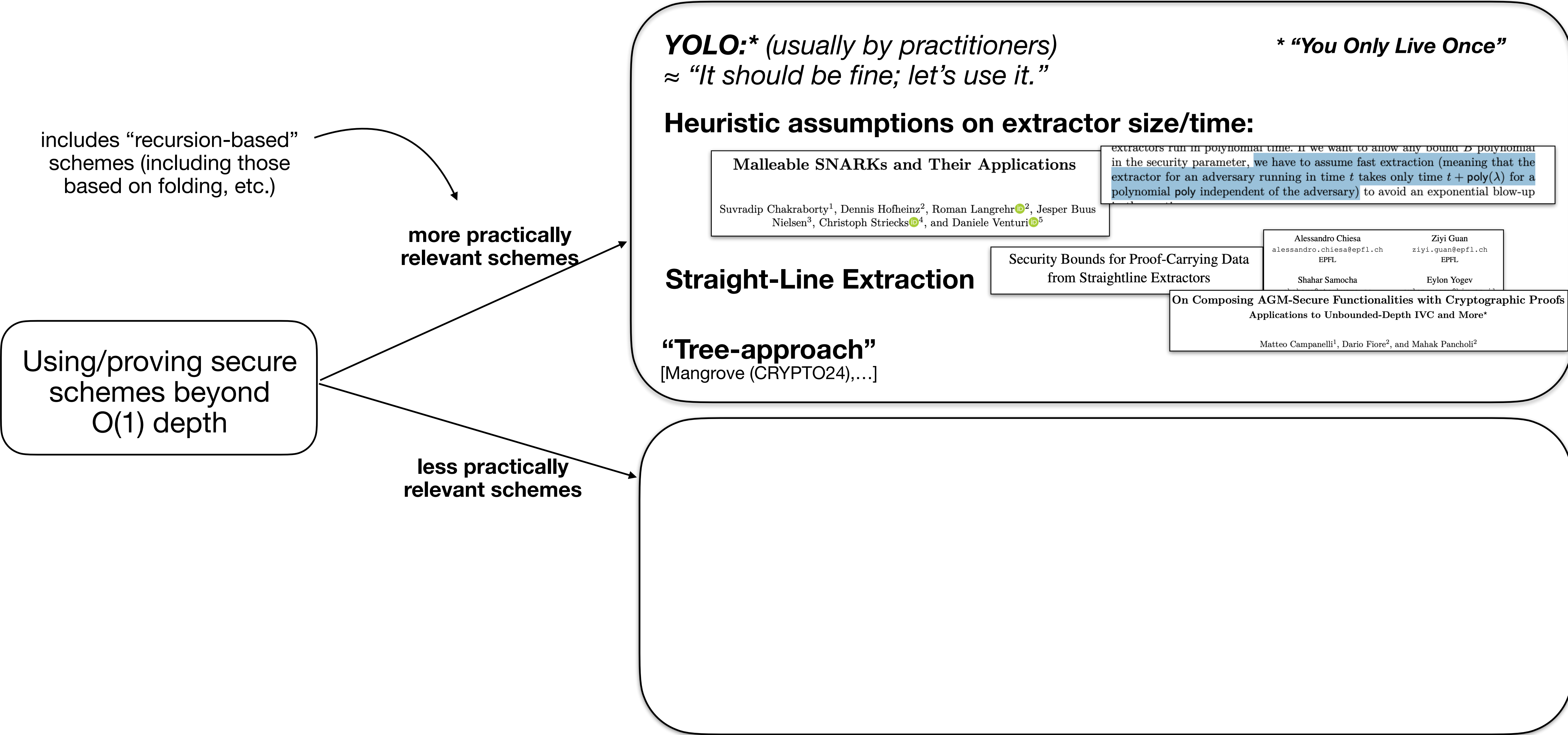
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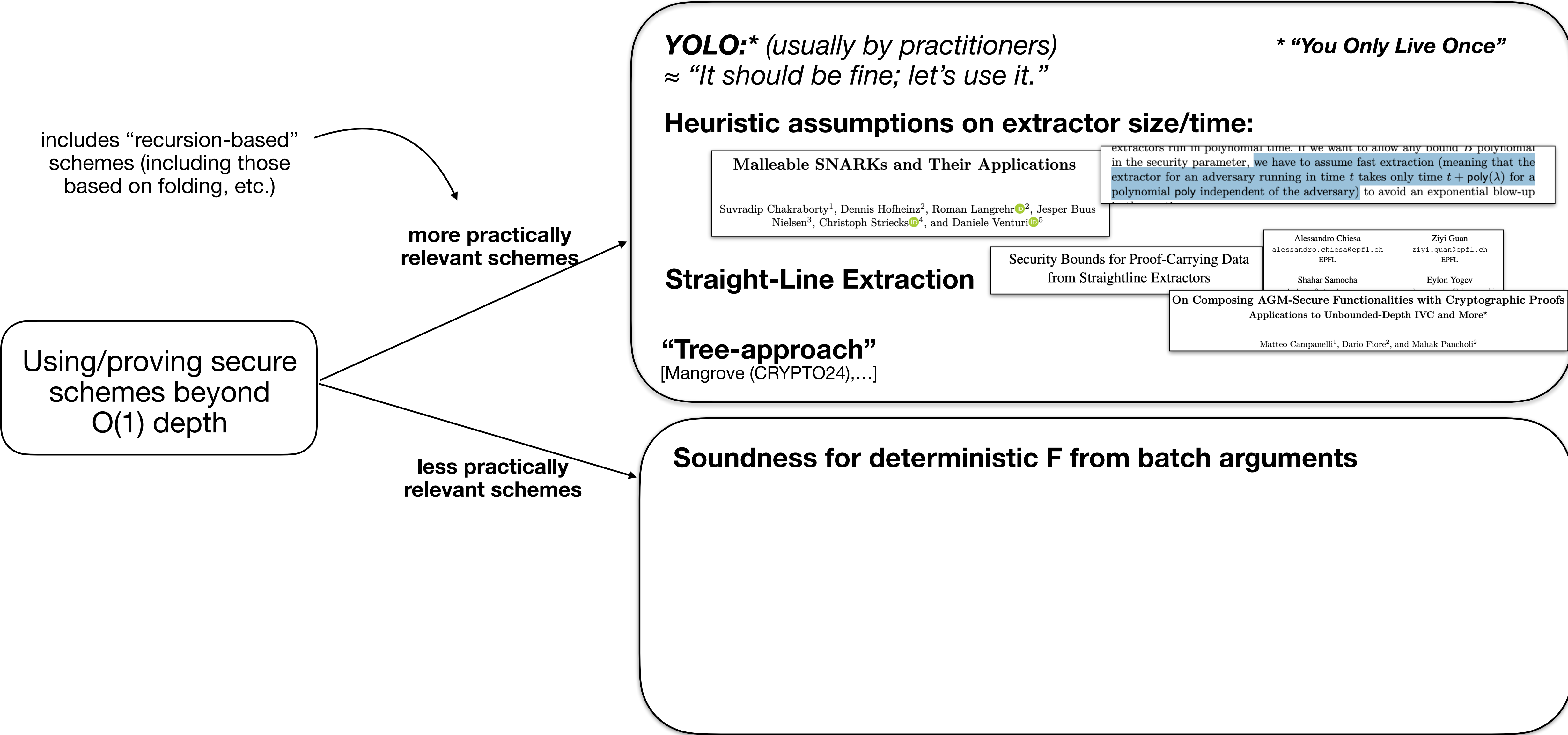
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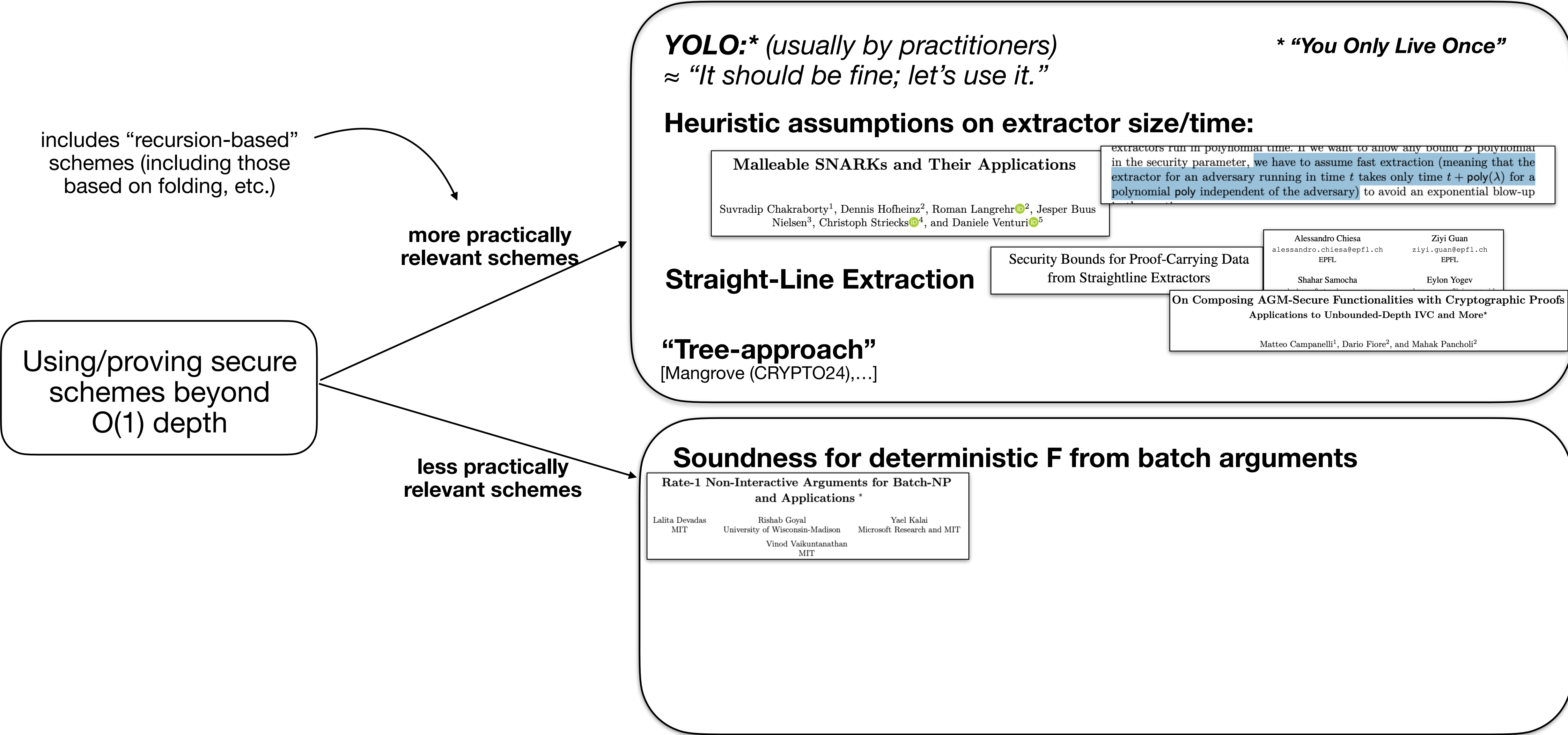
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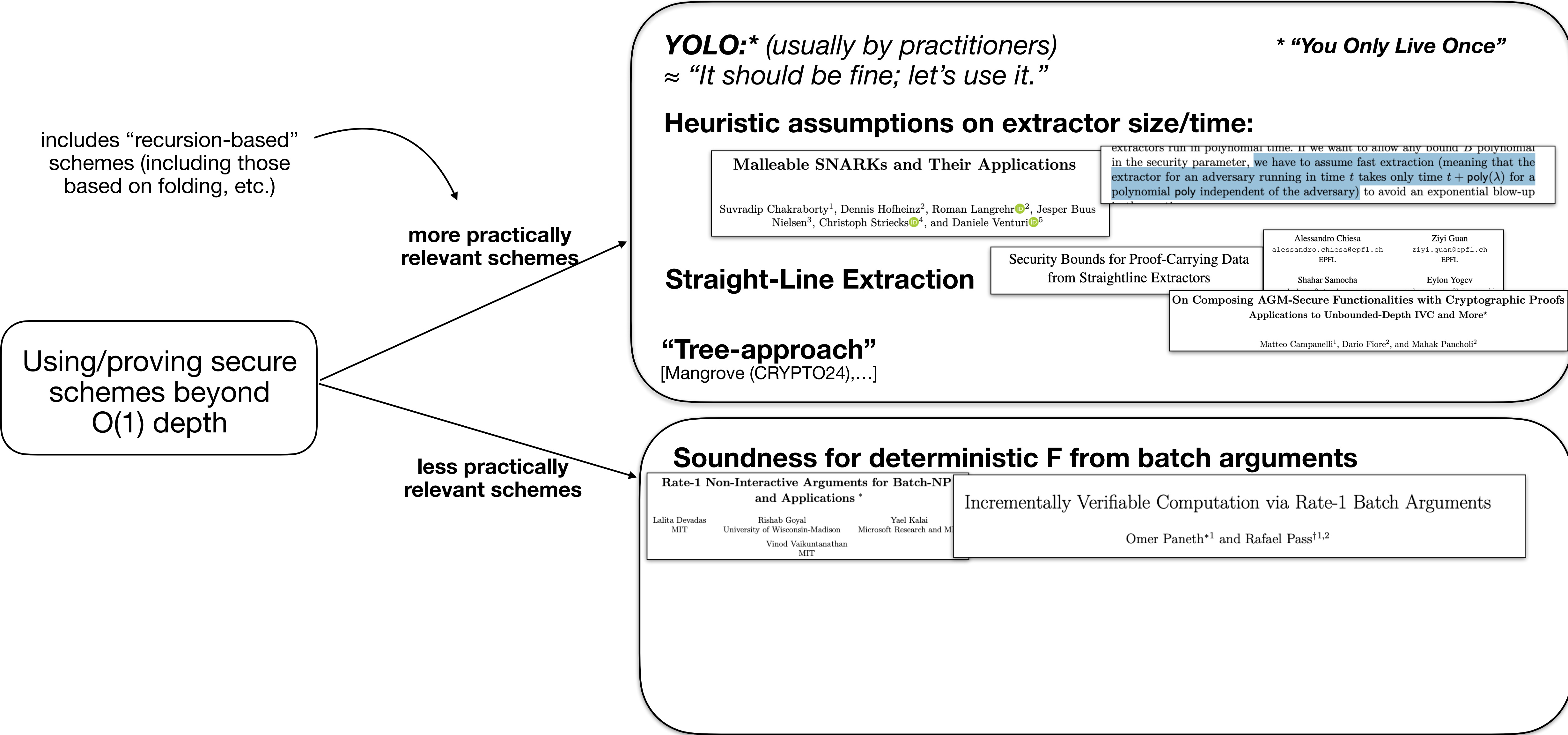
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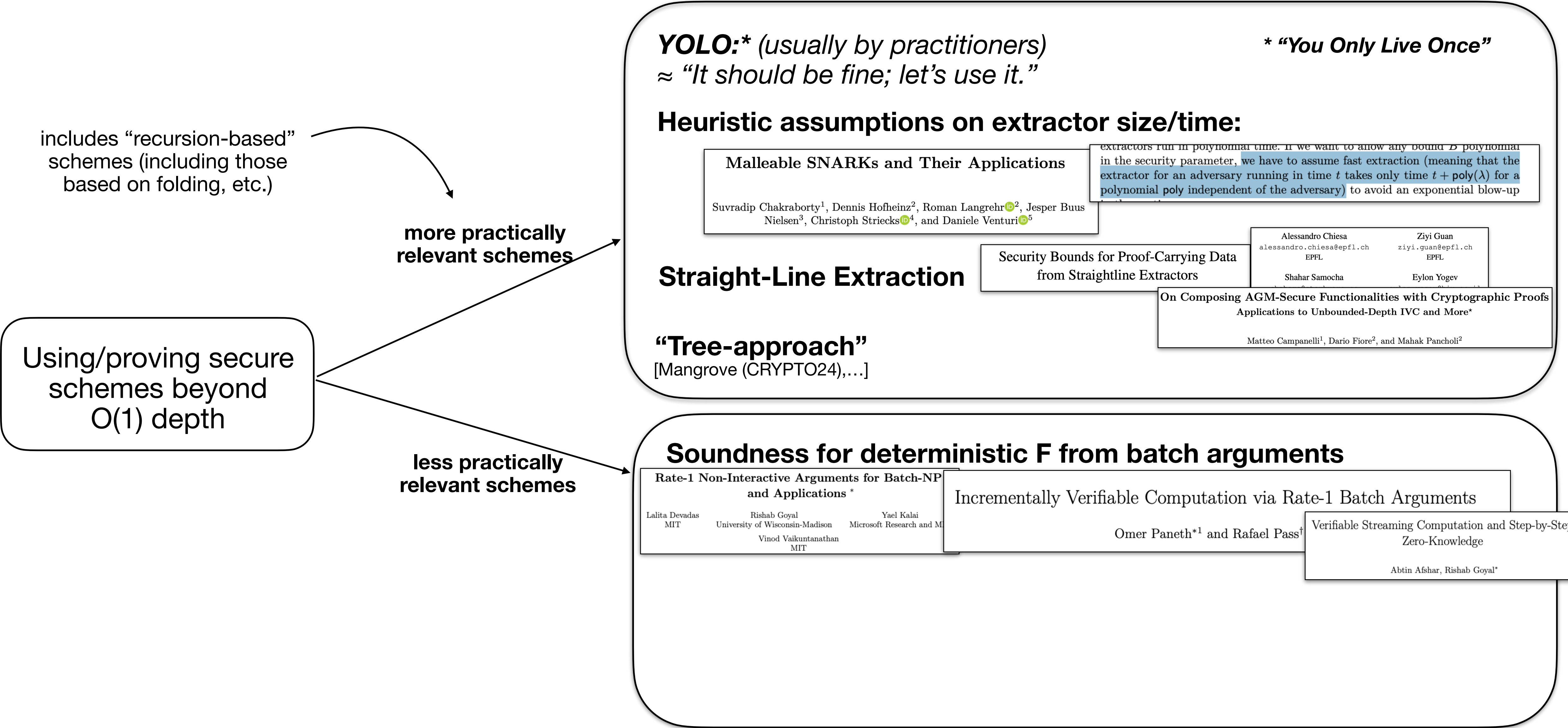
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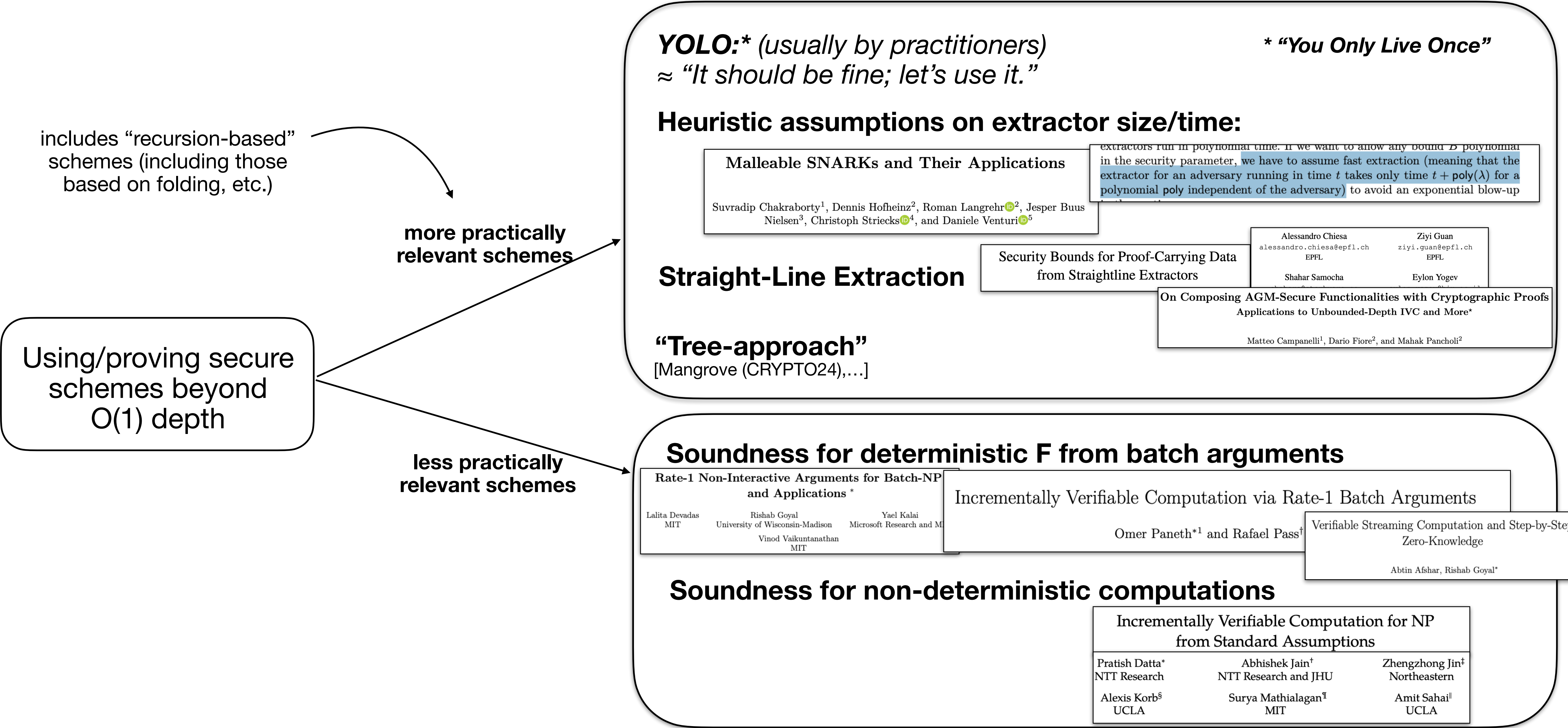
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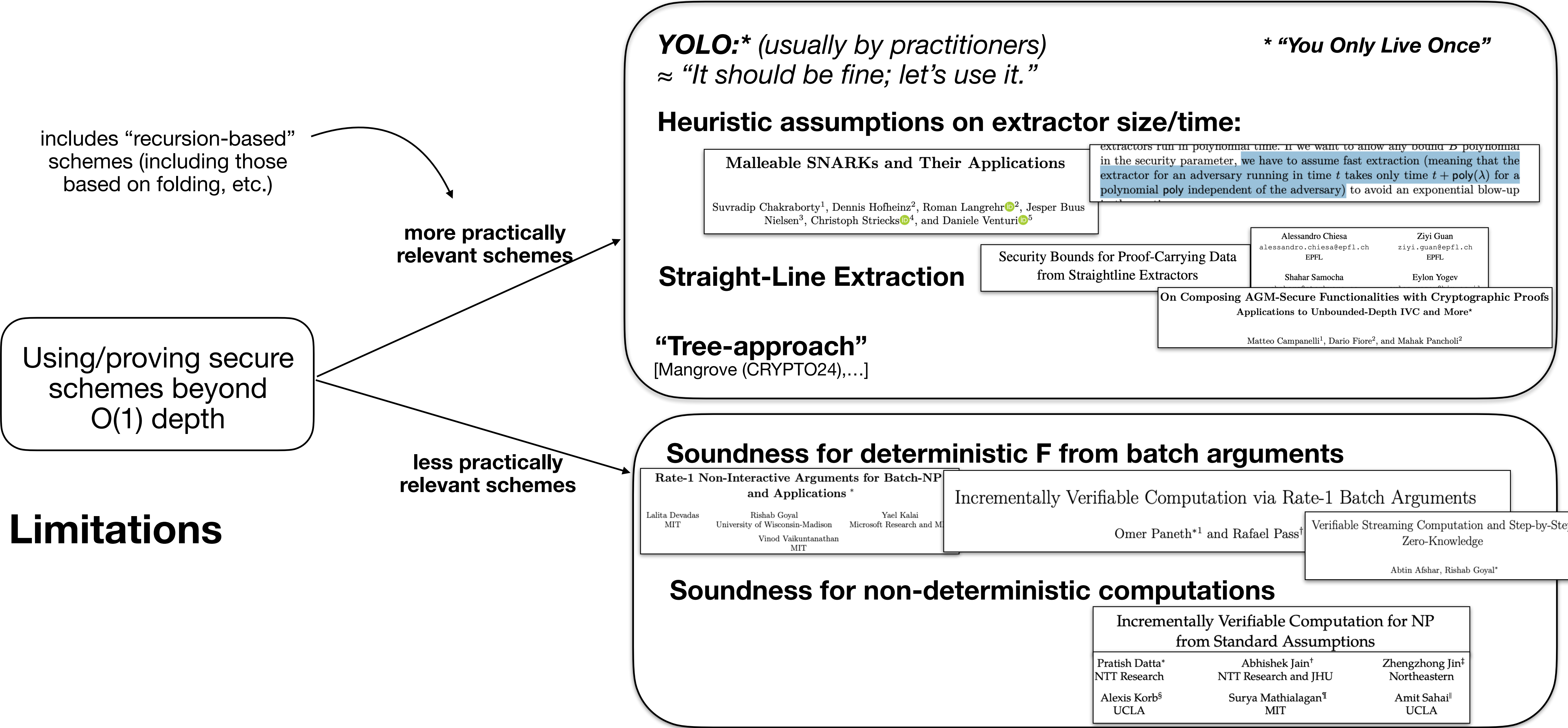
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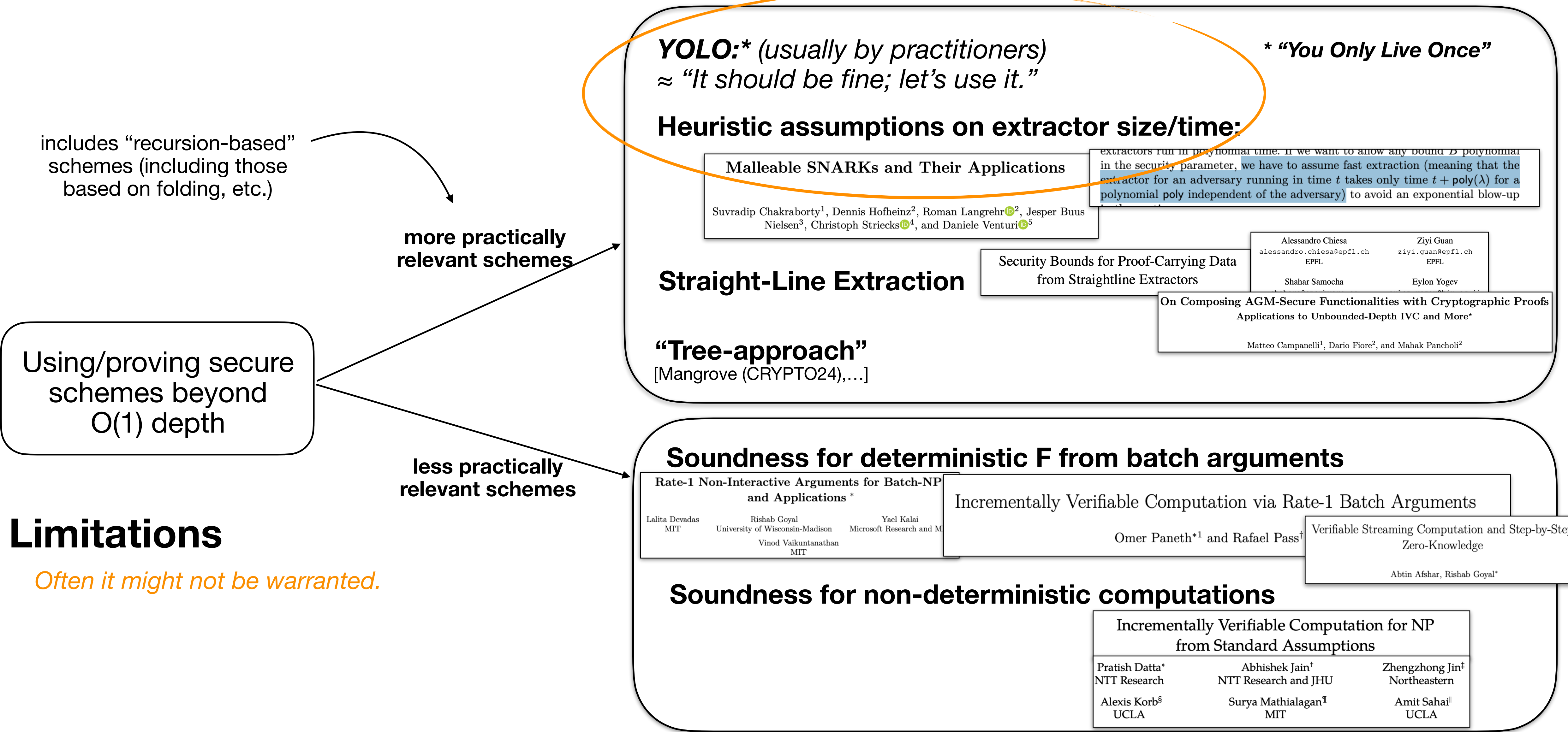


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Limitations

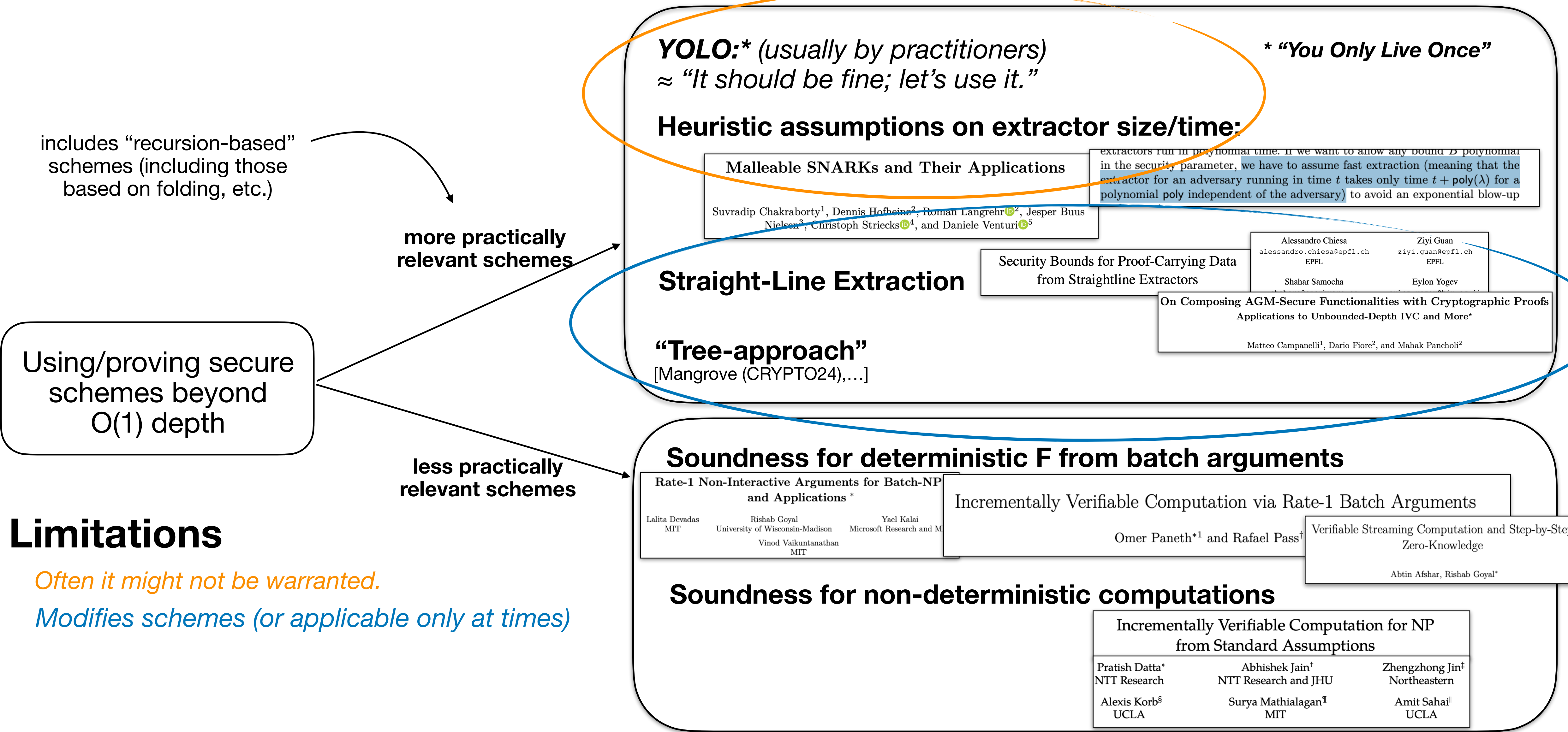
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Limitations

Often it might not be warranted.

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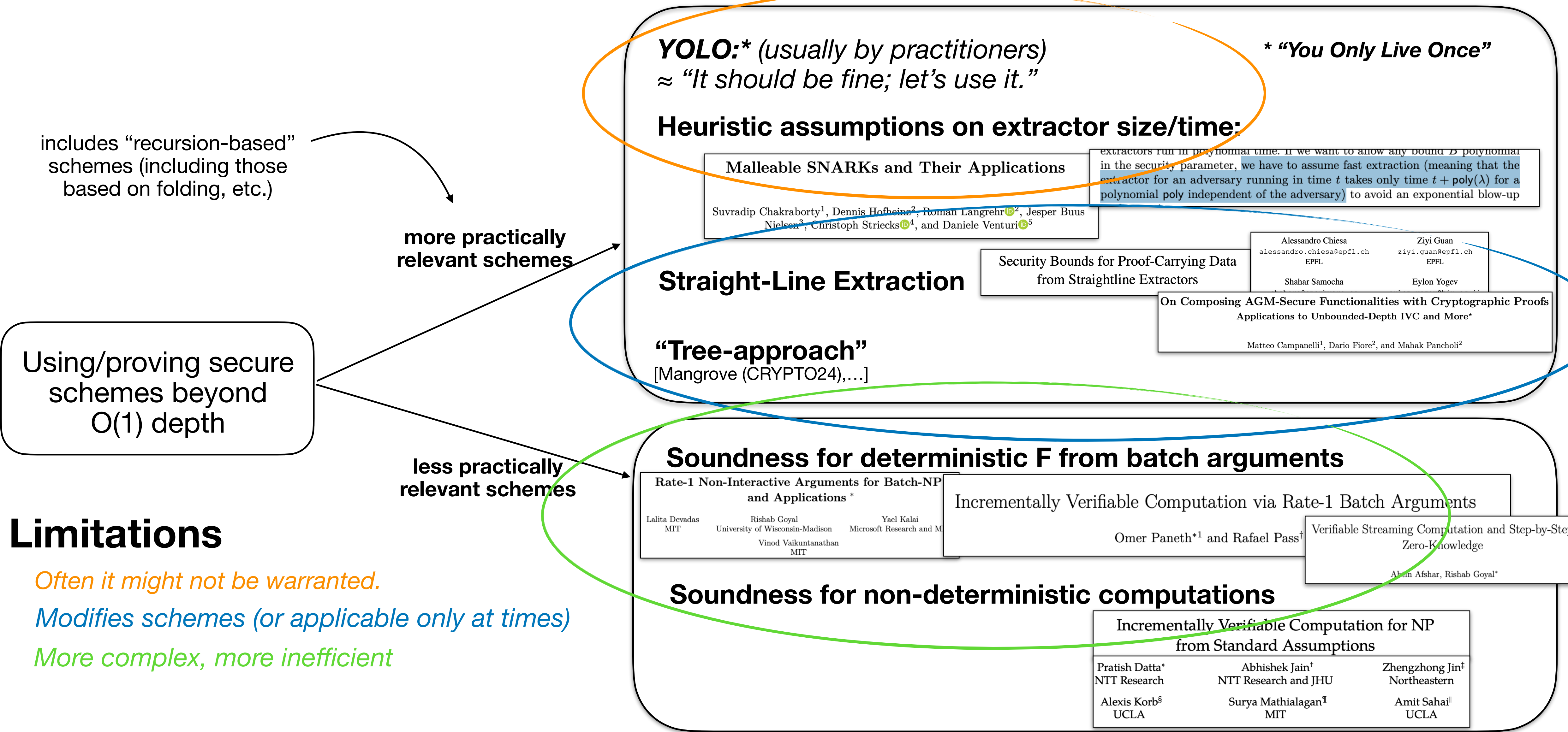


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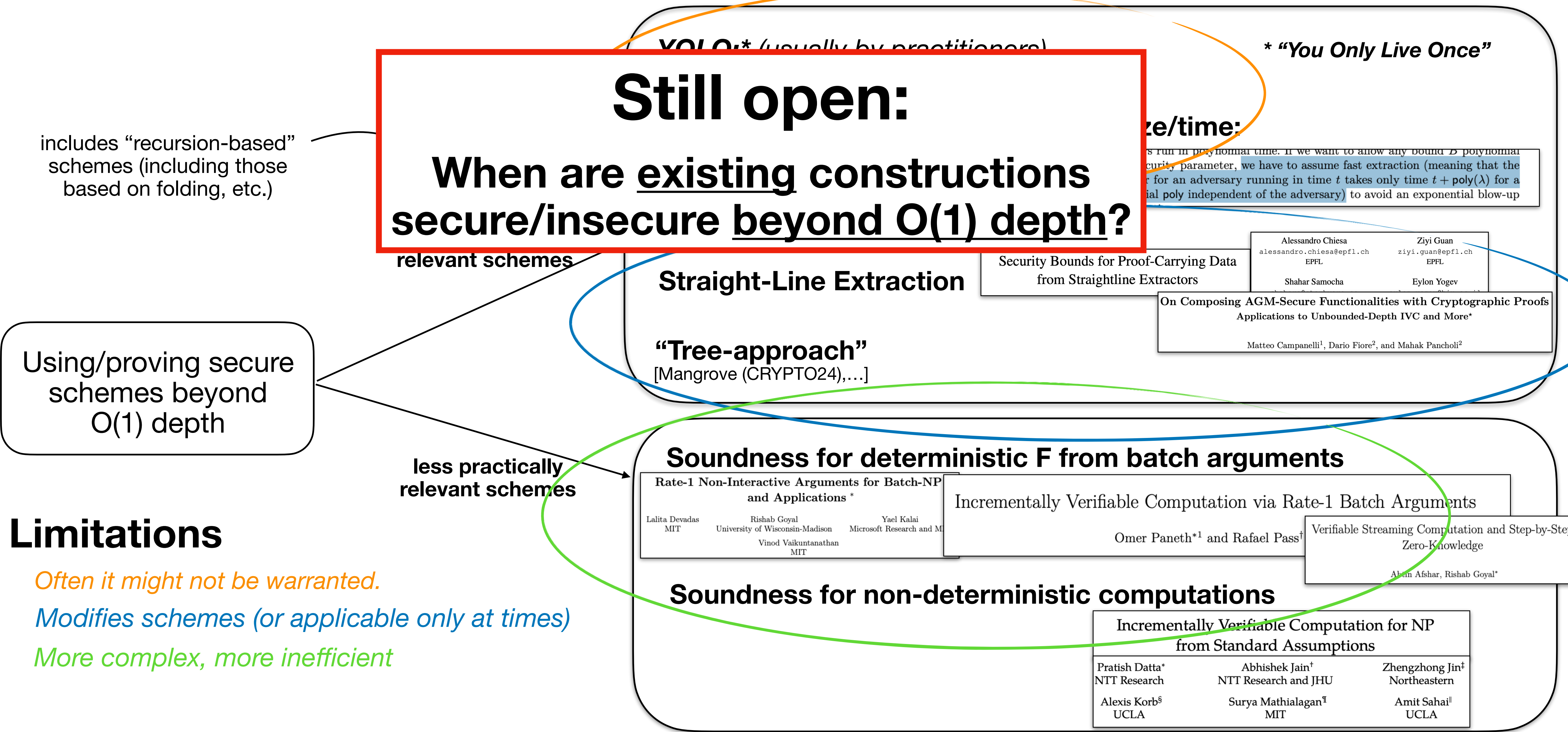
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Limitations

- Often it might not be warranted.
- Modifies schemes (or applicable only at times)
- More complex, more inefficient

How the Community Has Addressed This – A Landscape



This Work's Question

Still open:

When are existing constructions
secure/insecure beyond $O(1)$ depth?

The problem at hand

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**When is any construction secure/insecure
beyond $O(1)$ depth?**

We approach this question through two main conceptual lenses.

Lens 1: “*Depth*” as a Core Object of Study

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The asymptotic depth “line”.

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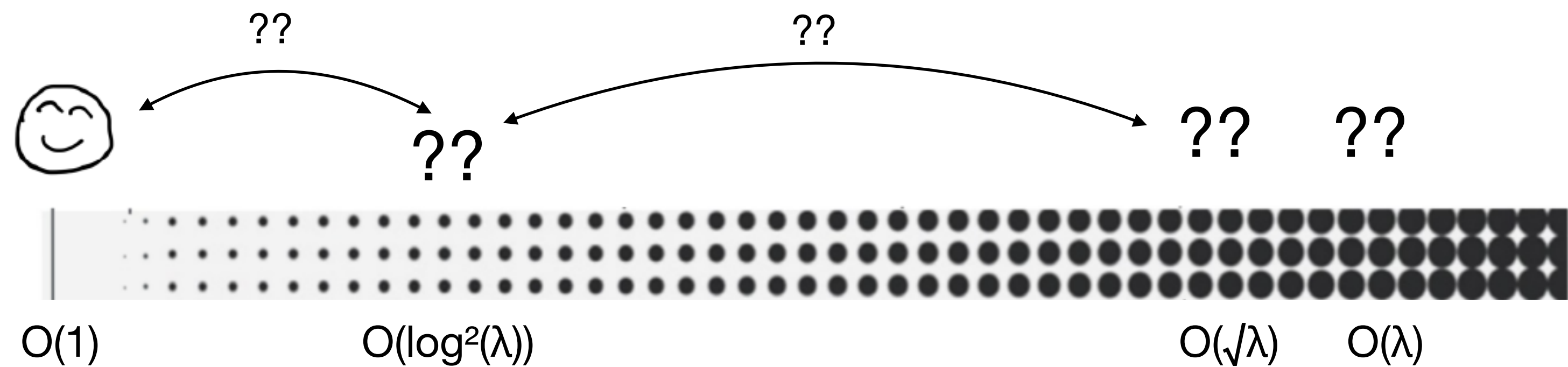
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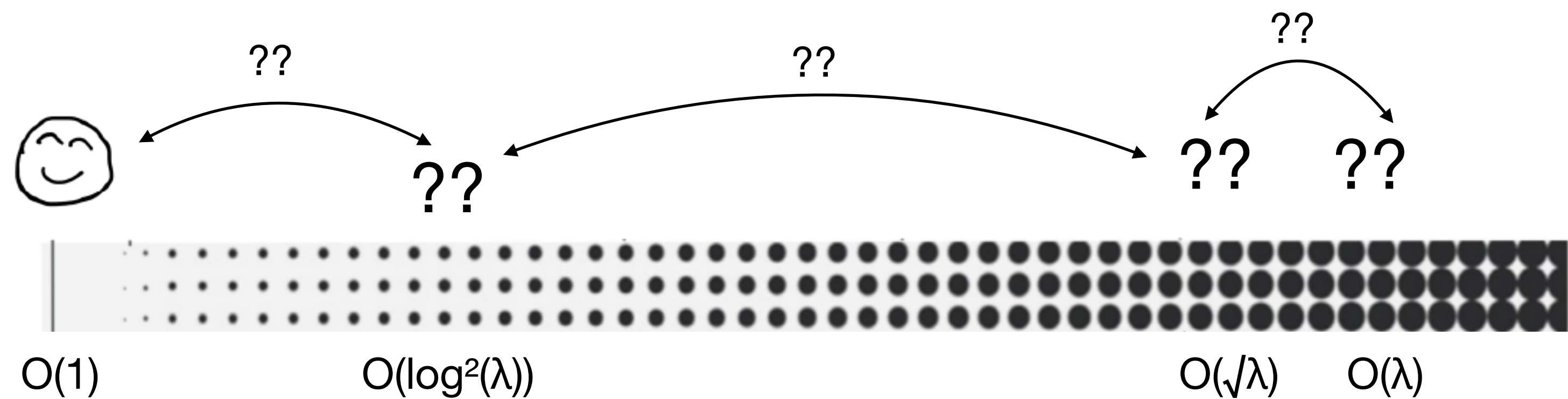
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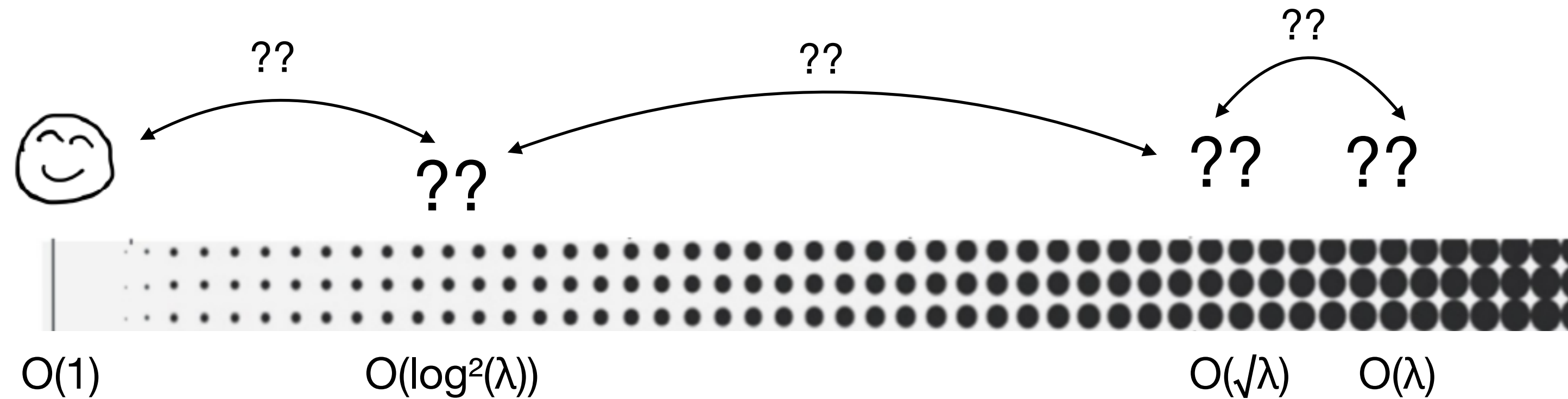
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A note on abuse of language:

I will say

“*big/bigger*” to mean “*fast/er growing*”;

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(And having in the foreground the relation [adversarial advantage] \leftrightarrow [depth])

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 - \approx incremental analogue of functional commitments

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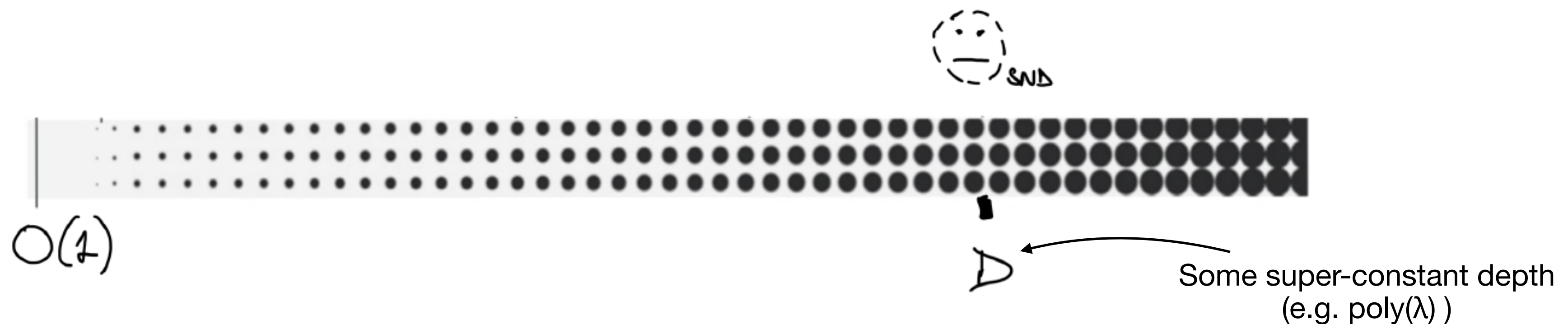
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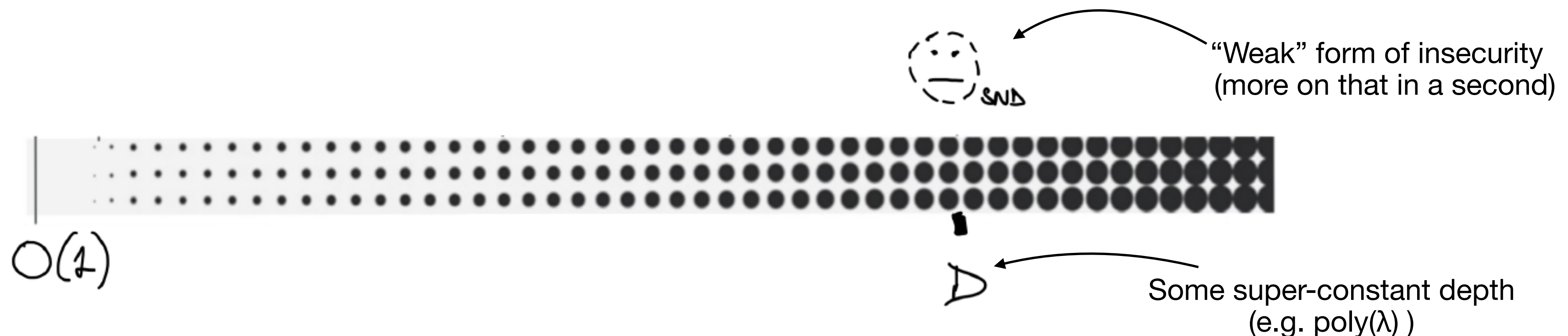
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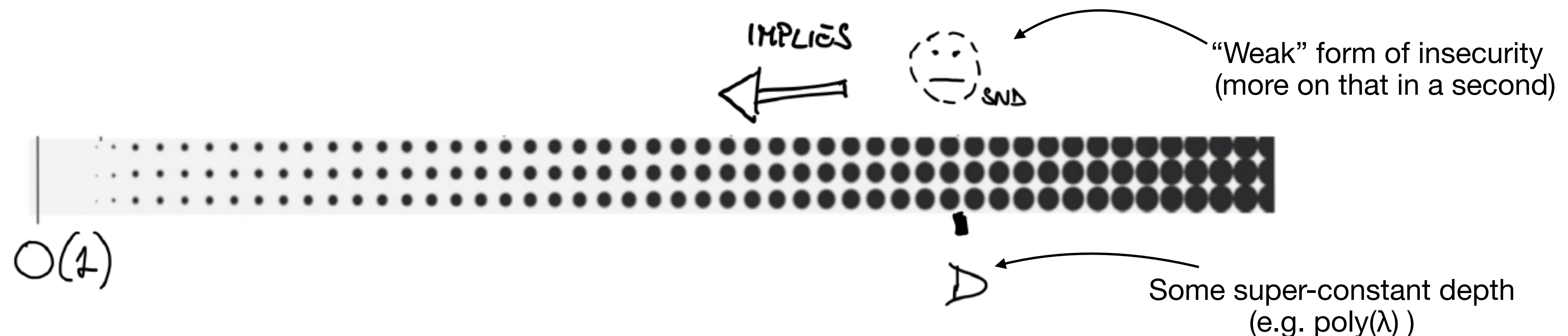
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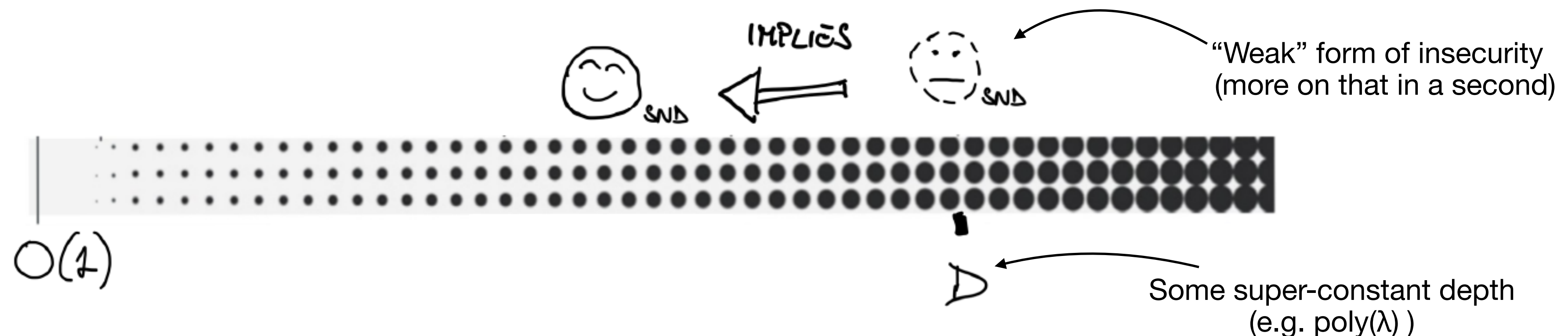
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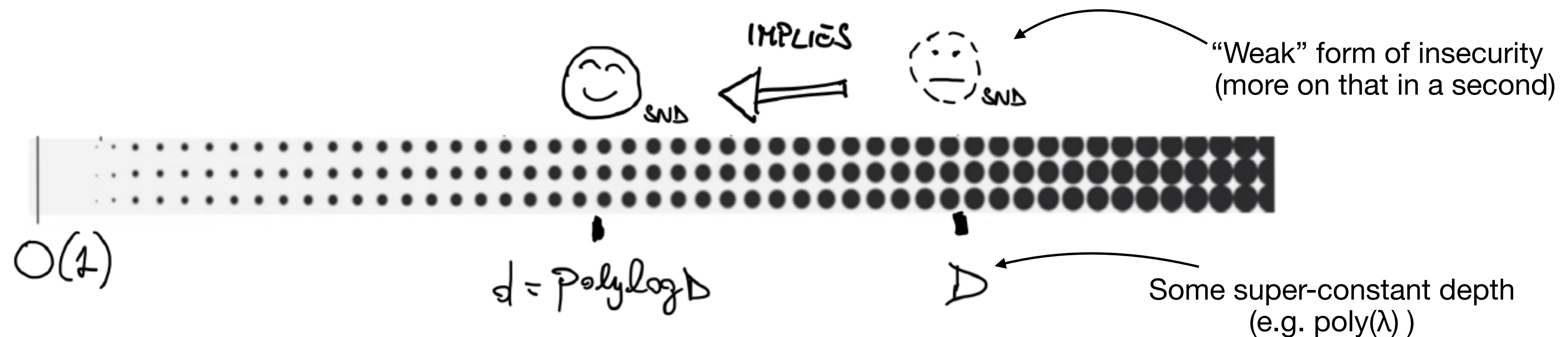
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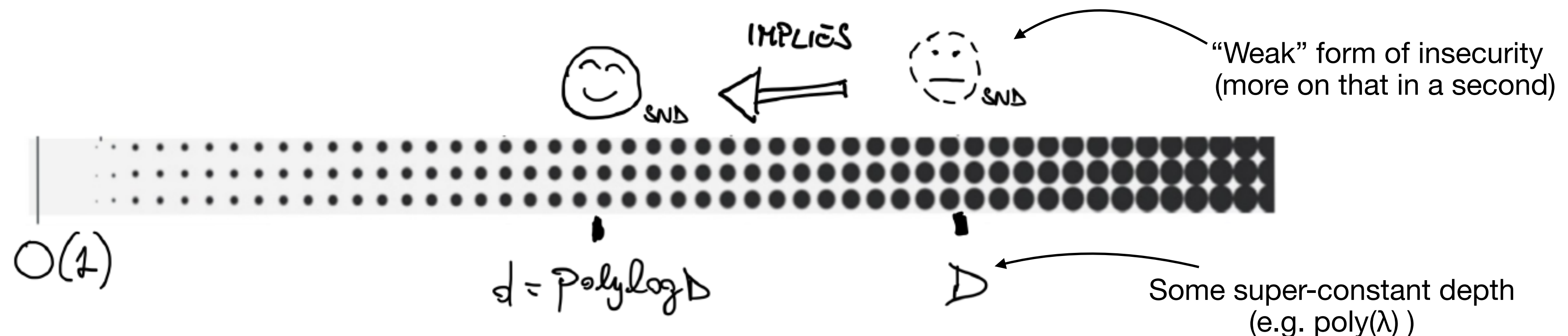
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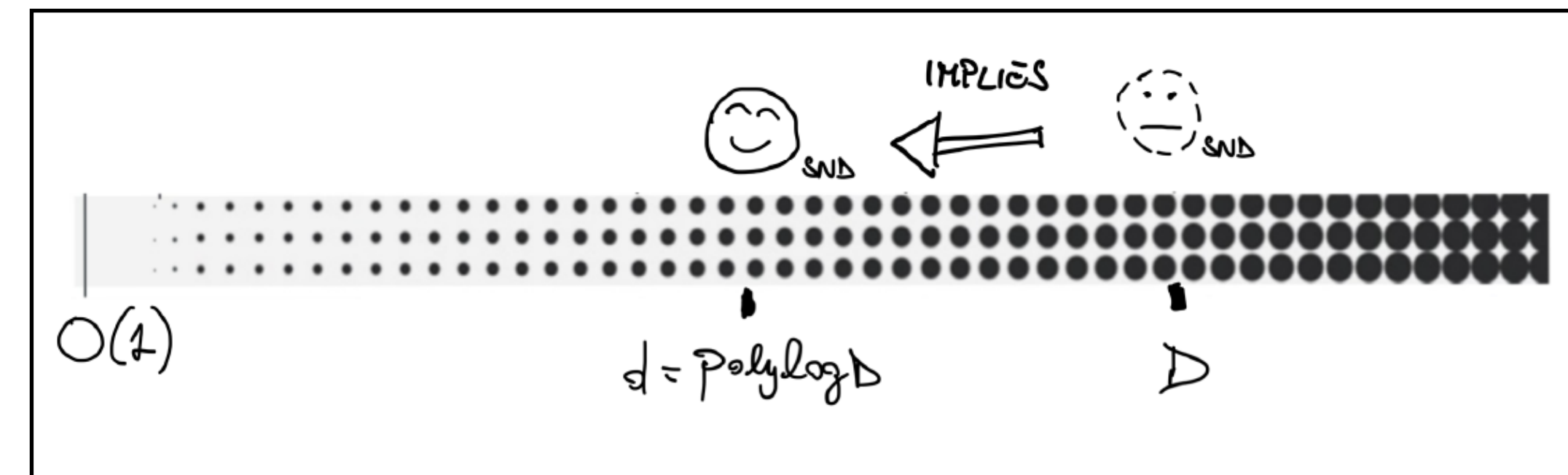
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Implication:

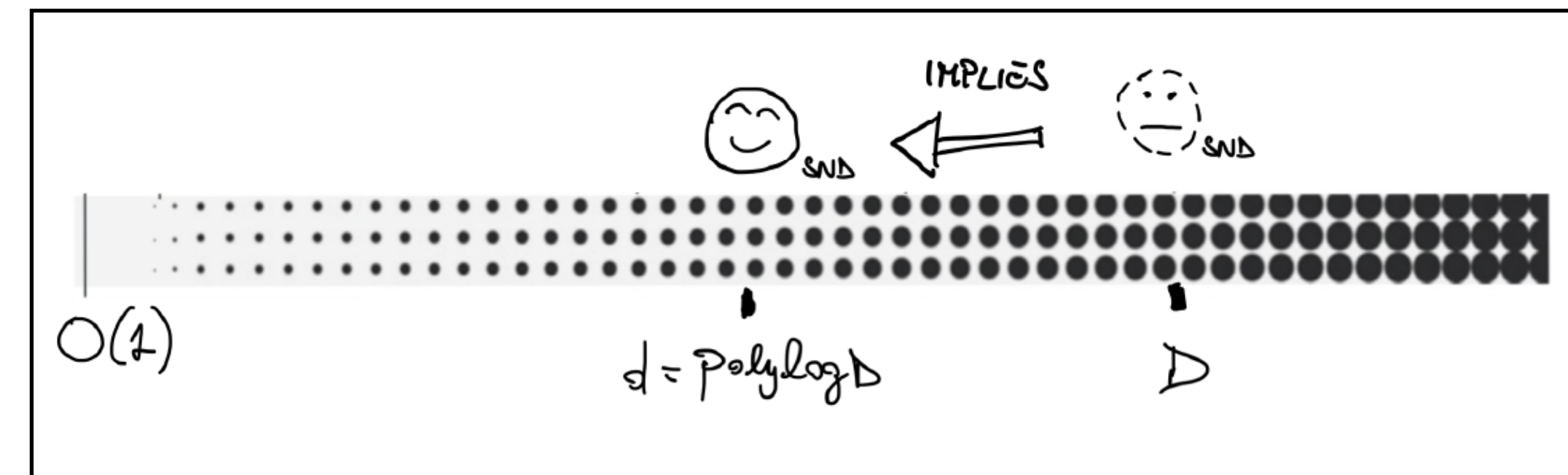
to prove security at some $\omega(1)$ depth d ,
show some $\omega(1)$ depth D where this weak property holds.

What Do We Mean by “Weak” Insecurity?



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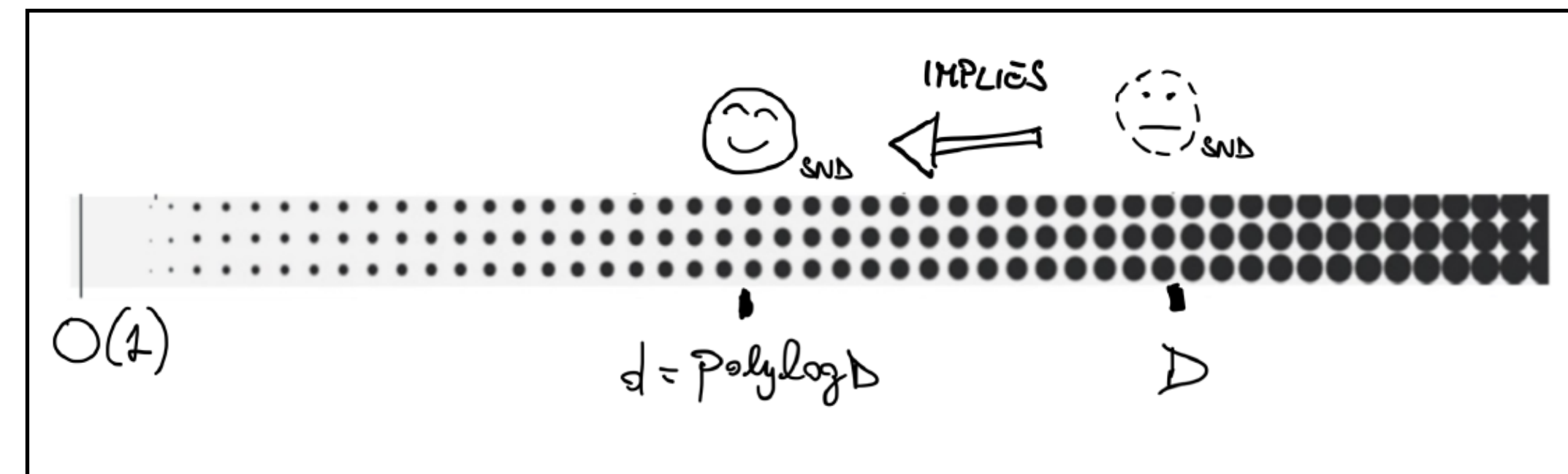


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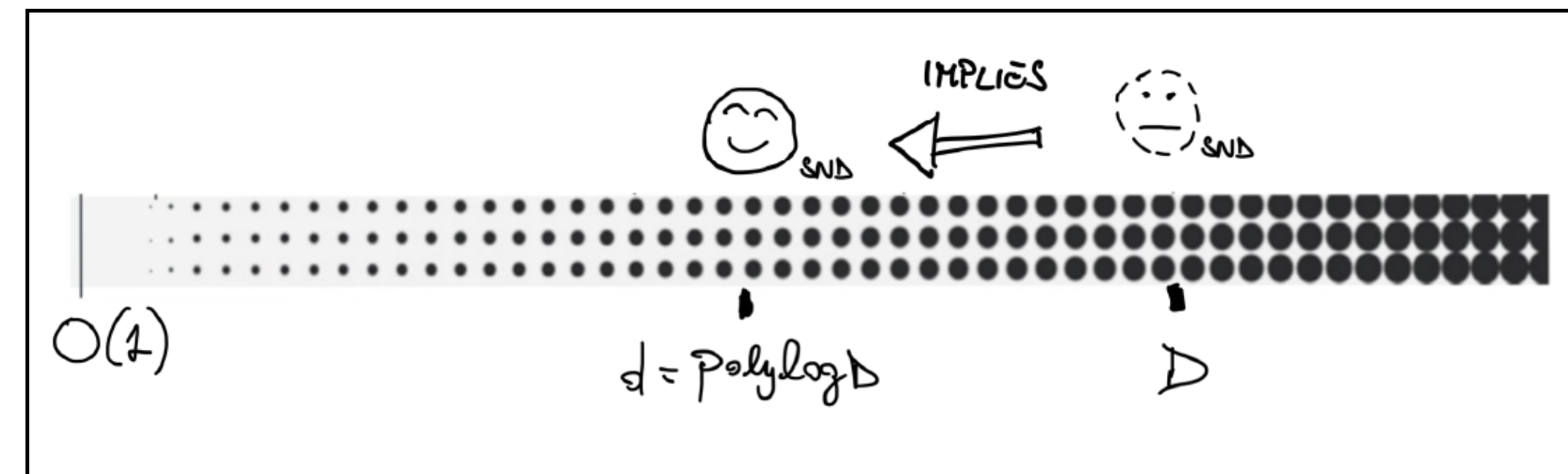


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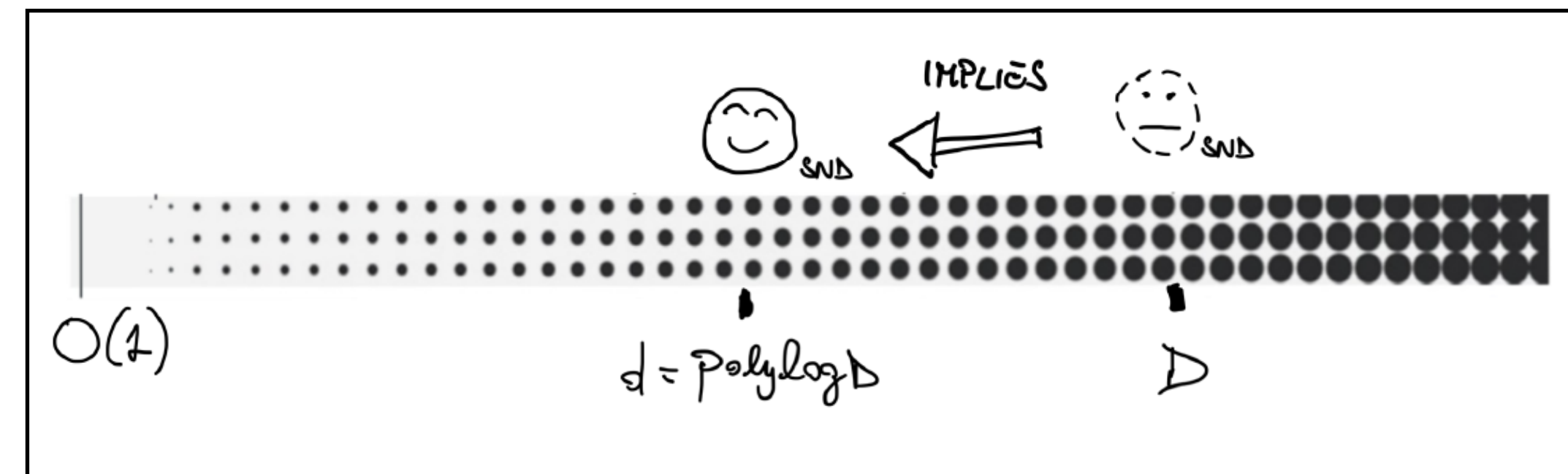
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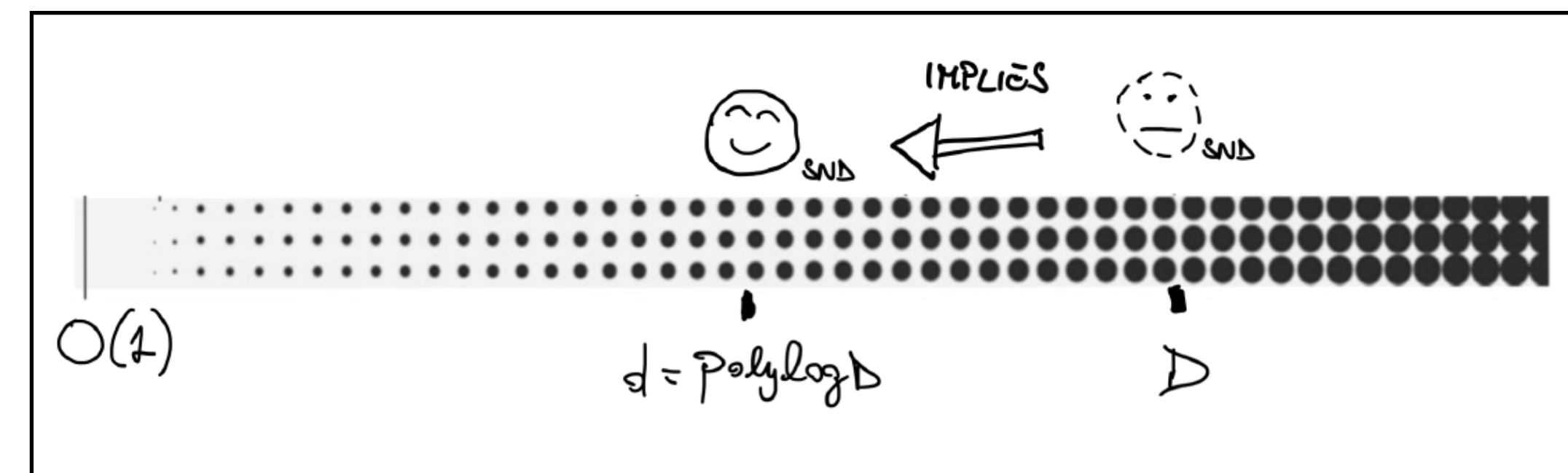
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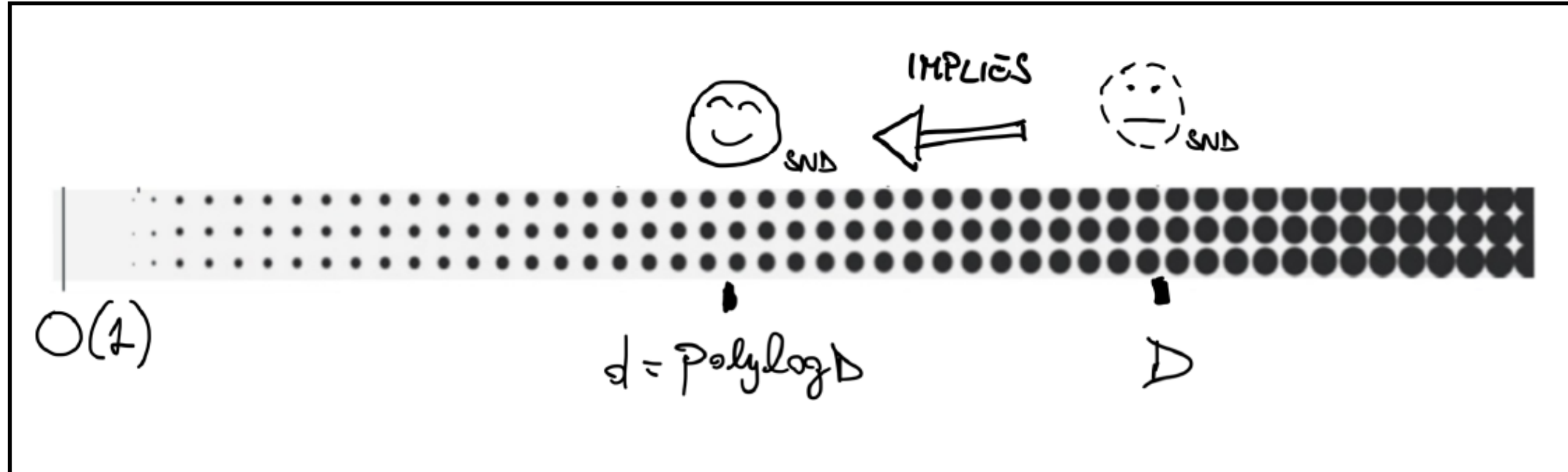
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↑ infinite set
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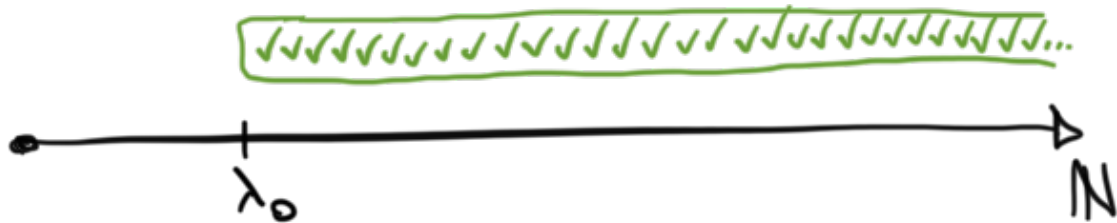
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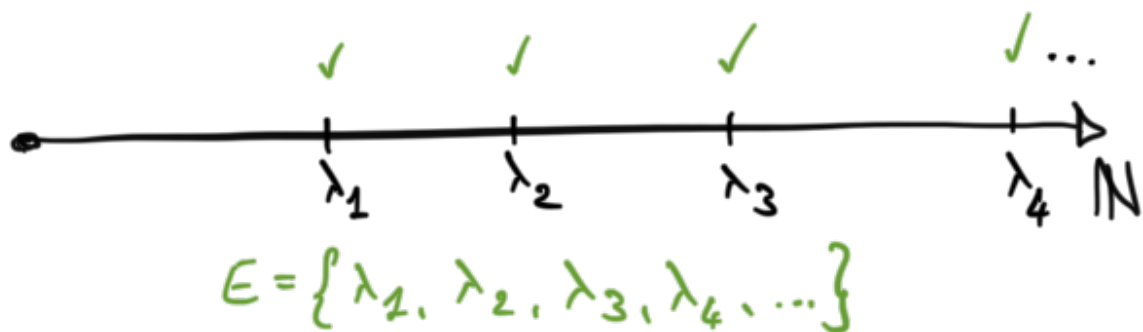
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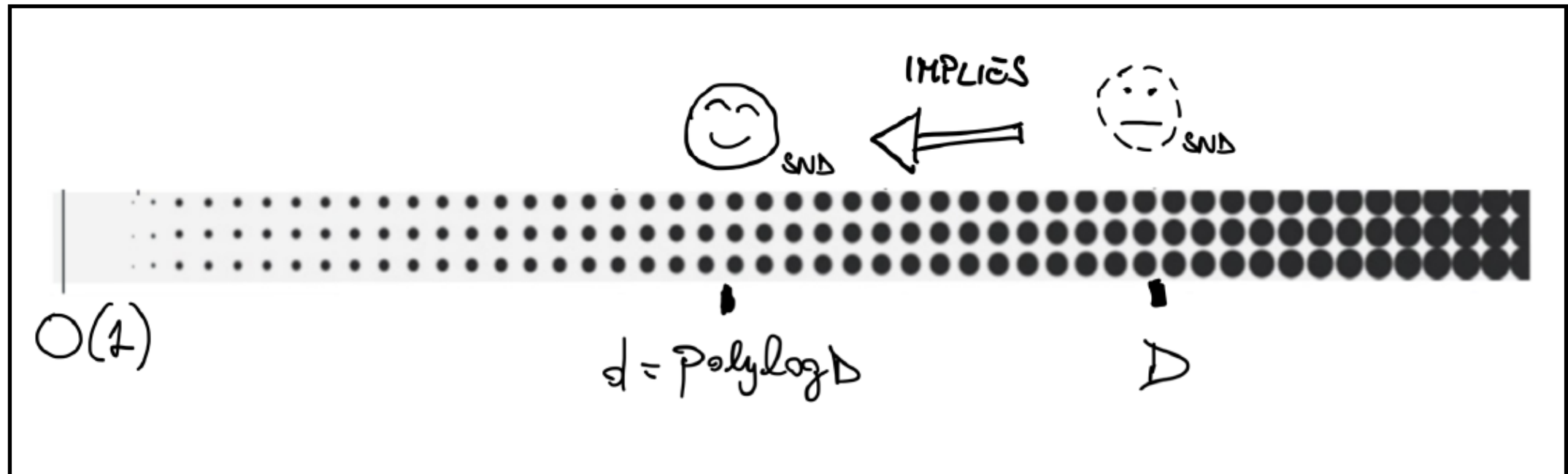
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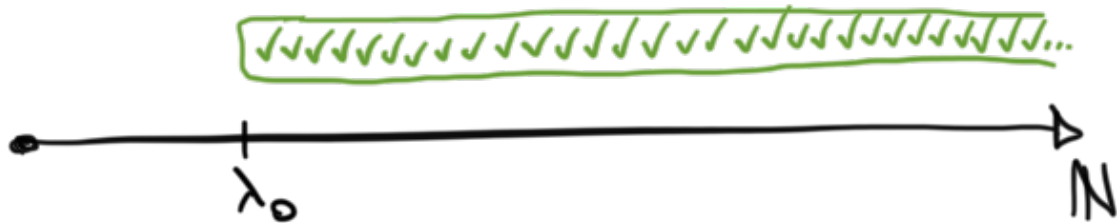
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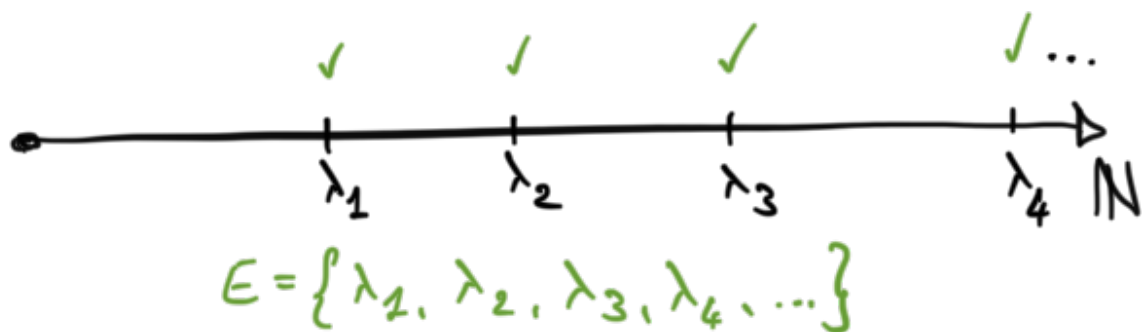


Cryptographers do find i.o. security interesting:

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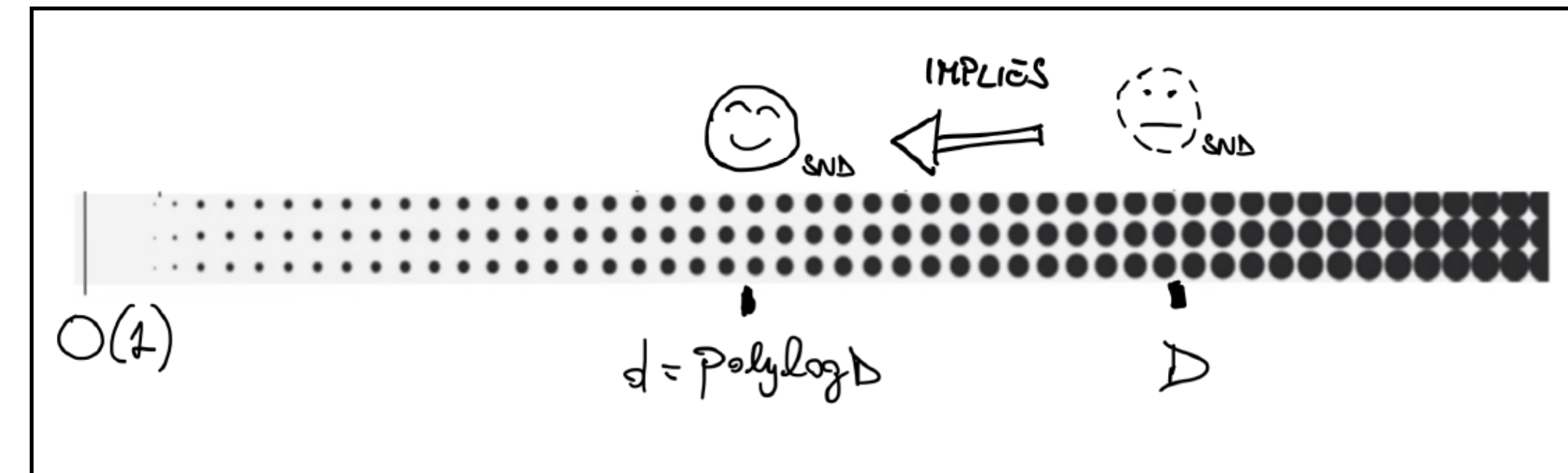
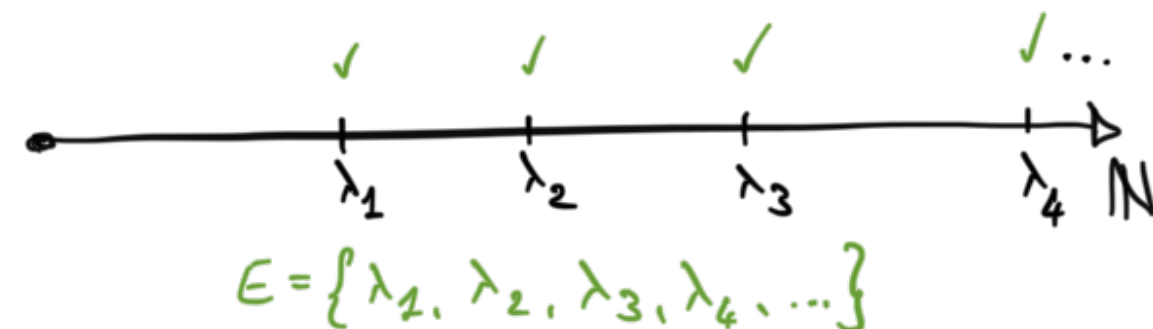
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A Note on Non-Interactive Zero-Knowledge from CDH

Geoffroy Couteau^{*}
Université Paris Cité, CNRS, IRIF

Abhishek Jain[†]
Johns Hopkins University

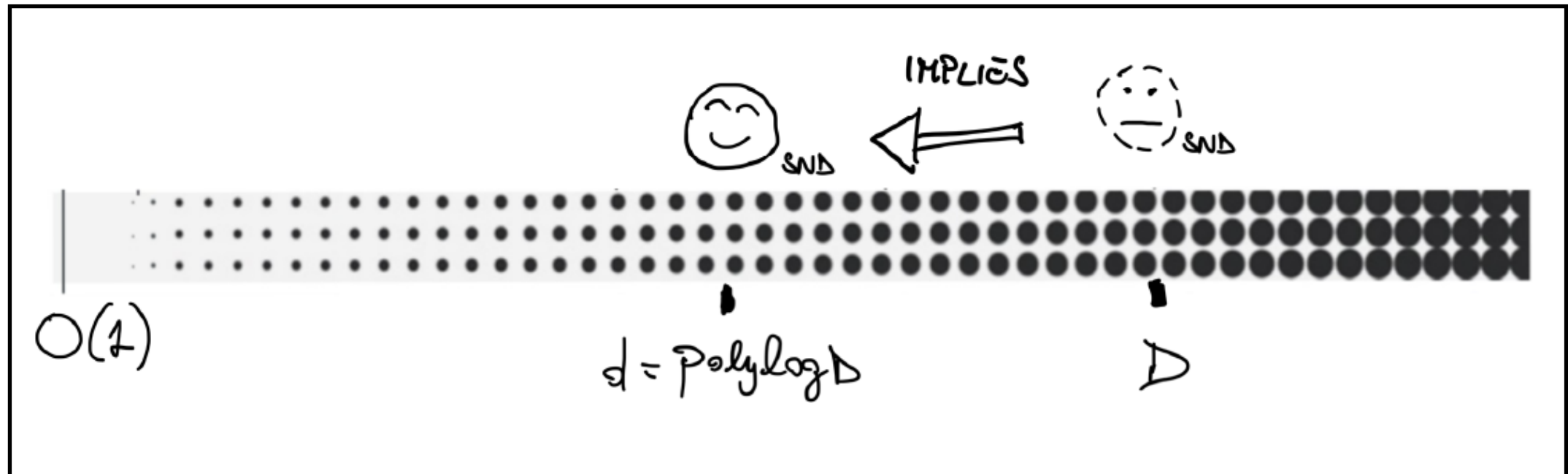
Zhengzhong Jin[‡]
MIT

Willy Quach[§]
Northeastern University

[CRYPTO '23]: builds i.o.-SND NIZKs from sub-exp CDH.

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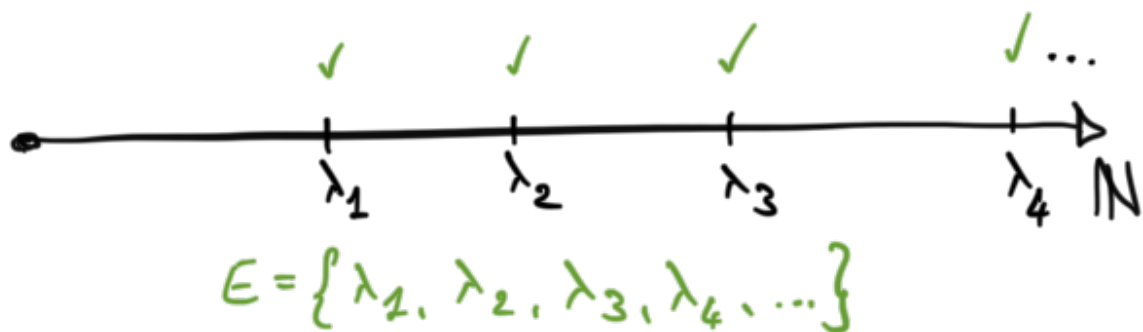
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Cryptographers do find i.o. security interesting:

A Note on Non-Interactive Zero-Knowledge from CDH

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MIT

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 Northeastern University

[CRYPTO '23]: builds i.o.-SND NIZKs from sub-exp CDH.

On the Possibility of Basing Cryptography on $\text{EXP} \neq \text{BPP}$ <div style="display: flex; justify-content: space-between;"> <div>Yanyi Liu Cornell University yl2866@cornell.edu</div> <div>Rafael Pass[*] Cornell Tech rafael@cs.cornell.edu</div> </div>	One-Way Functions and pKt Complexity <div style="display: flex; justify-content: space-between;"> <div>Shuichi Hirahara[*]</div> <div>Zhenjian Lu[†]</div> <div>Igor C. Oliveira[‡]</div> </div>
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[CRYPTO '21,TCC '24]:
connect \exists of i.o.-OWF to certain worst-case assumptions.

Our Results

(continued)

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(continued)

Motivating question for next result:

Let Π be an IVC (e.g., secure at $O(1)$ depth).

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(continued)

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(continued)

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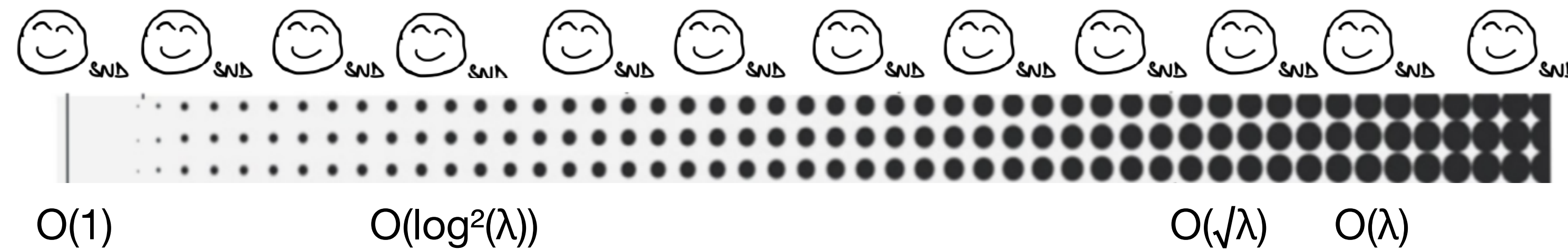
Case 1: security everywhere.

Our Results

(continued)

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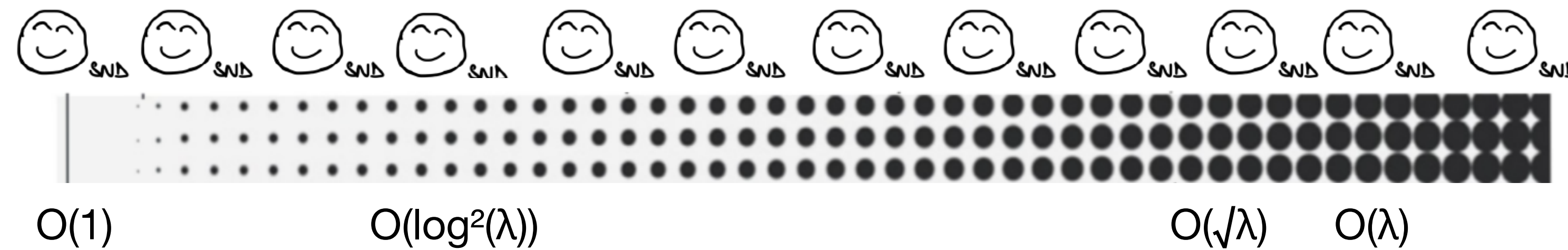
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Case 1: security everywhere.

Our Results

(continued)

Motivating question for next result:

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Our Results

(continued)

Motivating question for next result:

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Case 2: insecure somewhere.

Our Results

(continued)

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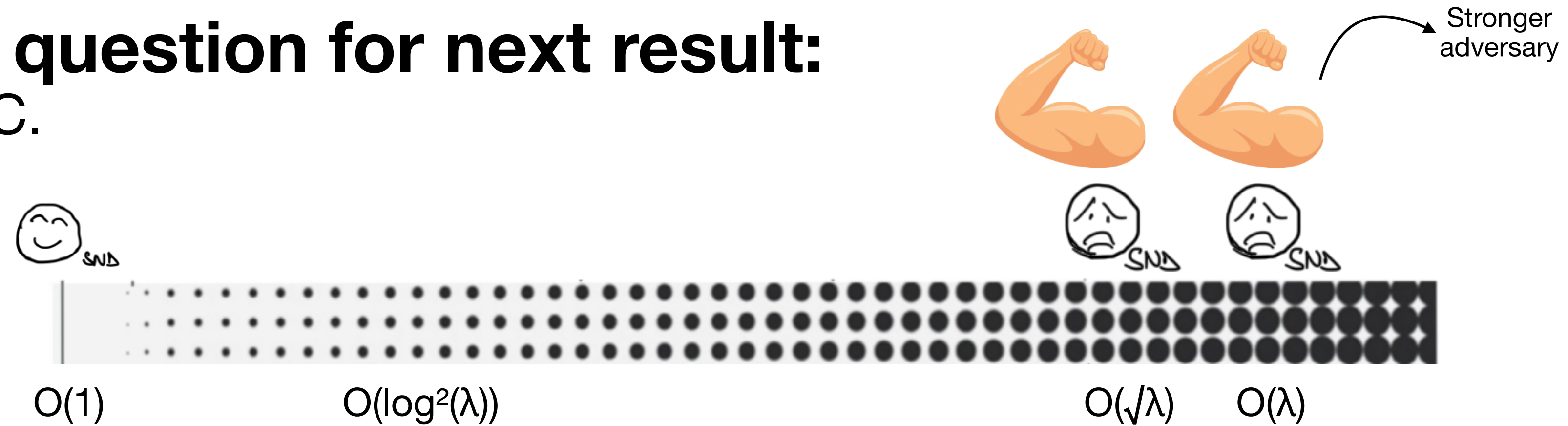
Case 2: insecure somewhere.

Our Results

(continued)

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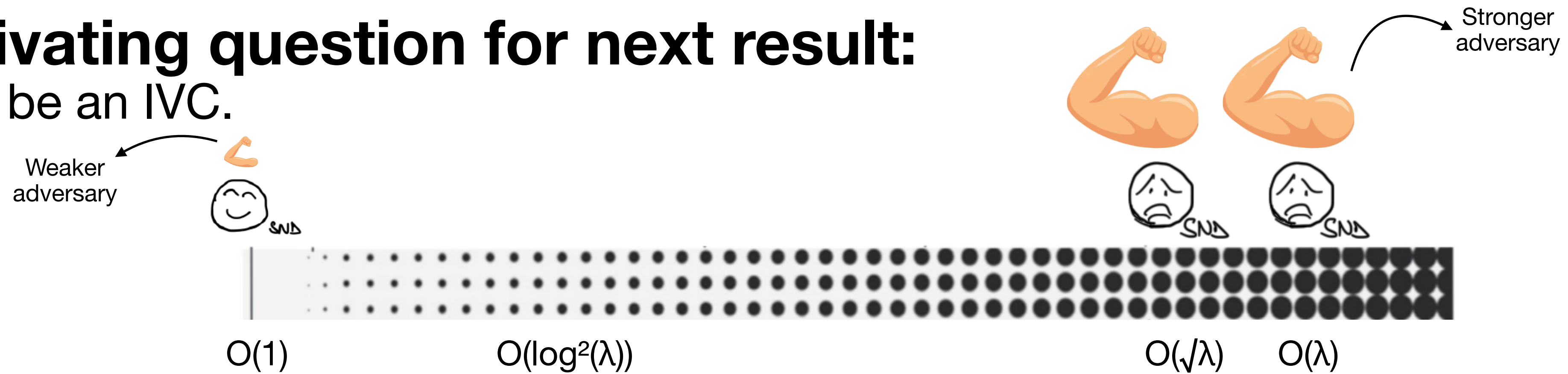
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(continued)

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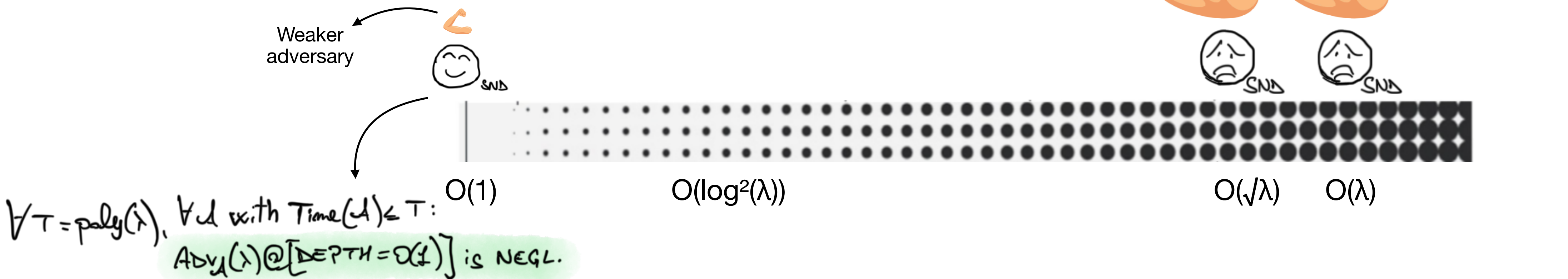
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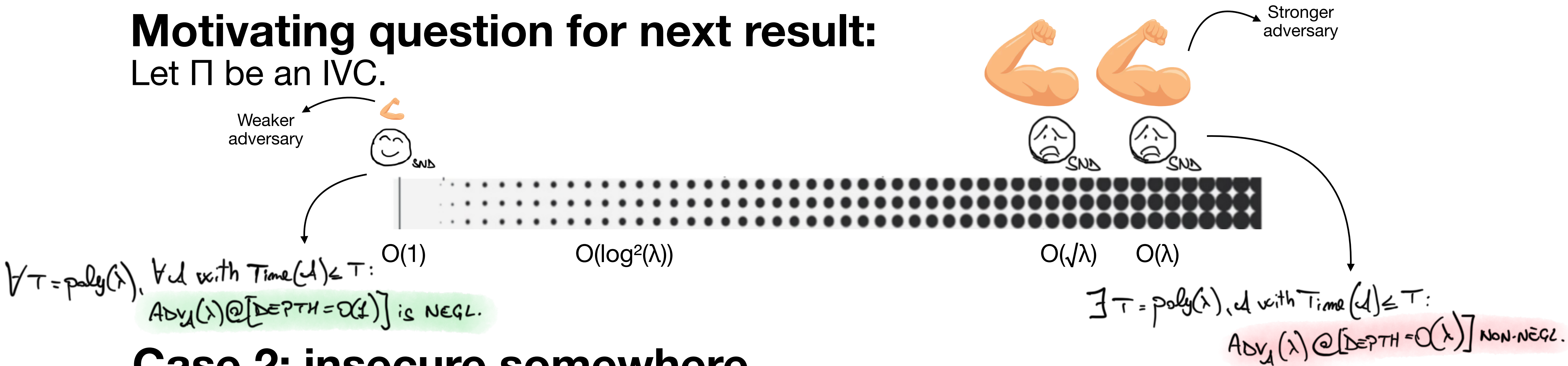
Case 2: insecure somewhere.

Our Results

(continued)

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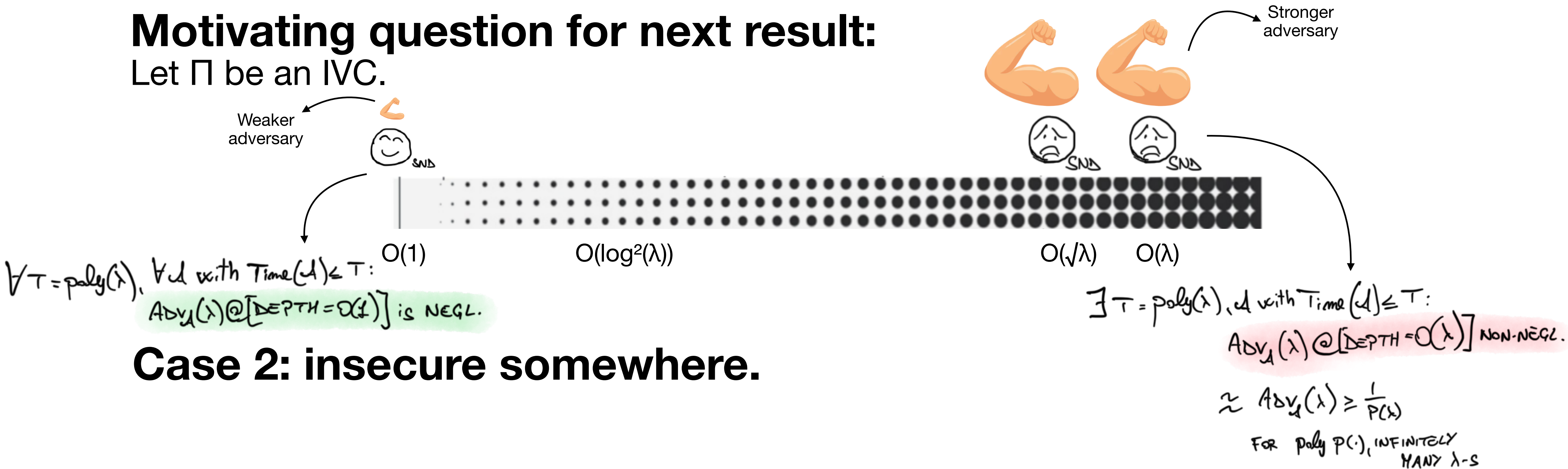
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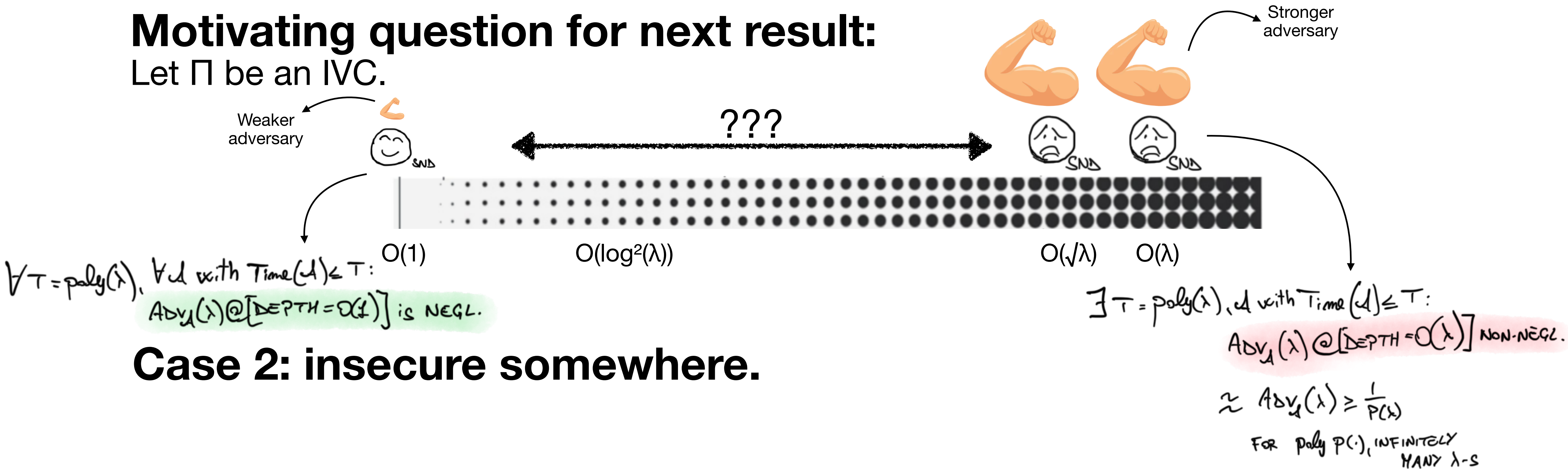


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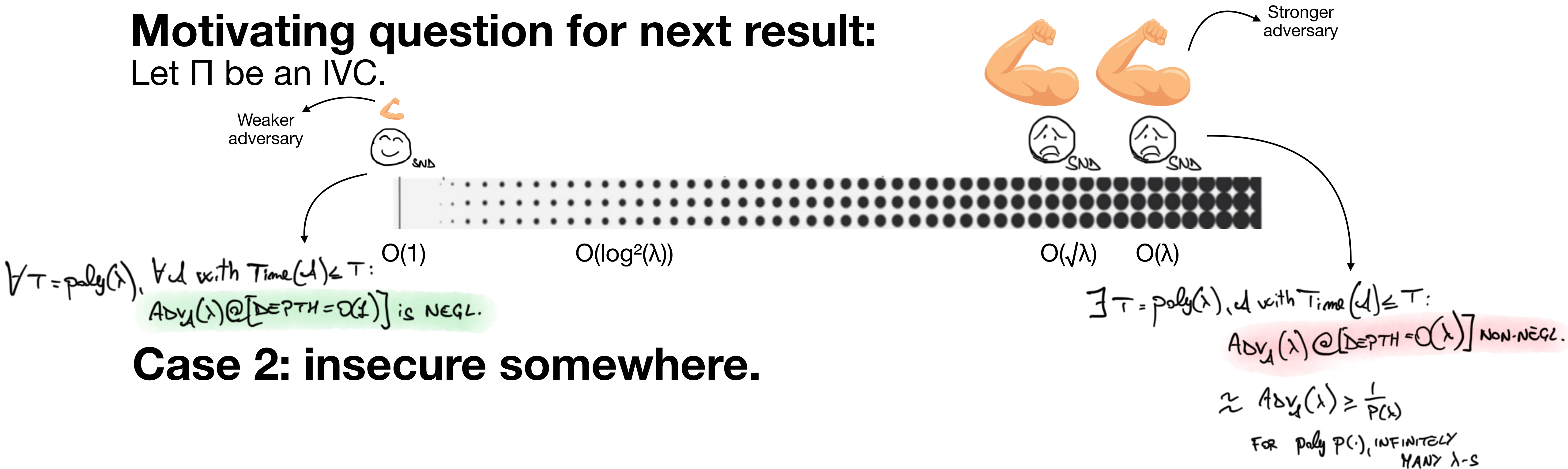


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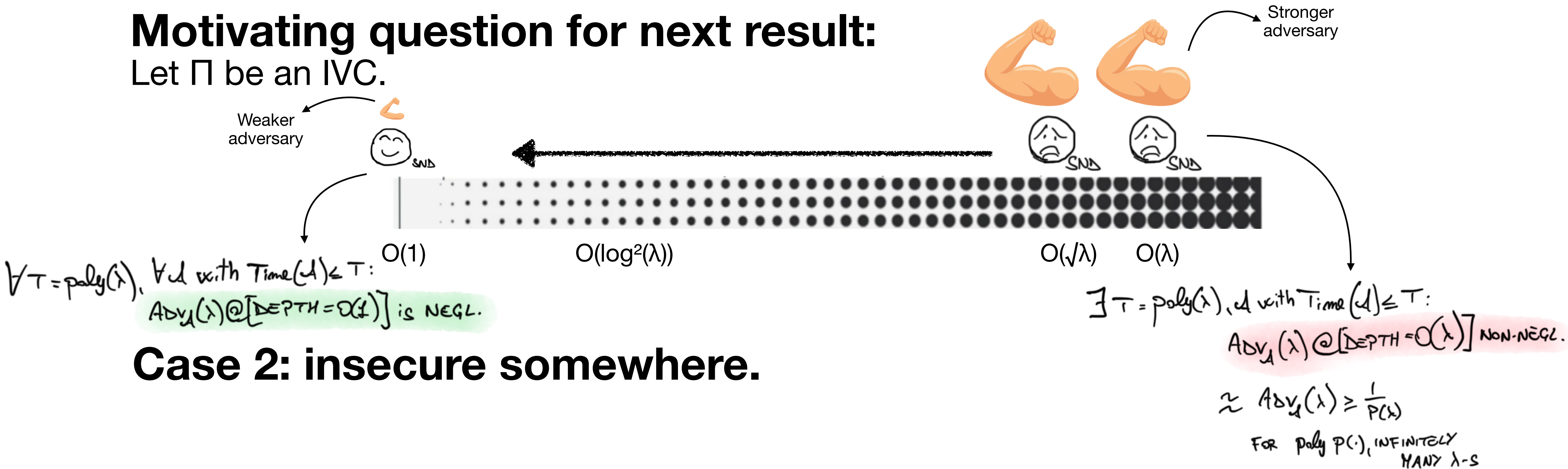
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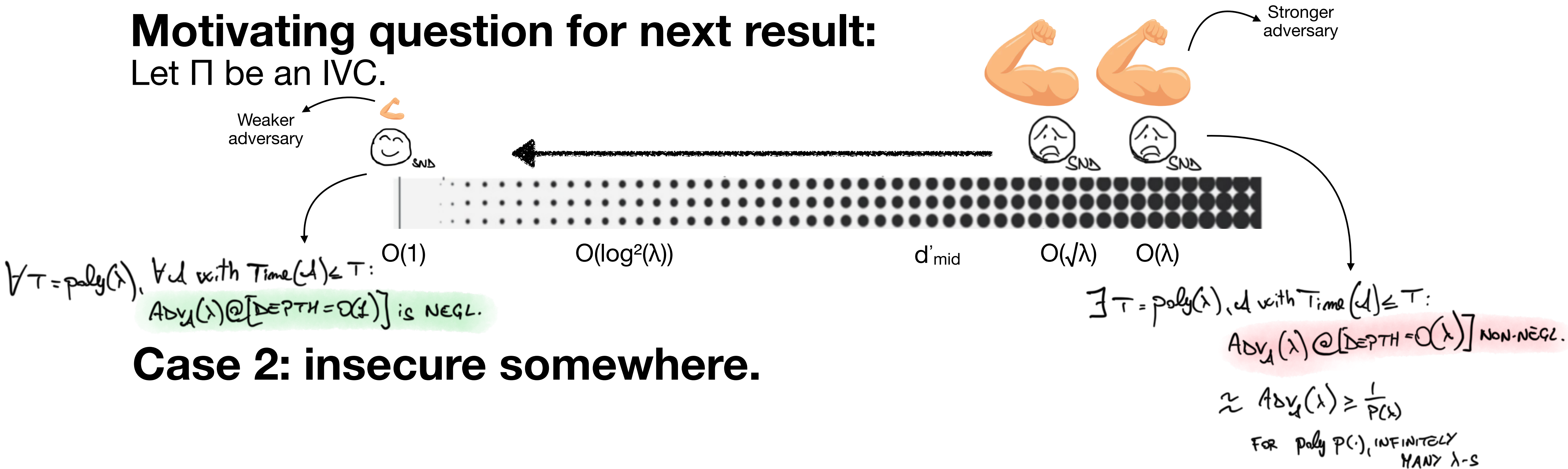


Case 2: insecure somewhere.

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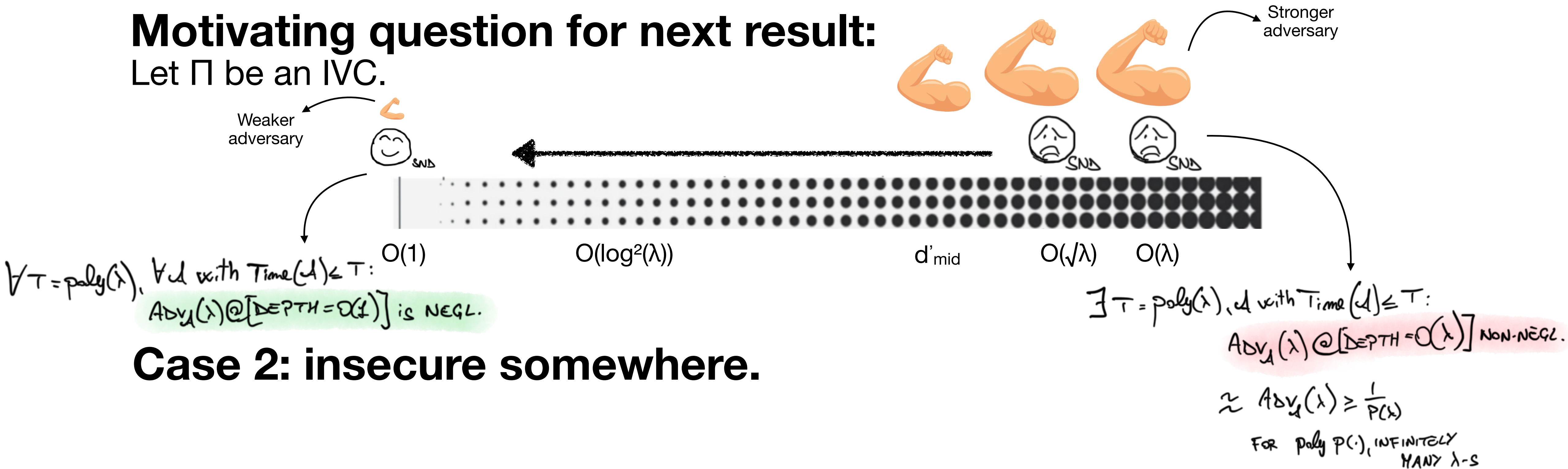


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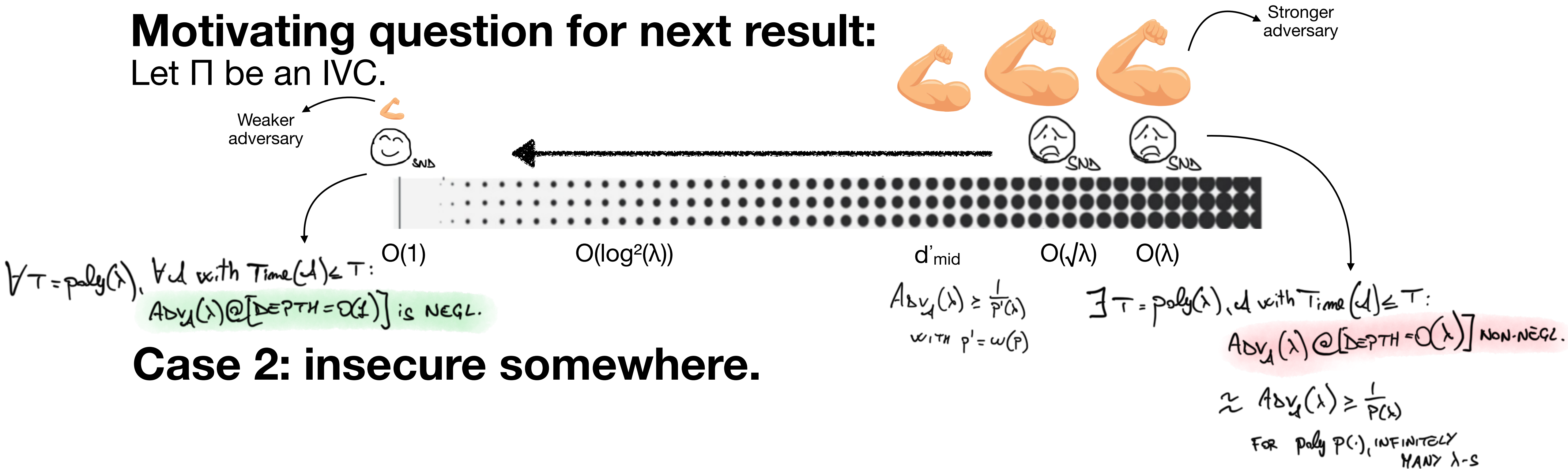


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Our Results

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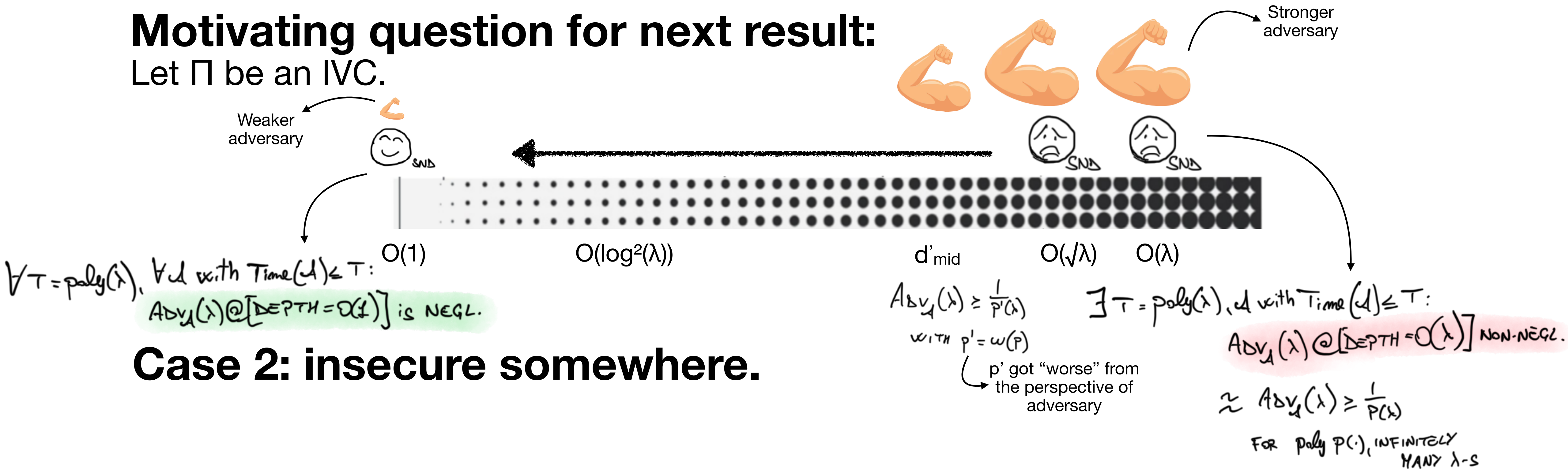


Case 2: insecure somewhere.

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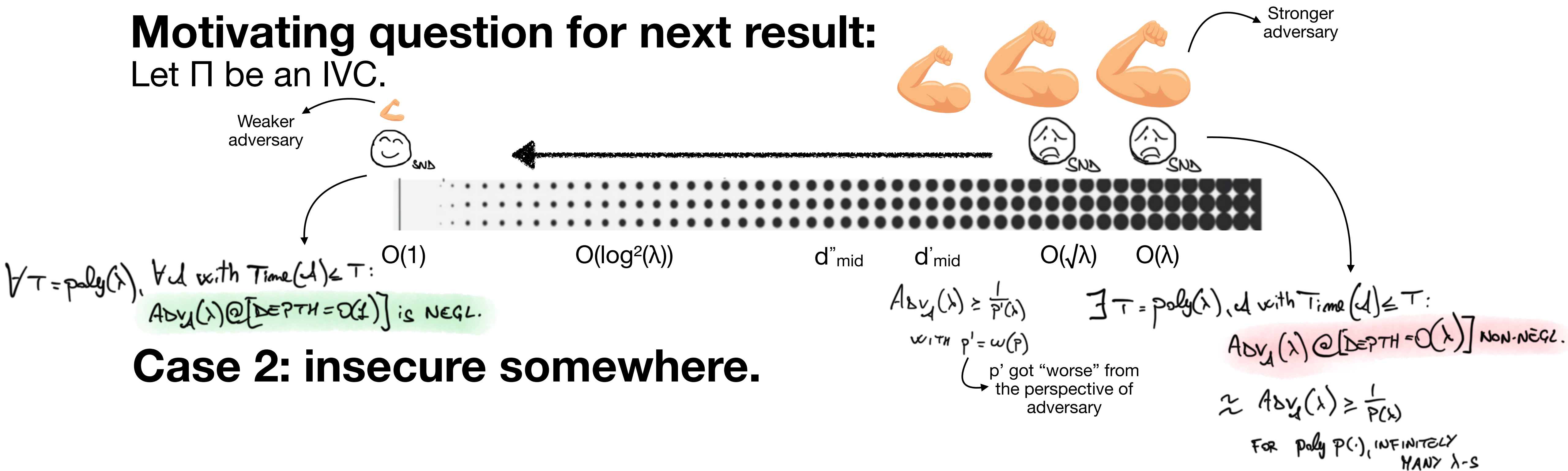
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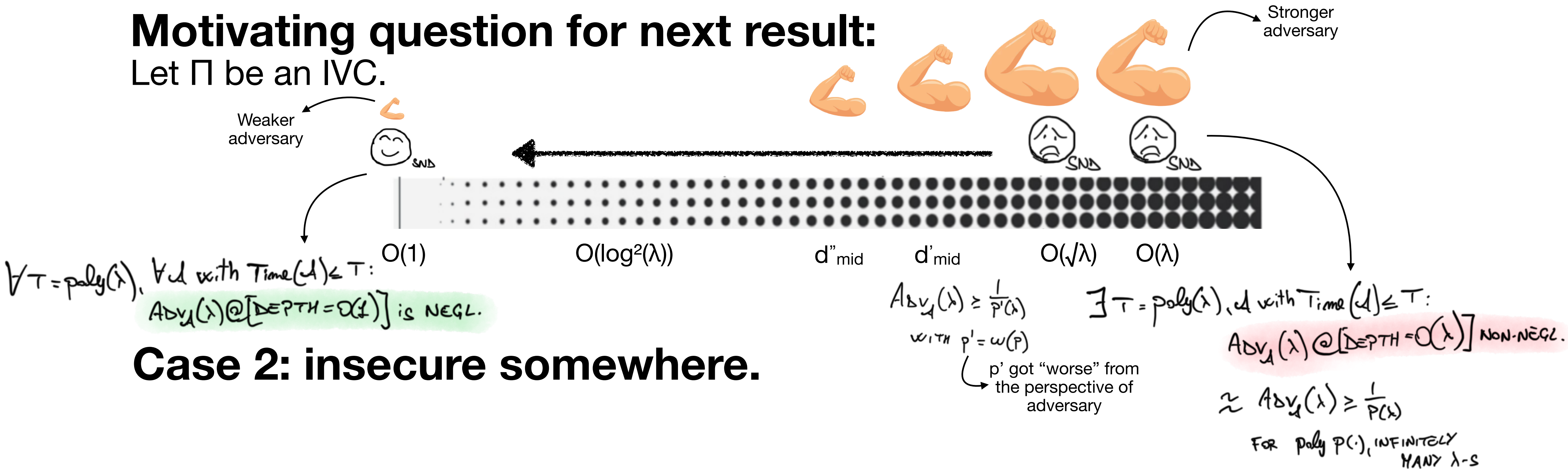
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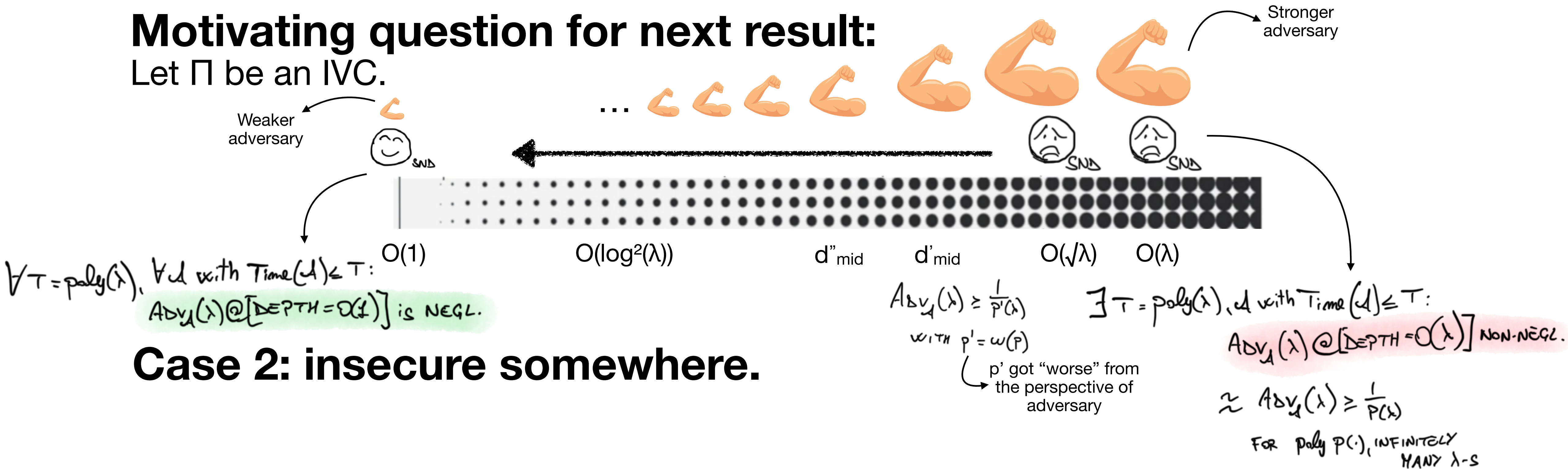


Case 2: insecure somewhere.

Our Results

(continued)

Motivating question for next result:
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Case 2: insecure somewhere.

Our Results (continued)

Motivating question for next result:
Let Π be an IVC.

We call this (potential) pattern in IVC
graceful security degradation

Weaker
adversary

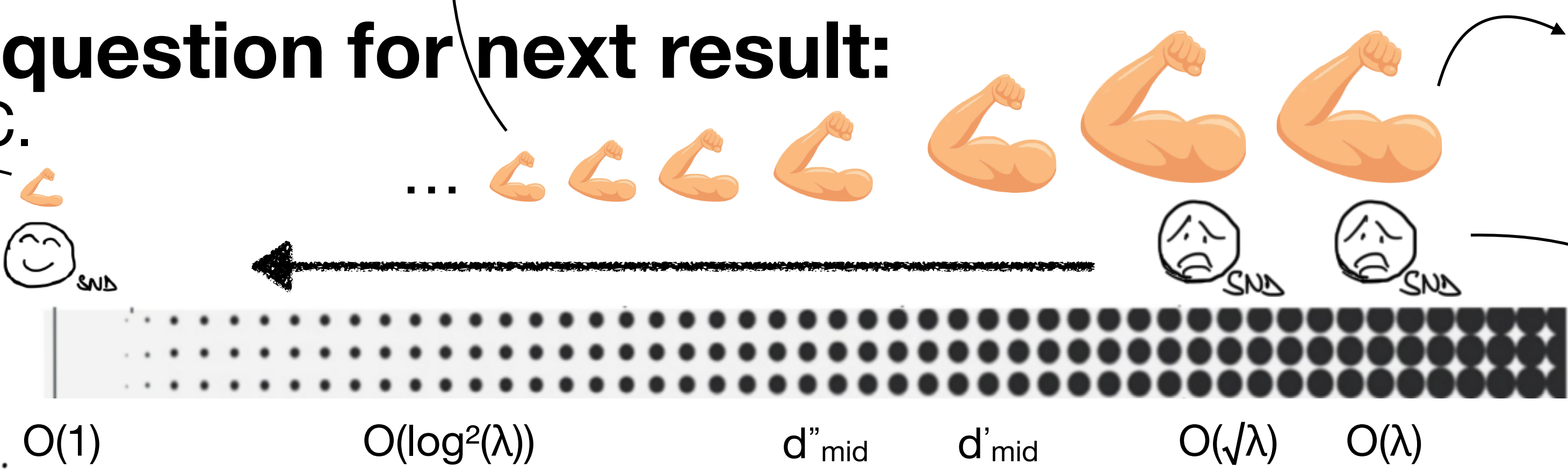
Stronger
adversary

$\forall T = \text{poly}(\lambda), \forall \mathcal{A}$ with $\text{Time}(\mathcal{A}) \leq T$:
 $\text{Adv}_{\mathcal{A}}(\lambda) @ [\text{DEPTH} = O(1)]$ is NEGL.

Case 2: insecure somewhere.

$\text{Adv}_{\mathcal{A}}(\lambda) \geq \frac{1}{p(\lambda)}$
with $p' = \omega(p)$
p' got "worse" from
the perspective of
adversary

$\exists T = \text{poly}(\lambda), \mathcal{A}$ with $\text{Time}(\mathcal{A}) \leq T$:
 $\text{Adv}_{\mathcal{A}}(\lambda) @ [\text{DEPTH} = O(\lambda)]$ NON-NEGL.
 $\approx \text{Adv}_{\mathcal{A}}(\lambda) \geq \frac{1}{p(\lambda)}$
FOR $\text{poly } p(\cdot)$, INFINITELY
MANY λ -S



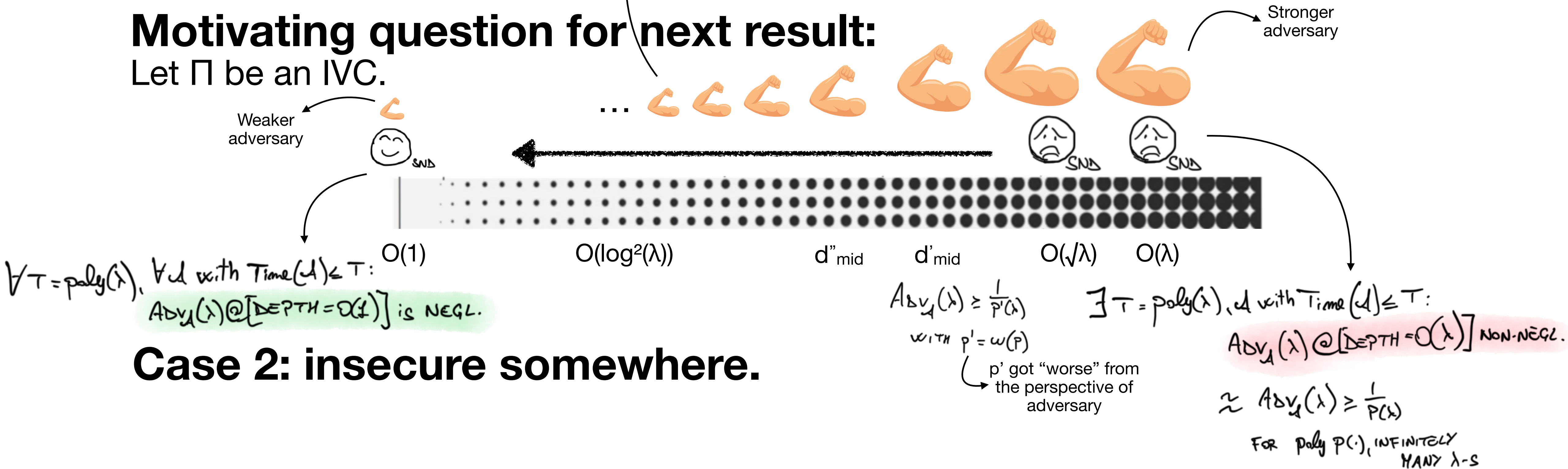
Our Results (continued)

Q: Can an IVC exhibit it?

We call this (potential) pattern in IVC graceful security degradation

Motivating question for next result:

Let Π be an IVC.



Case 2: insecure somewhere.

Our Results (continued)

Q: Can an IVC exhibit it?

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graceful security degradation

A practical framing around graceful sec. degradation:

Motivating question for next result:

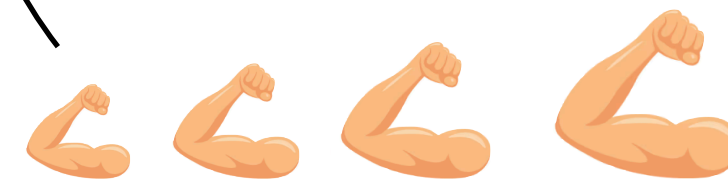
Let Π be an IVC.

Weaker
adversary



SND

...



Stronger
adversary



SND



SND

$O(1)$

$O(\log^2(\lambda))$

d''_{mid}

d'_{mid}

$O(\sqrt{\lambda})$

$O(\lambda)$

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MANY λ -S

Our Results (continued)

Q: Can an IVC exhibit it?

We call this (potential) pattern in IVC
graceful security degradation

A practical framing around graceful sec. degradation:

 \approx better and better inverse poly-s

Motivating question for next result:

Let Π be an IVC.

Weaker
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Our Results (continued)

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graceful security degradation

A practical framing around graceful sec. degradation:

🦵🦵🦵🦵🦵 ≈ better and better inverse poly-s

And cryptographers do sometimes work with inverse poly sec.

Motivating question for next result:

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Our Results (continued)

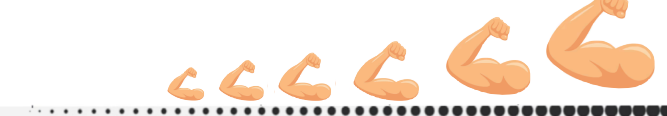

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MANY λ -S

Case 2: insecure somewhere.

Result ("no free snack" theorem):

Let Π be an IVC. Then:

Our Results (continued)

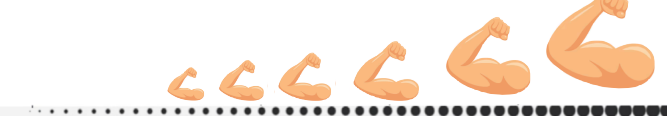
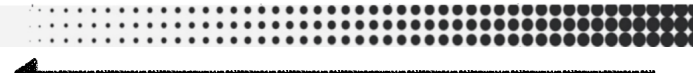
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A practical framing around graceful sec. degradation:

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 may offer tradeoffs to practitioners.

Motivating question for next result:

Let Π be an IVC.

Weaker
adversary



...



Stronger
adversary



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FOR $\text{poly } p(\cdot)$, INFINITELY
MANY λ -S

Case 2: insecure somewhere.

Result ("no free snack" theorem):

Let Π be an IVC. Then:

- either Π is secure at arbitrary polynomial depths,

Our Results (continued)

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...



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FOR $\text{poly } p(\cdot)$, INFINITELY
MANY λ -S

Case 2: insecure somewhere.

Result ("no free snack" theorem):

Let Π be an IVC. Then:

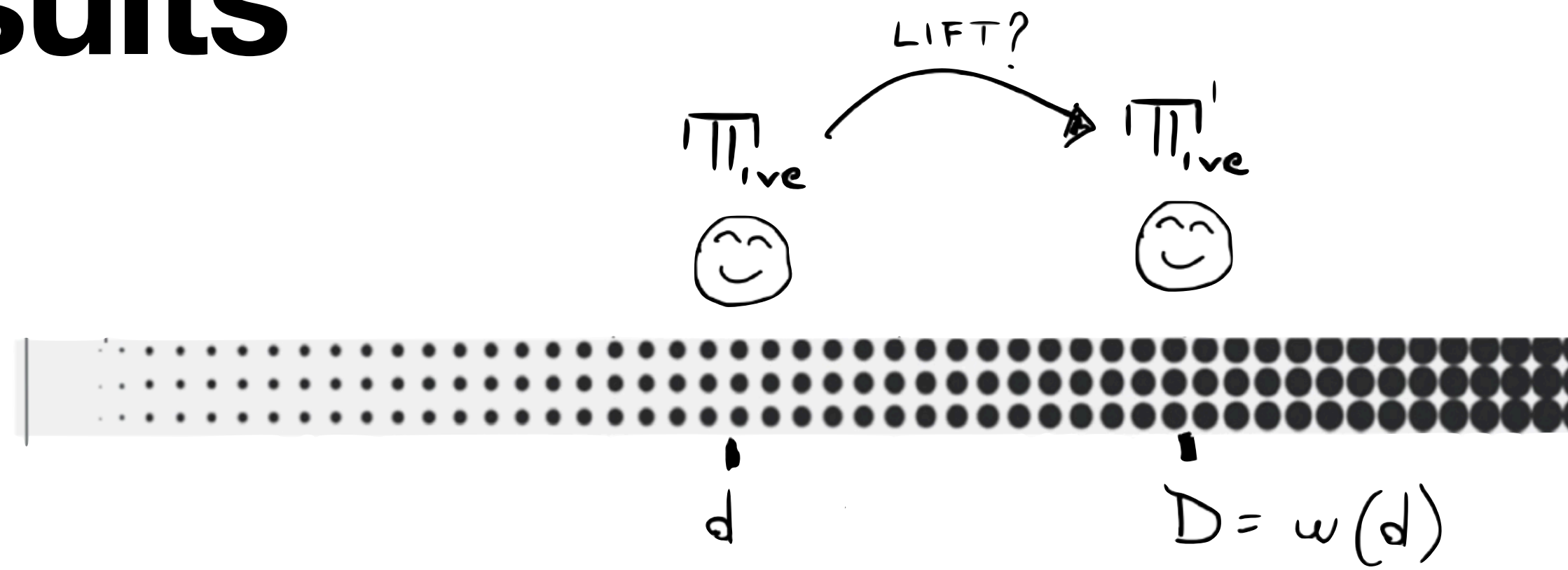
- either Π is secure at arbitrary polynomial depths,
- or Π cannot exhibit graceful security degradation.

Our Results

(continued)

Our Results

(continued)

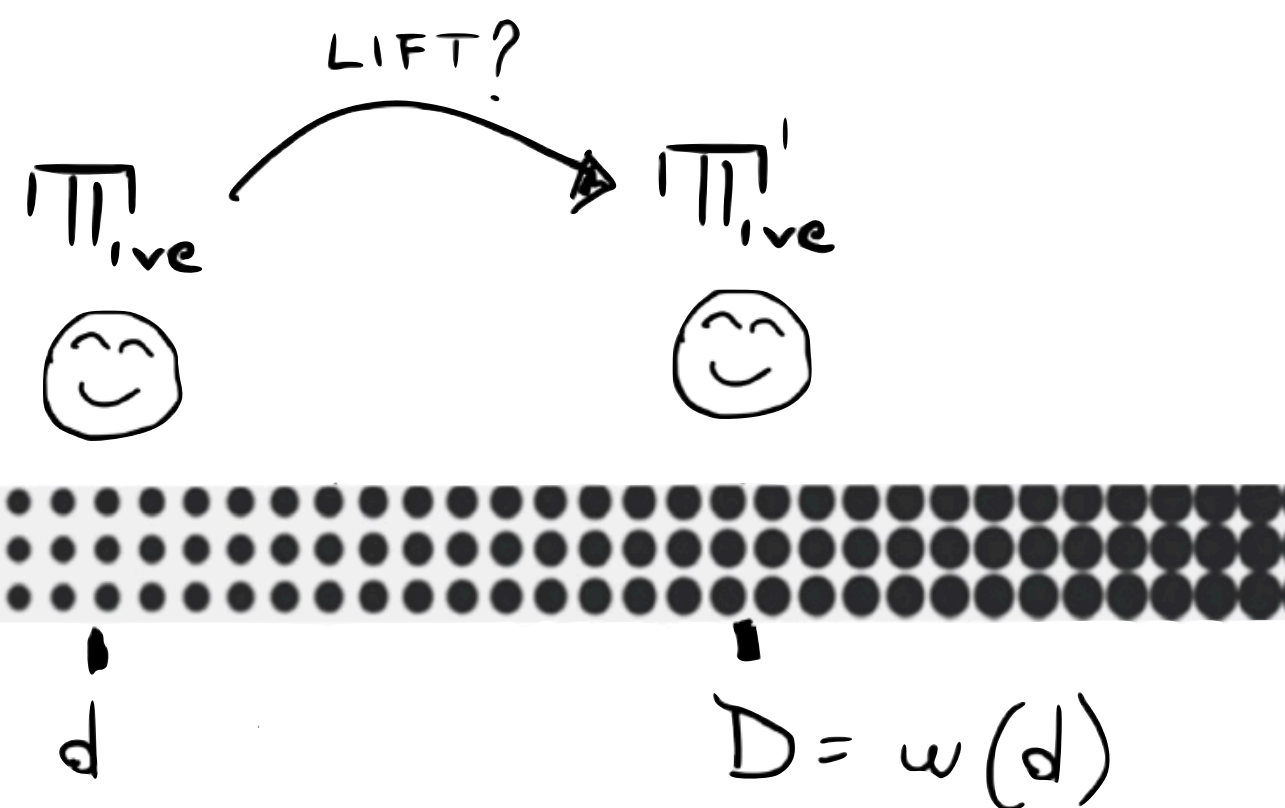


Our Results

(continued)

NB: We are interested in:

- black-box lifting results,
- and that preserve performance.

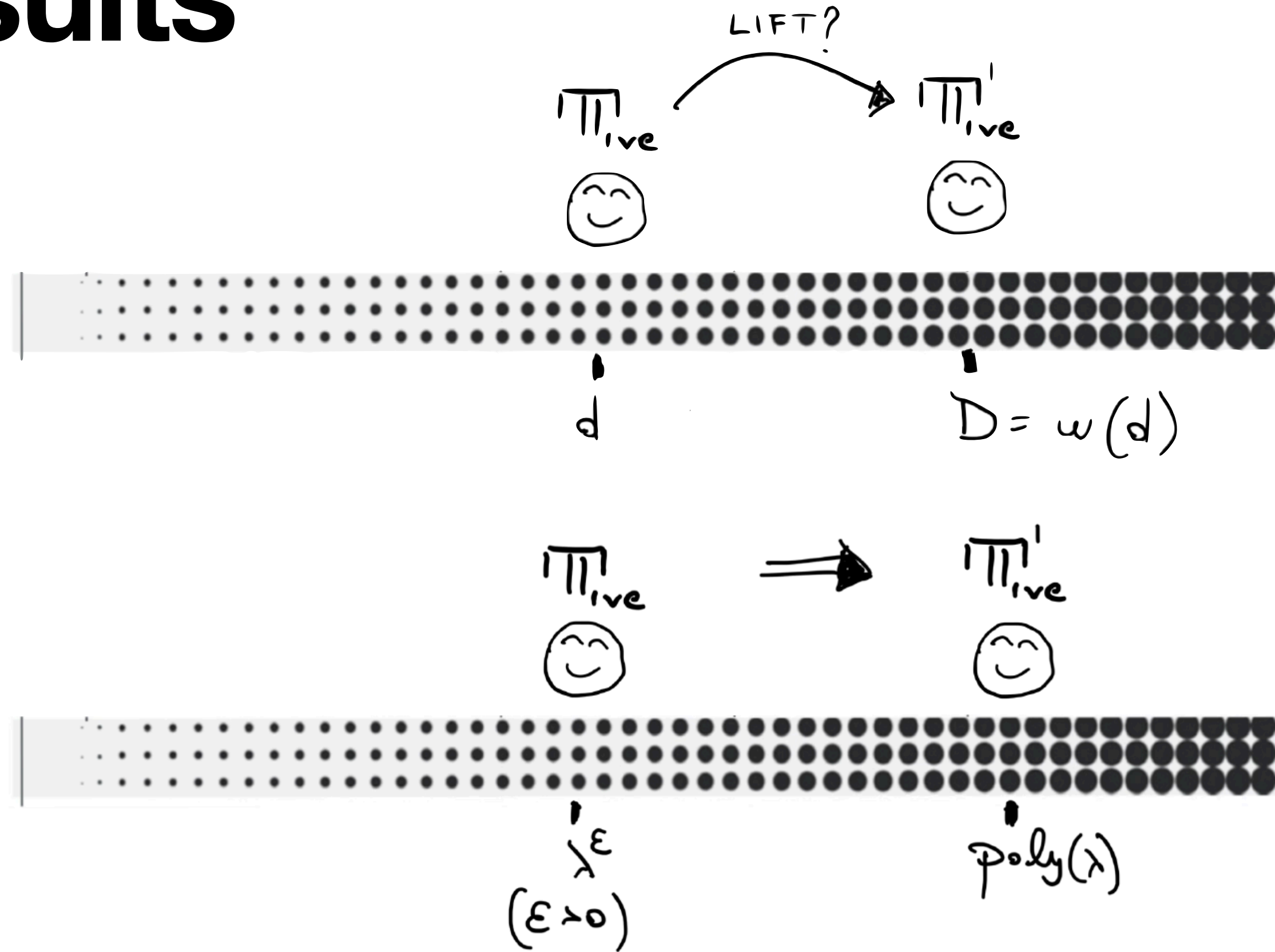


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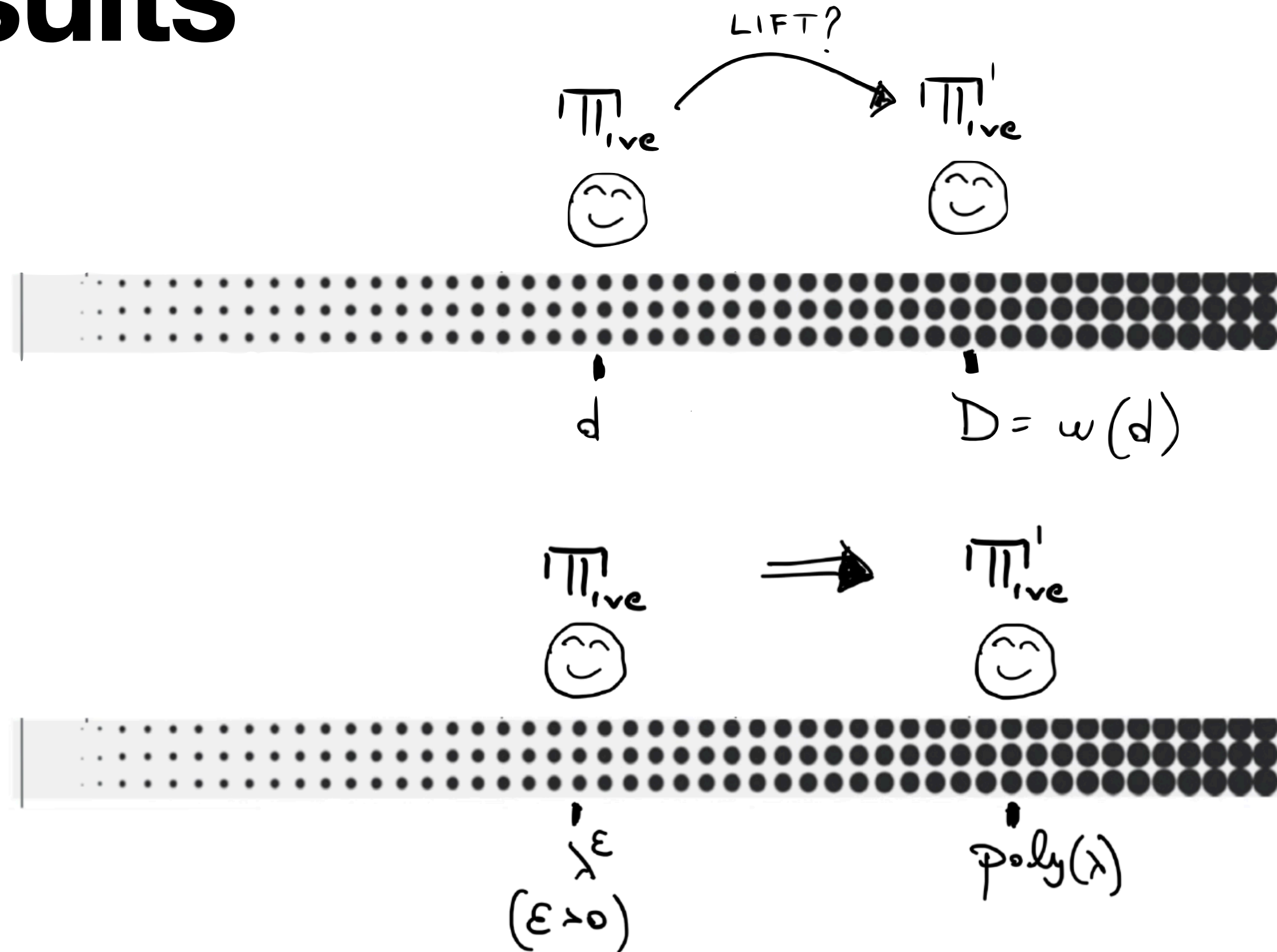


Our Results

(continued)

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Theorem (sublinear depths):

\exists IVC Π SND at depth λ^ϵ (for some $\epsilon > 0$)
 $\Rightarrow \exists$ IVC Π' SND at arbitrary depth.

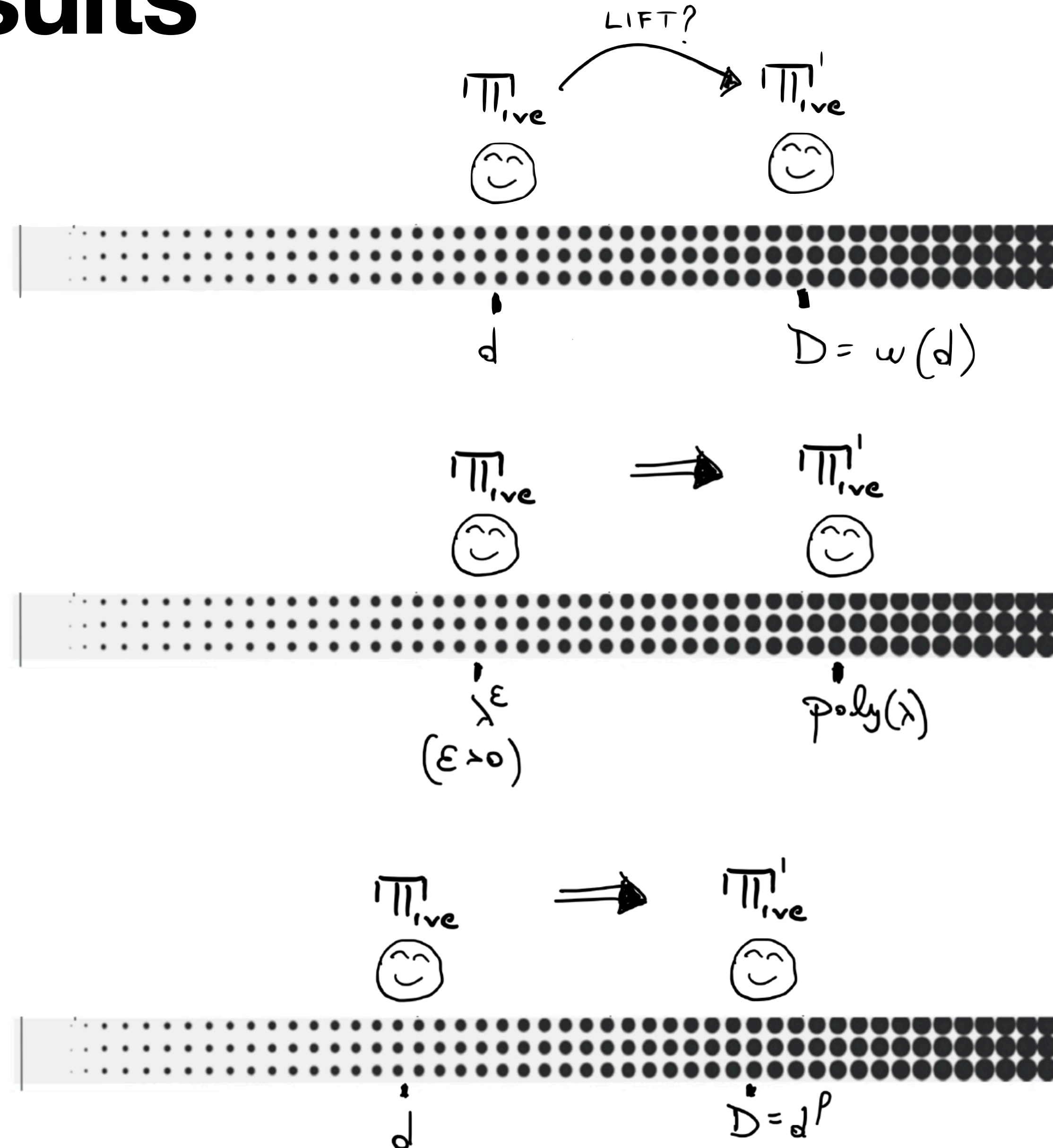
Overhead for P/V/proof size in Π' is $O_\lambda(1)$.

Our Results

(continued)

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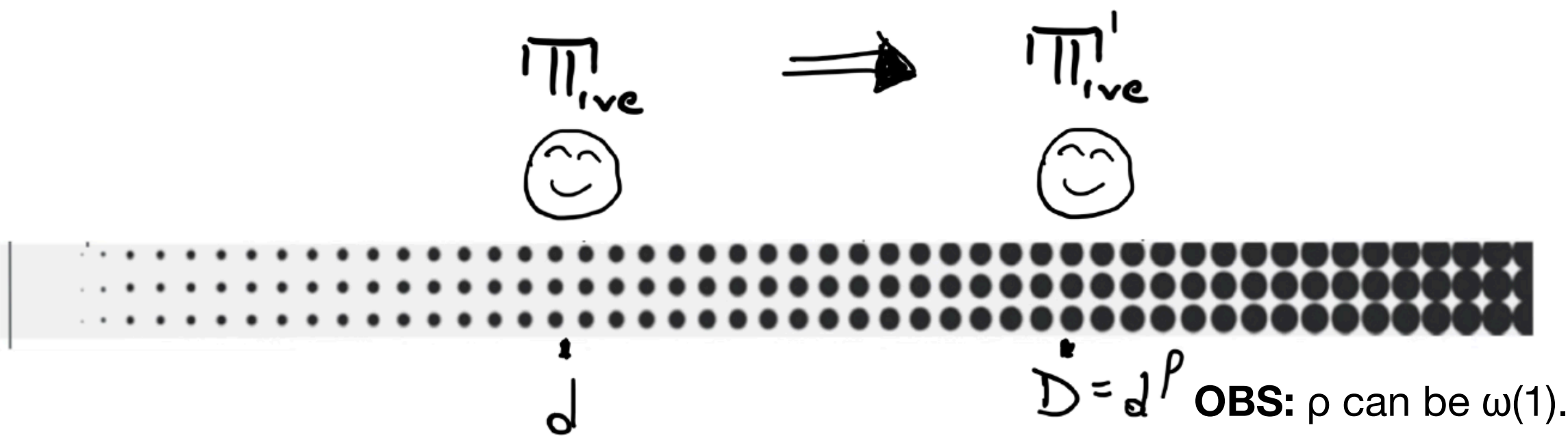
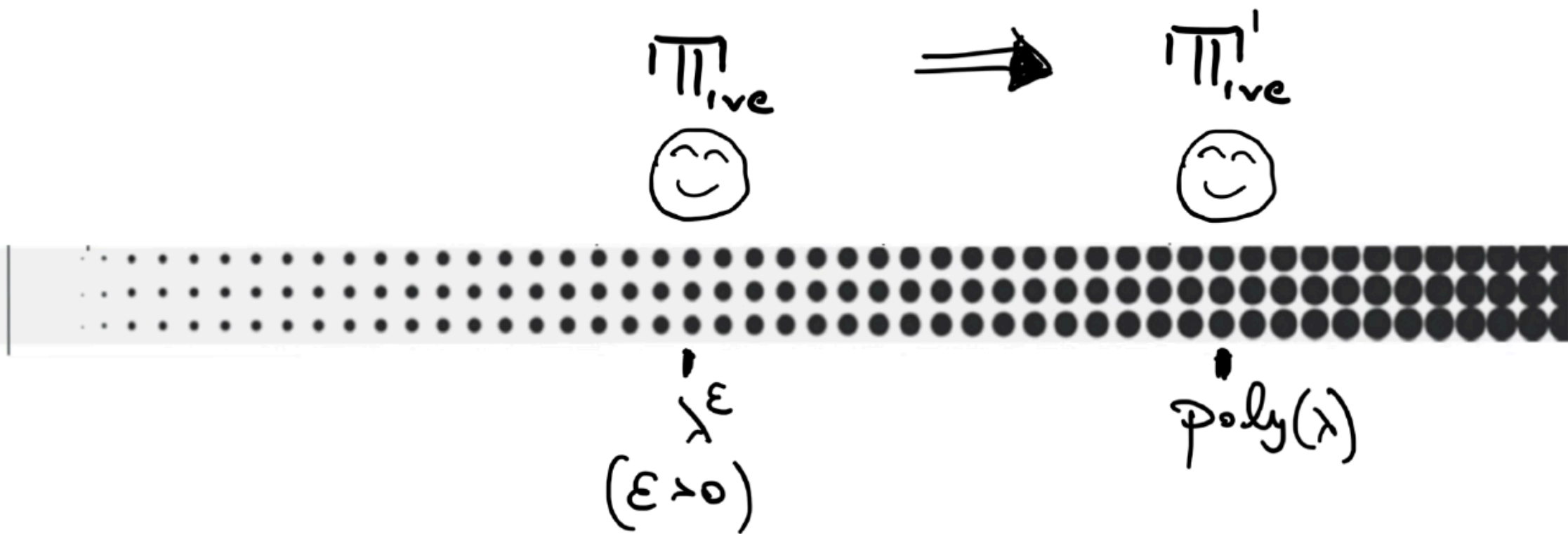
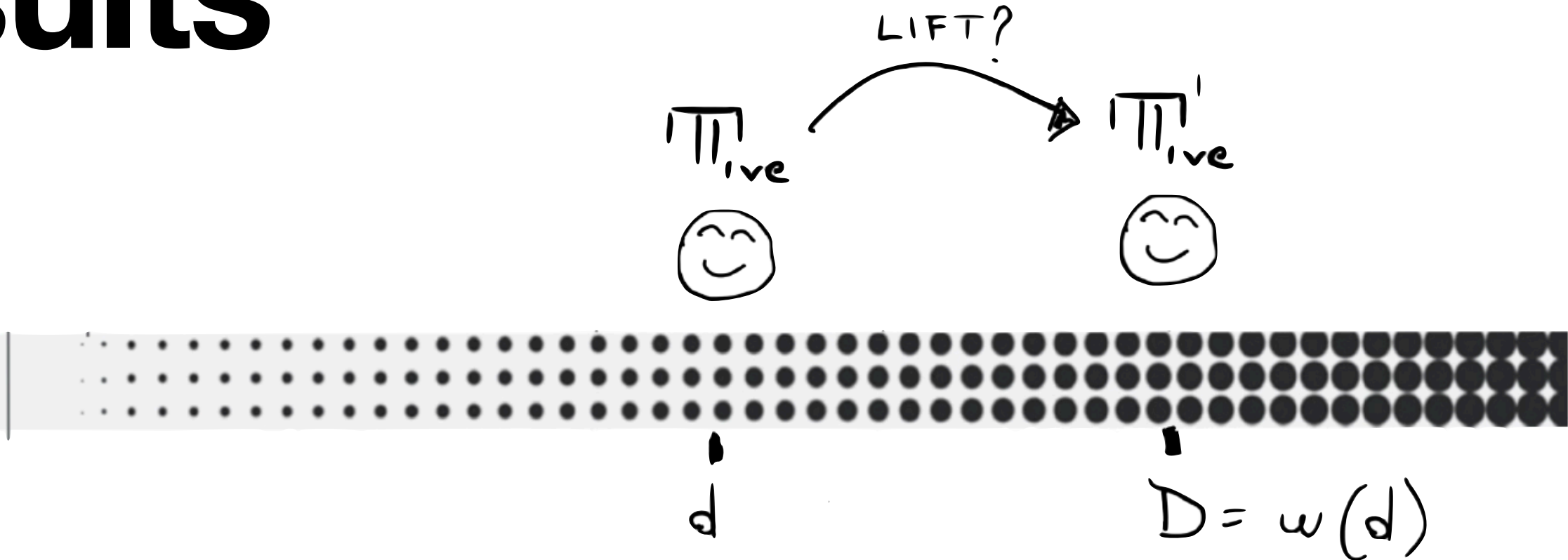
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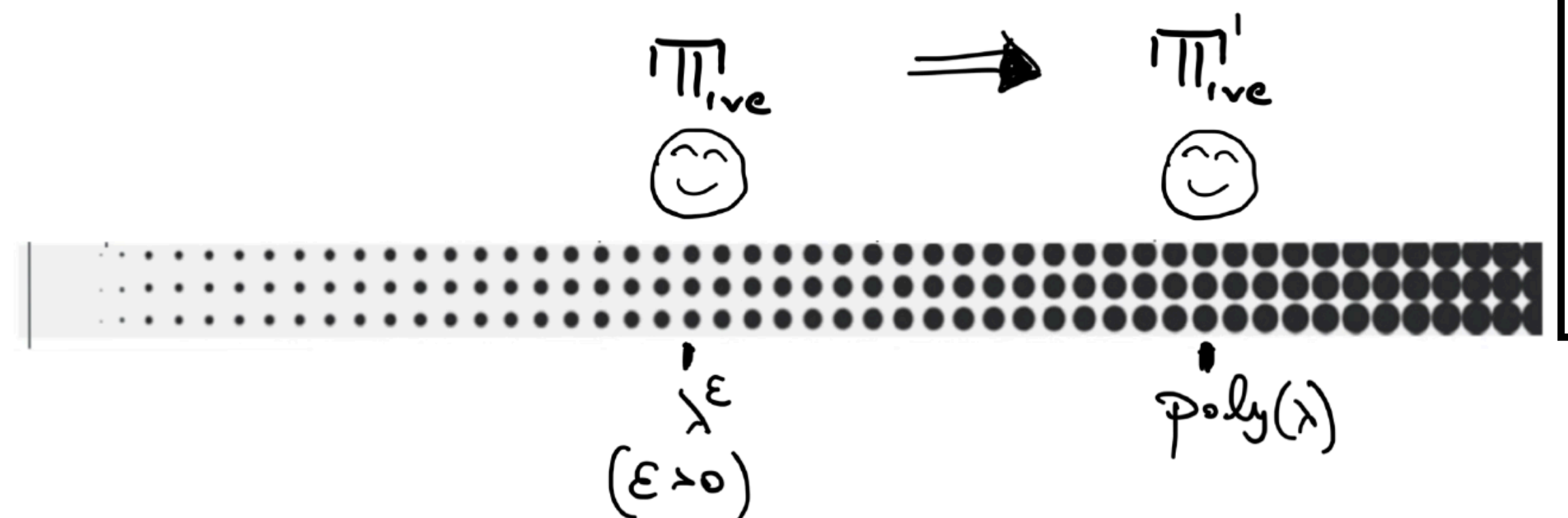
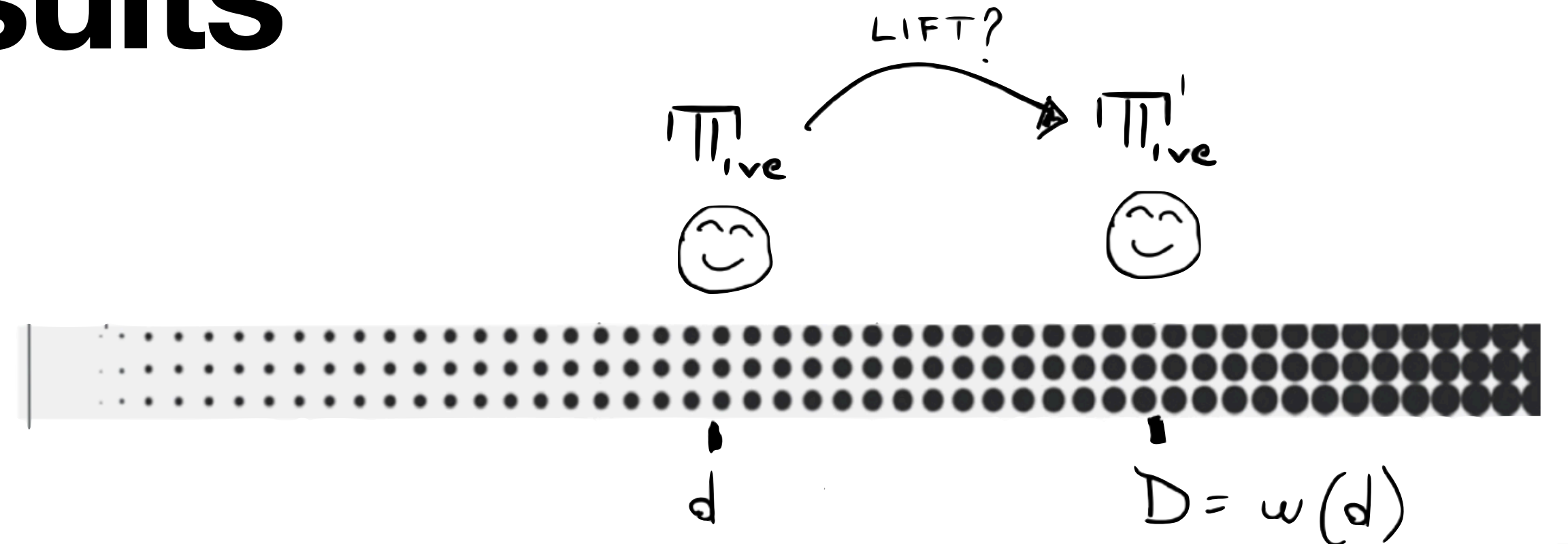
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Our Results

(continued)

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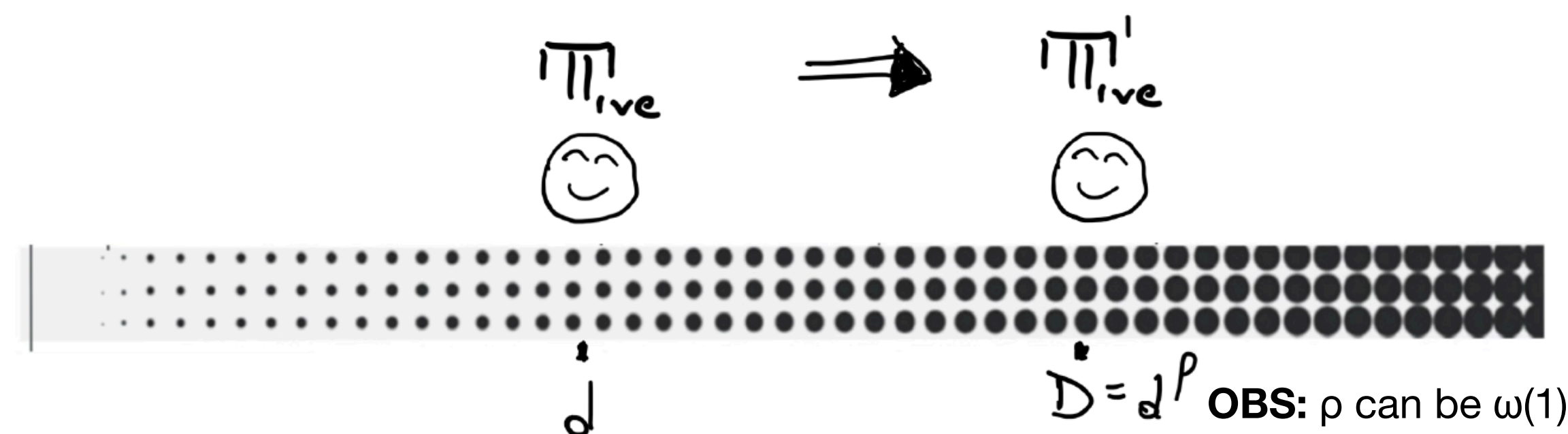
- black-box lifting results,
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Theorem (sublinear depths):

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 $\Rightarrow \exists$ IVC Π' SND at arbitrary depth.

Overhead for P/V/proof size in Π' is $O_\lambda(1)$.



Theorem (general lifting):

\exists IVC Π SND at depth d
 $\Rightarrow \exists$ IVC Π' SND at depth $D = d^\rho$.

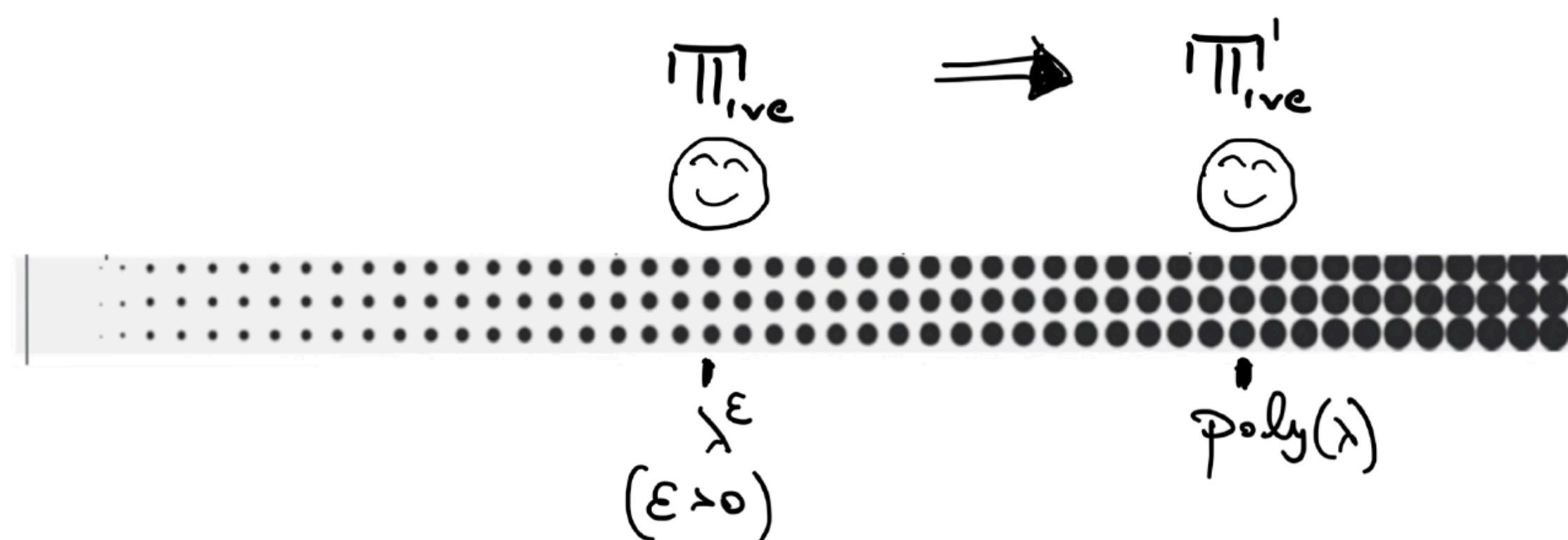
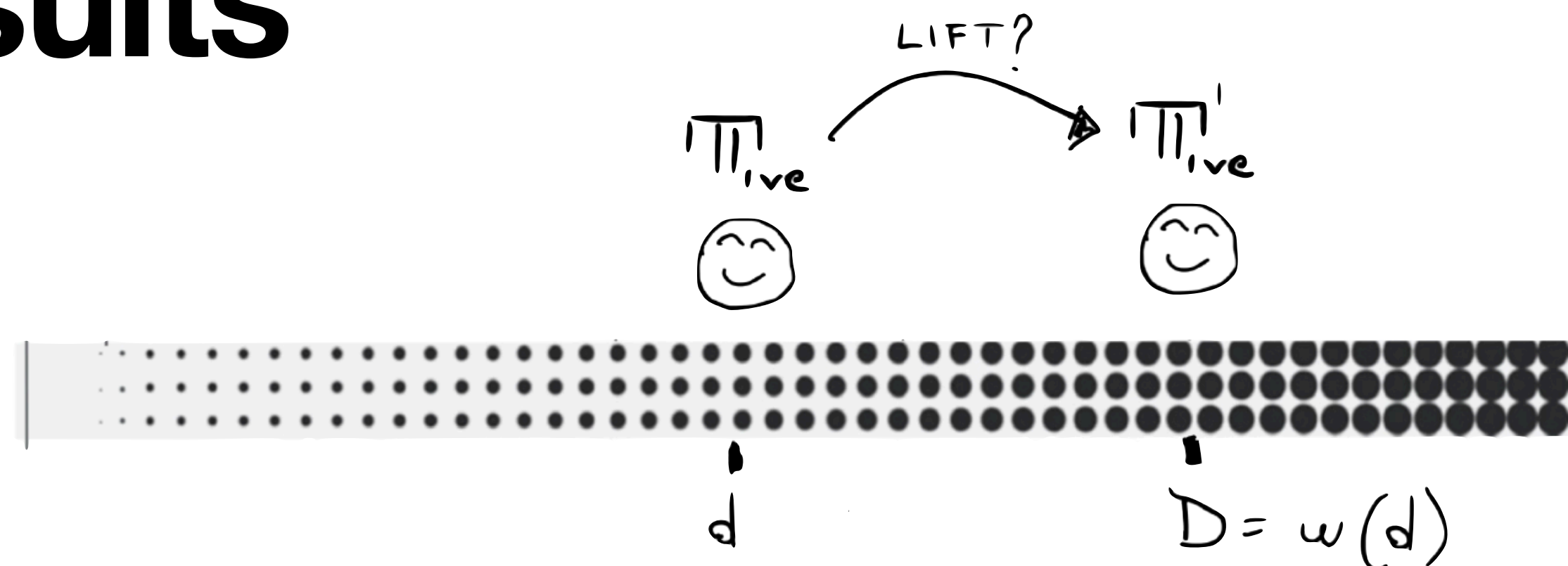
Overhead* for P/V/proof size in Π' is $O_\lambda(\rho)$

Our Results

(continued)

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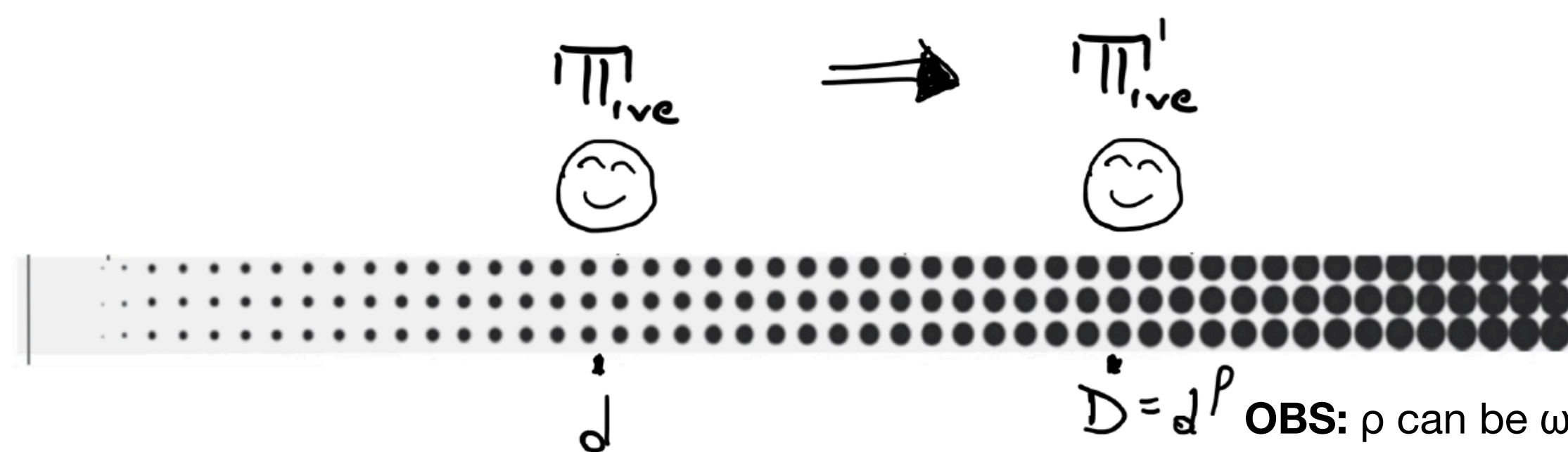
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\exists IVC Π SND at depth d
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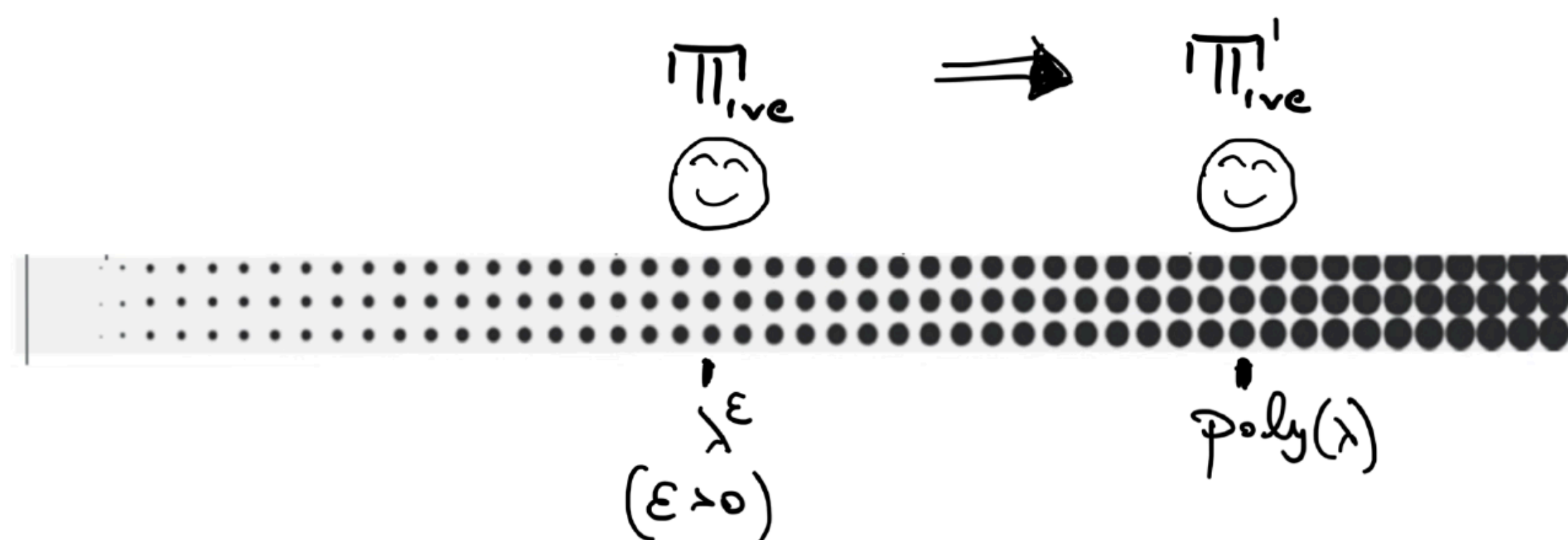
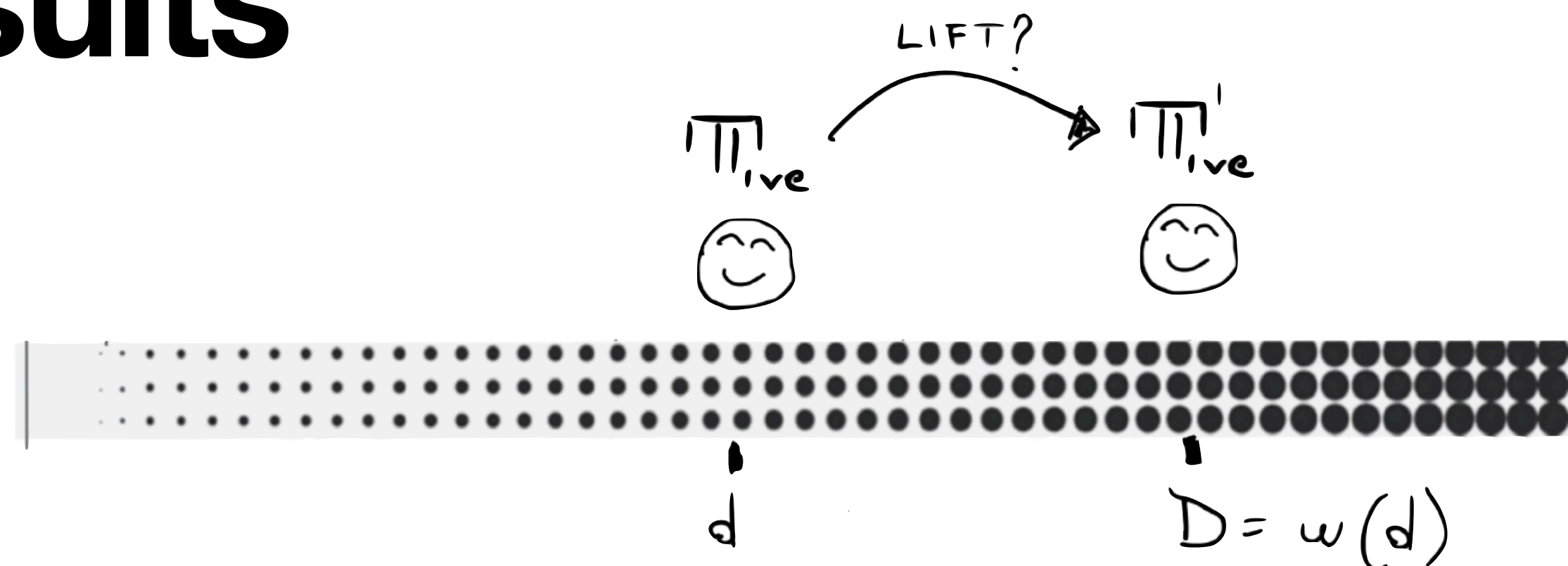
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Our Results (continued)

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- black-box lifting results,
- and that preserve performance.



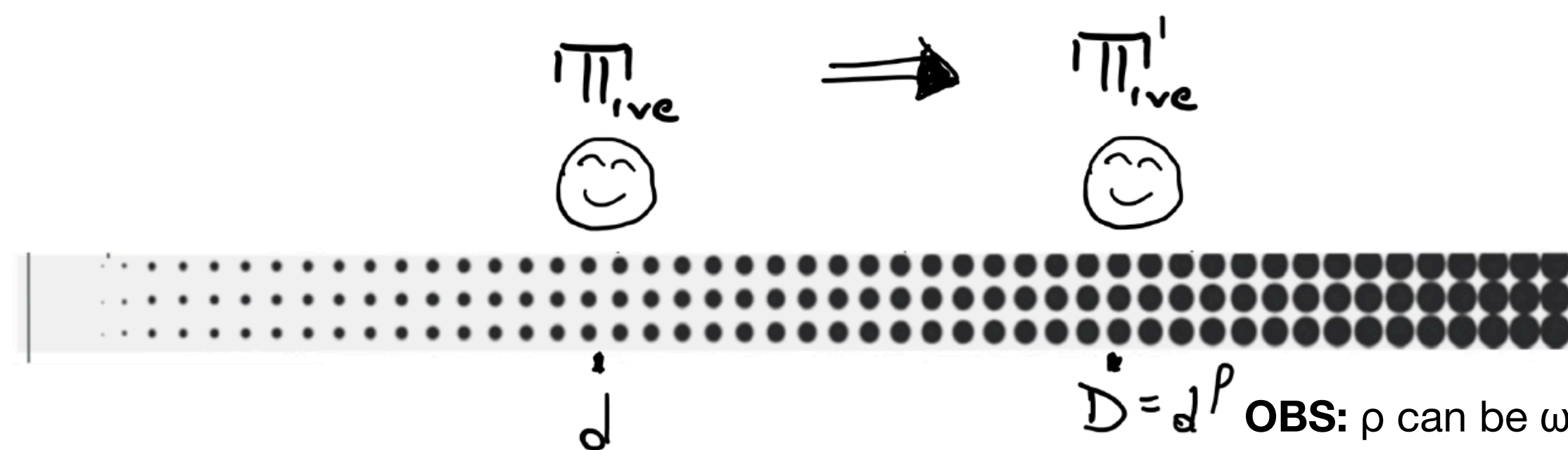
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Corollary:

\exists IVC SND at $O(1) \Rightarrow$
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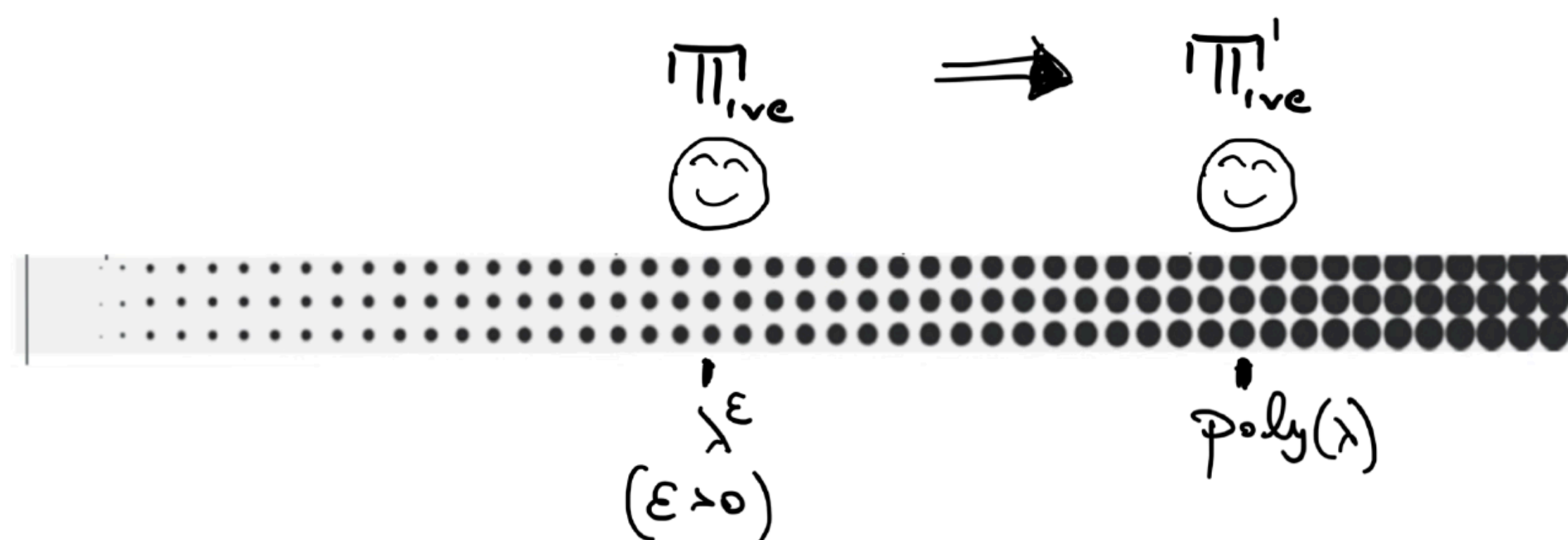
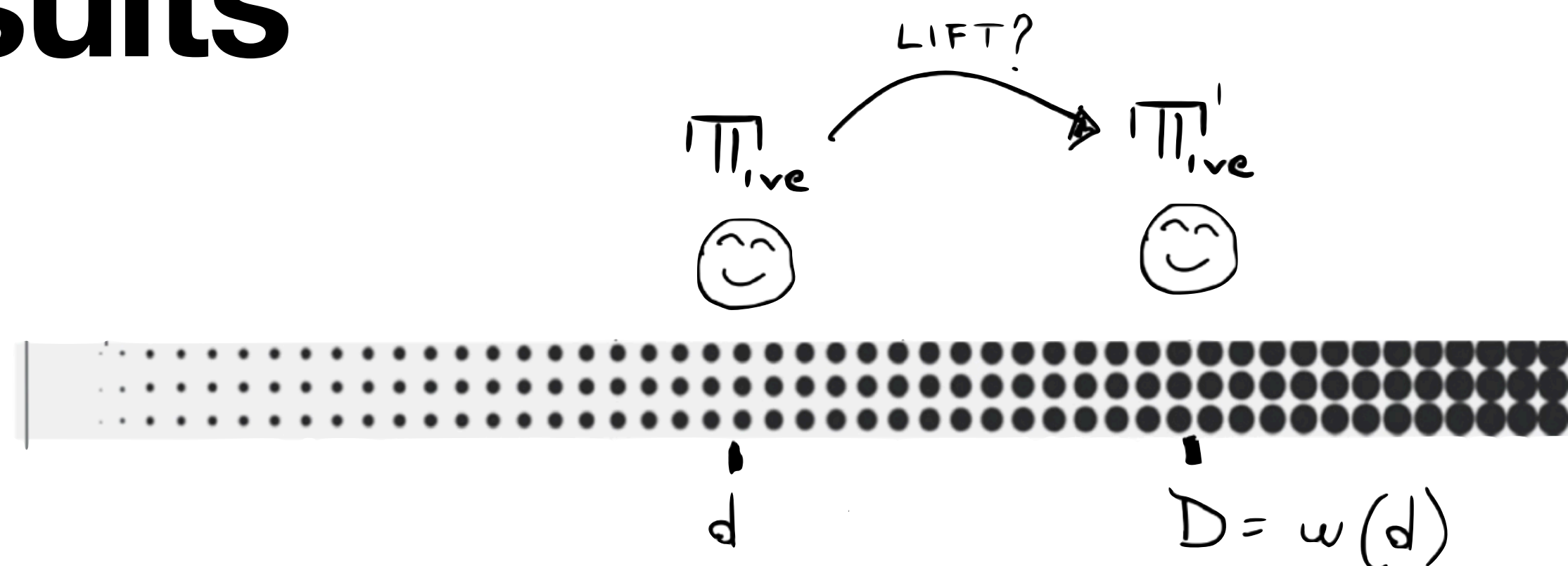
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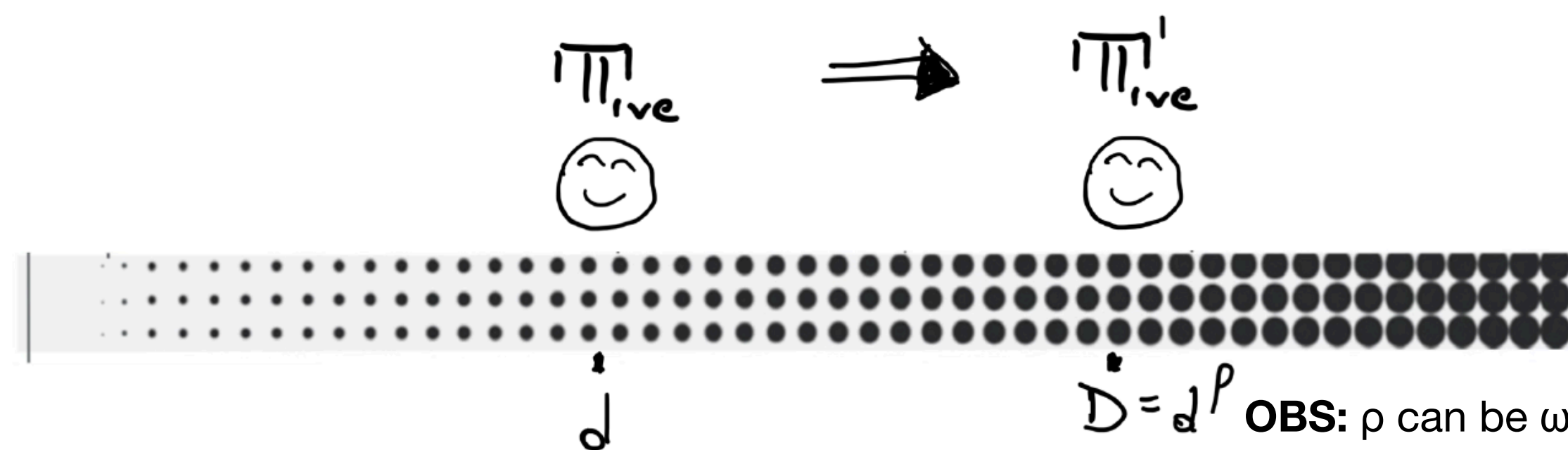
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Special case: $d = O(1)$; $\rho = O(\log \lambda)$



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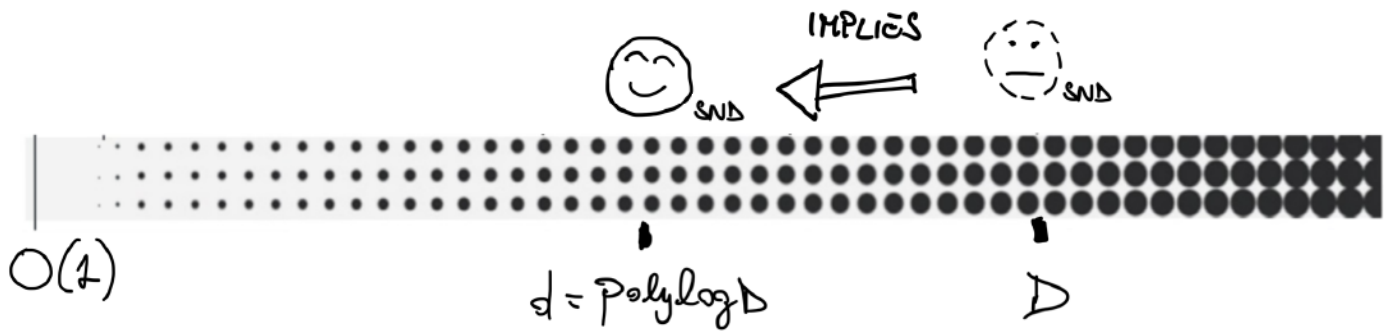
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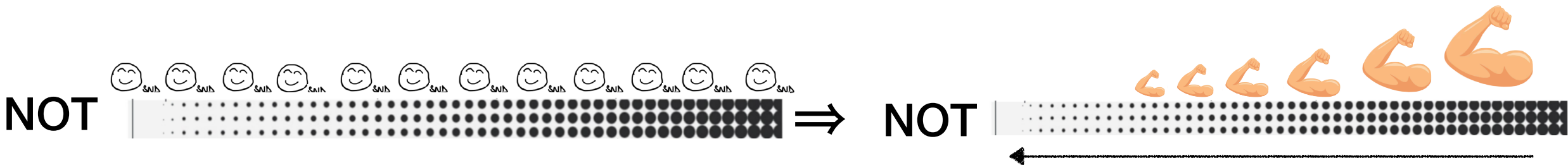
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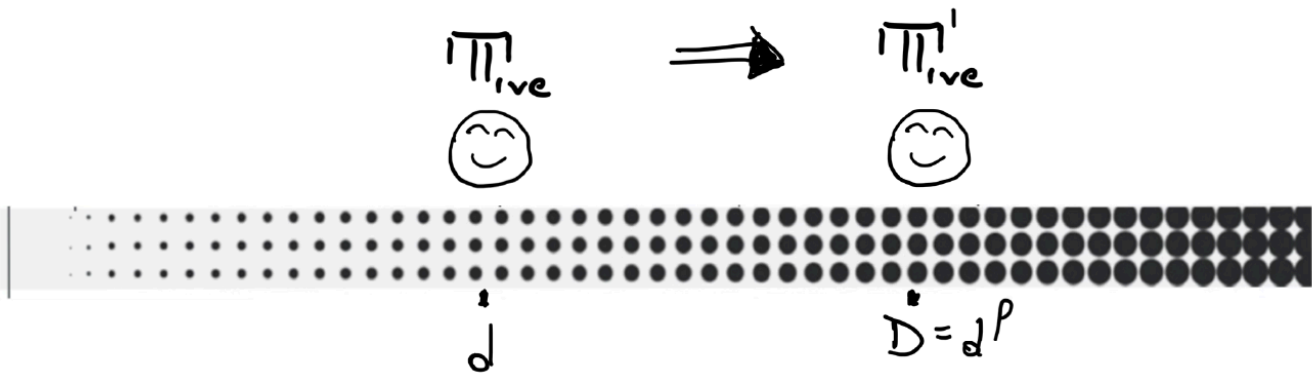
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Insecure IVCs cannot exhibit graceful sec. degradation



Black-box lifting with low overhead

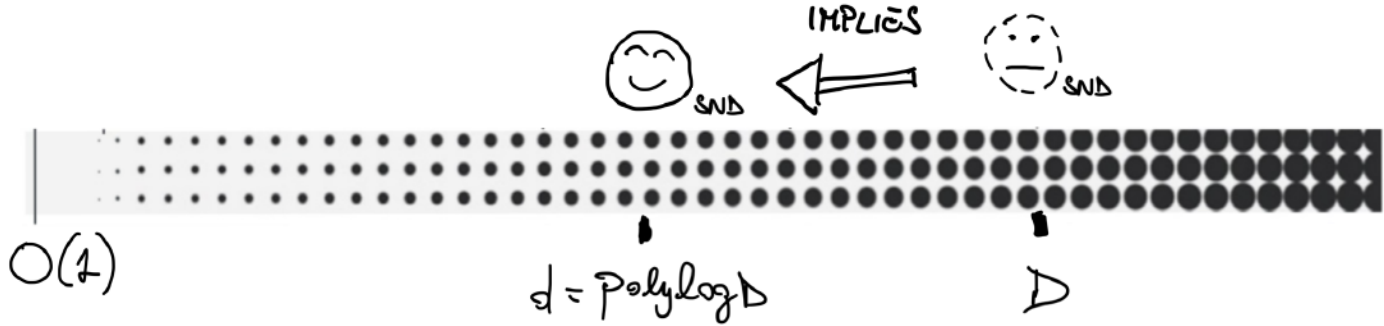


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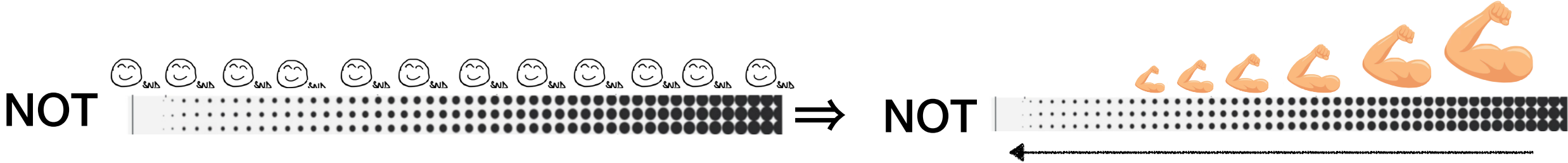
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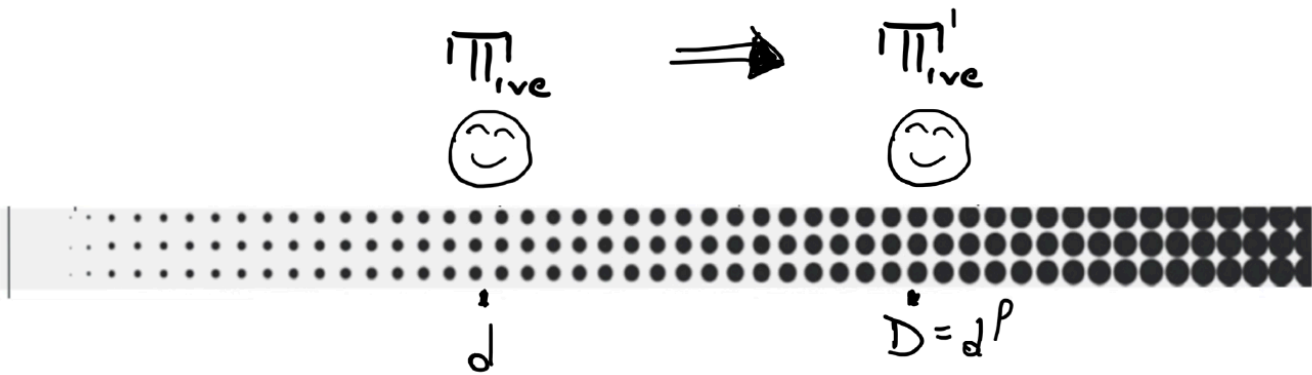
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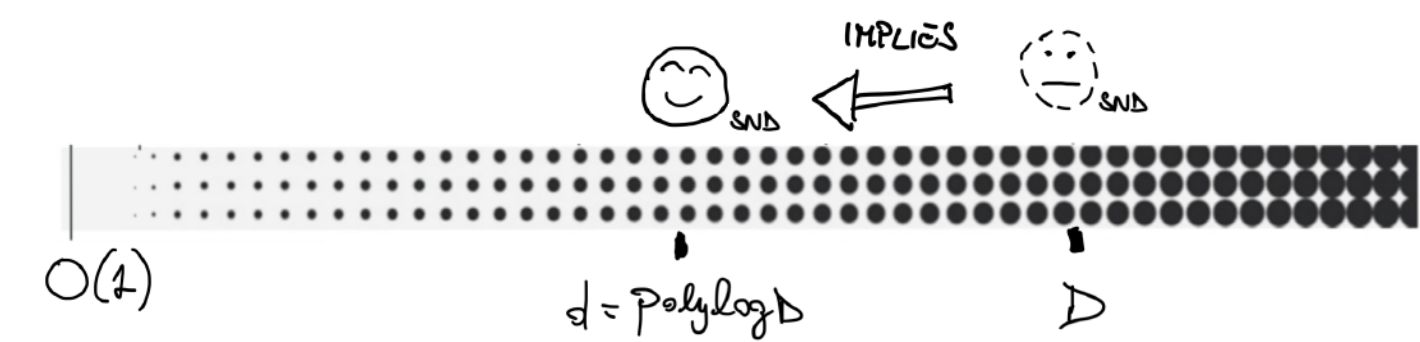


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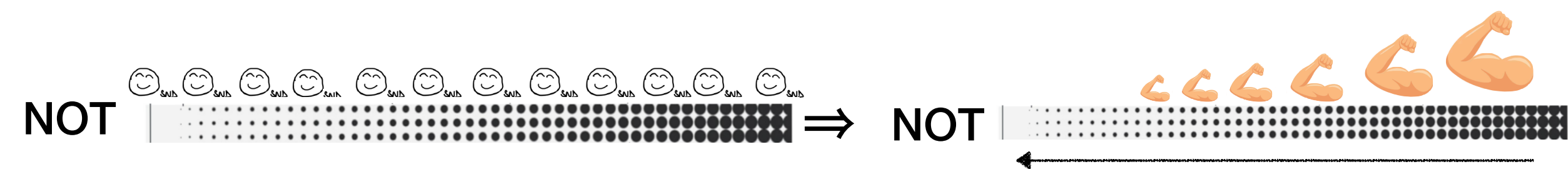
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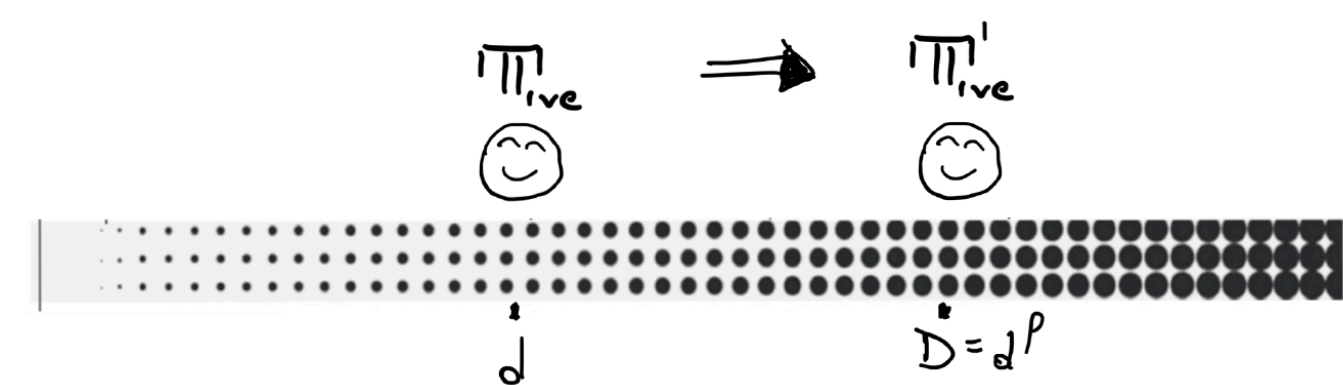
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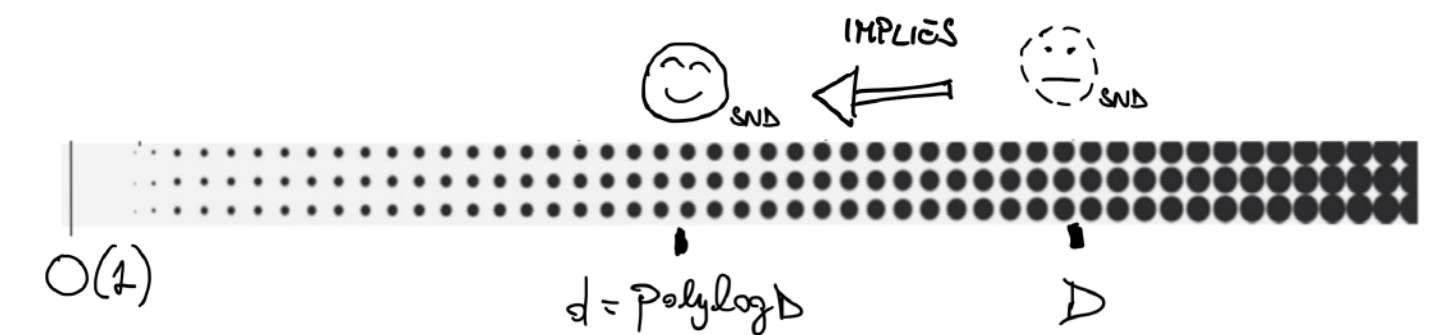


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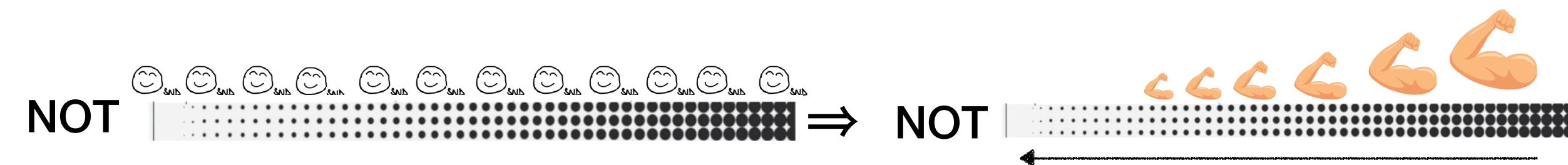
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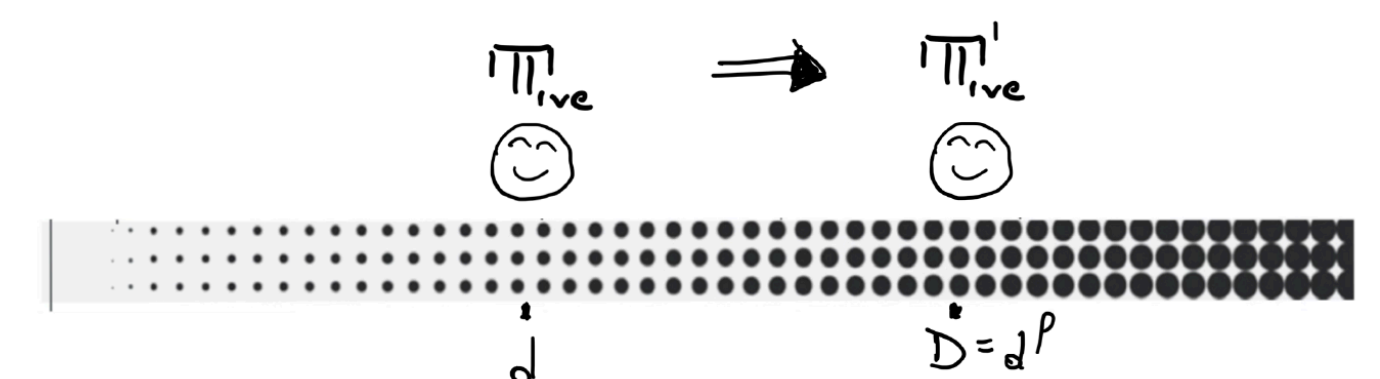
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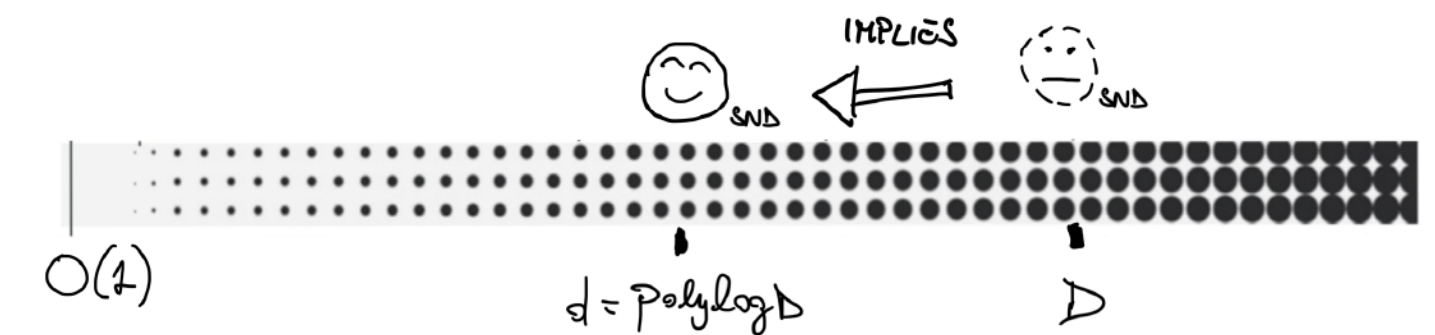


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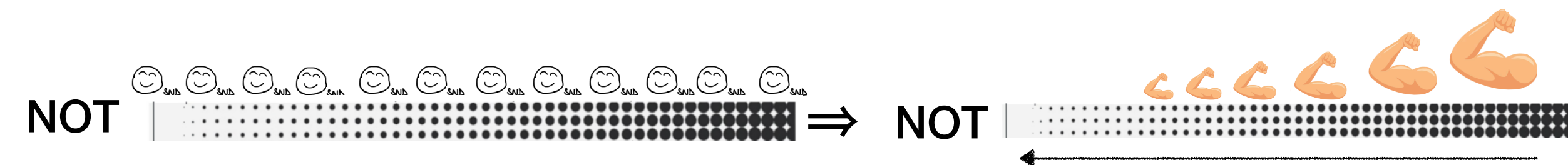
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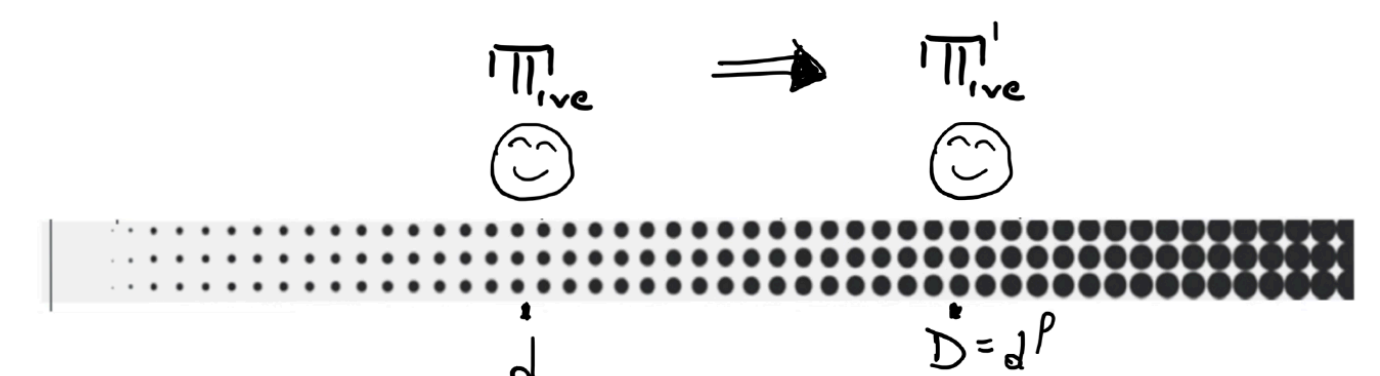
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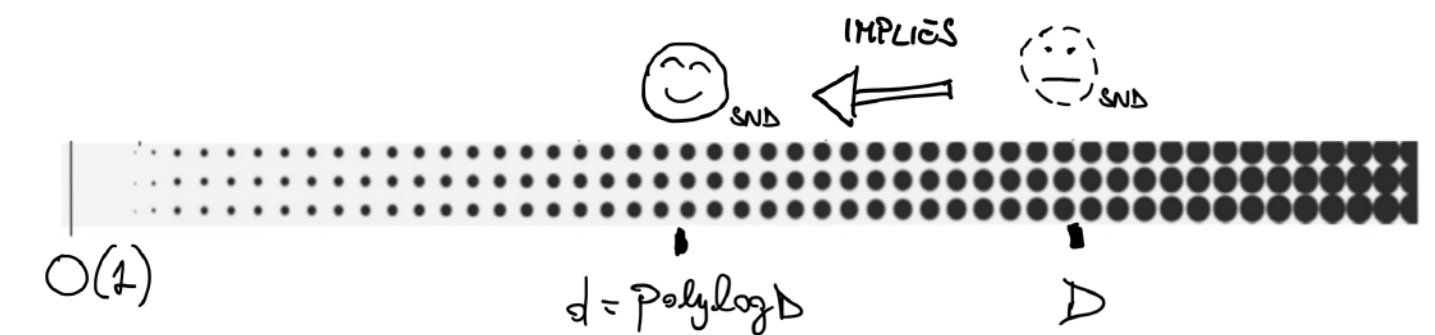


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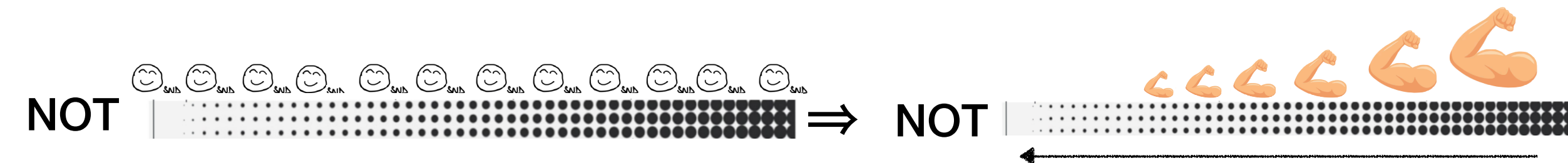
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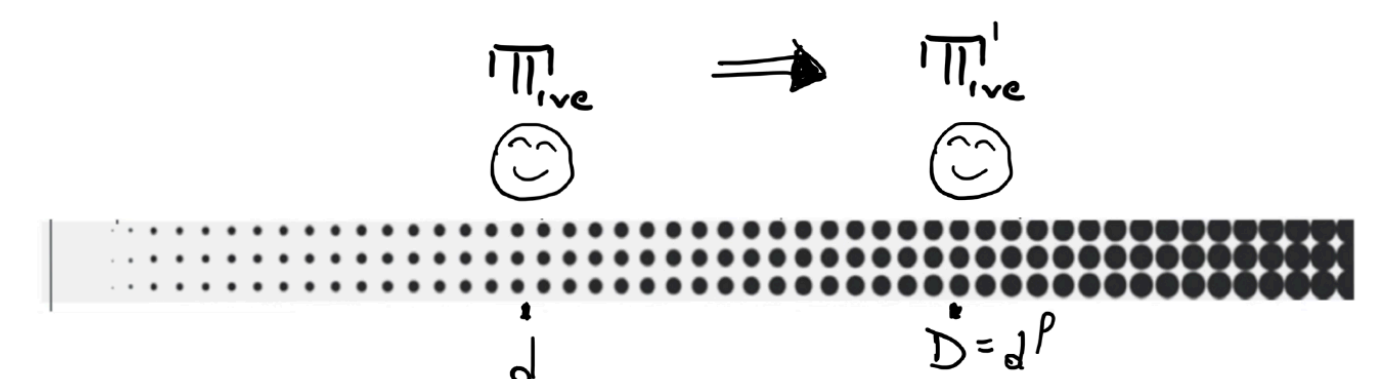
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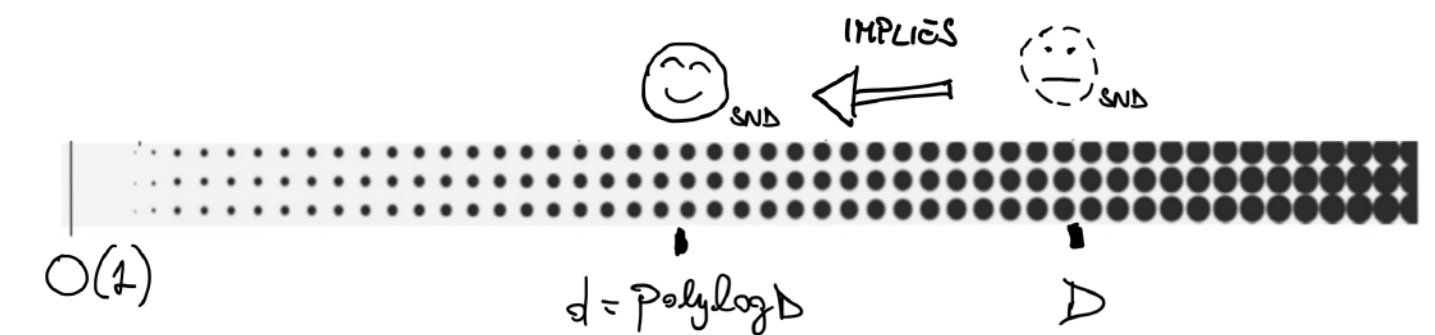


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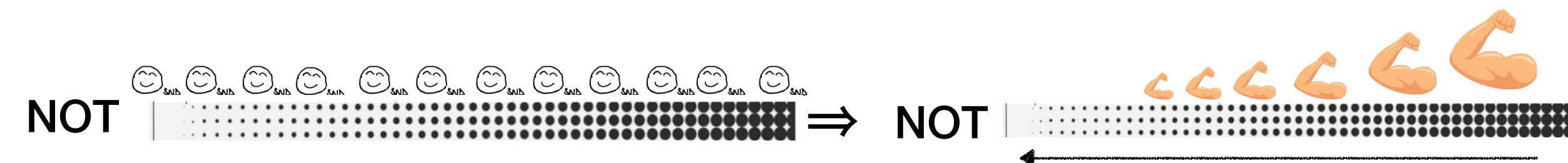
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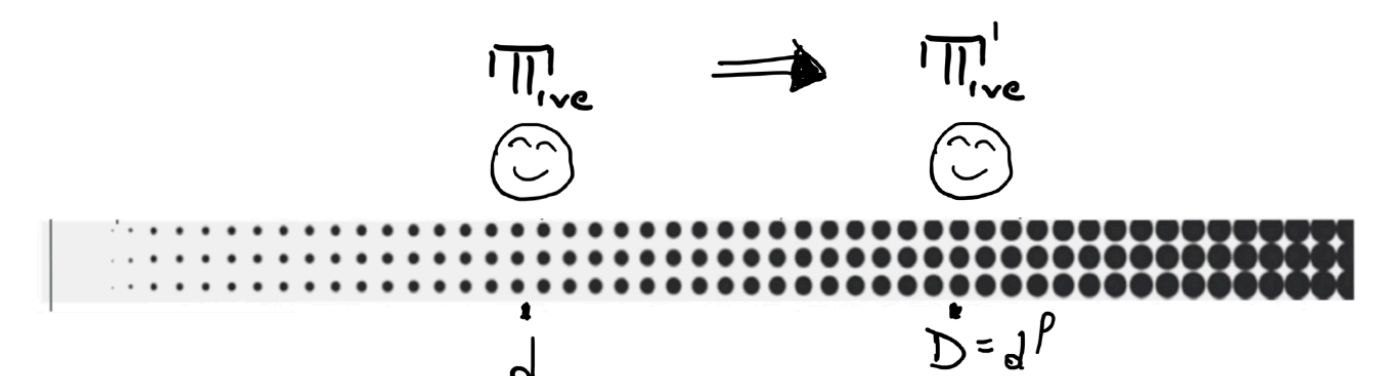
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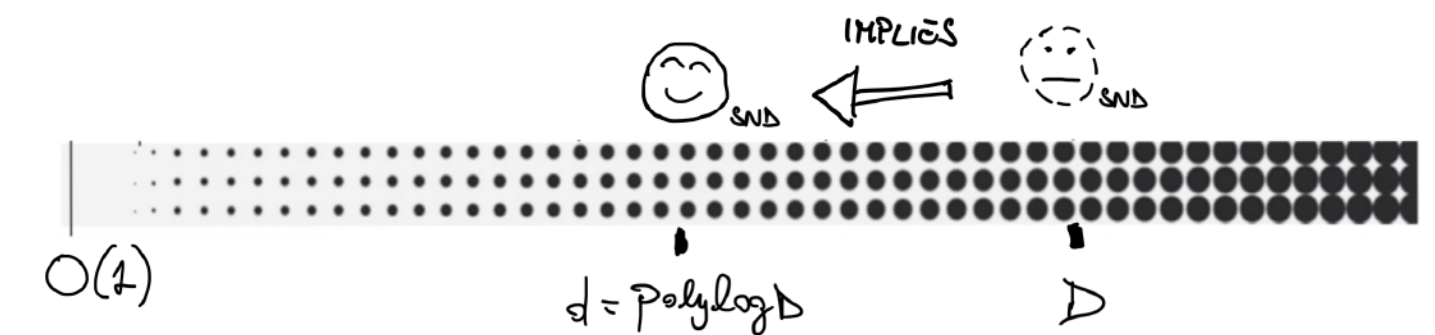


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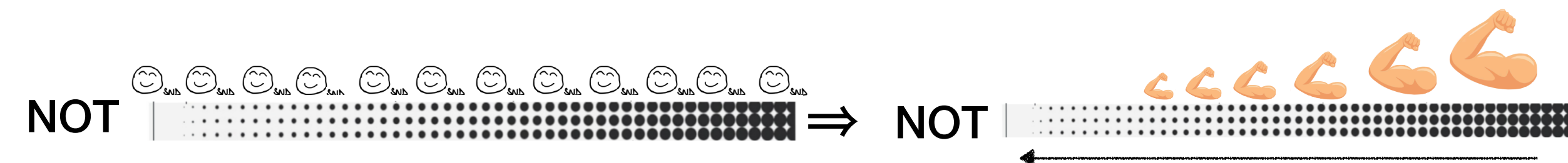
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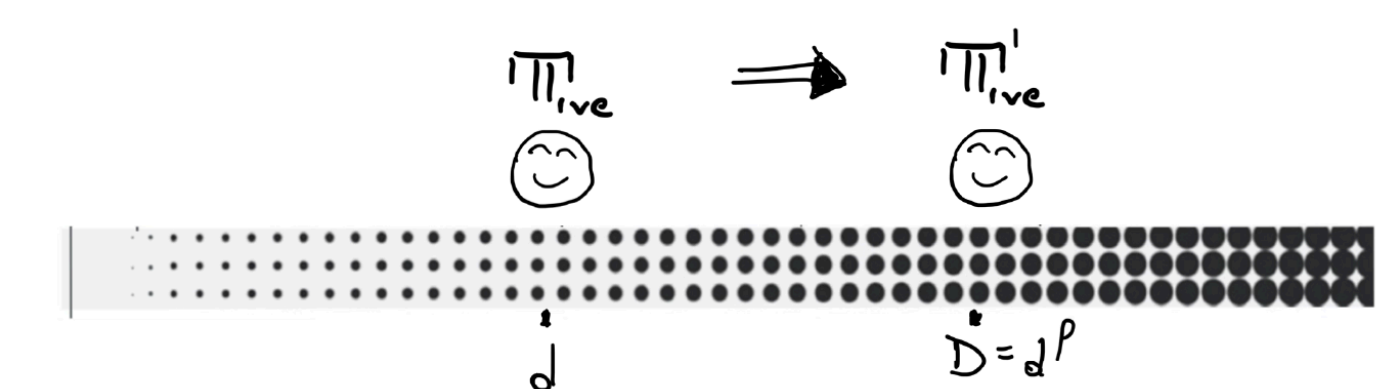
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New Notion: Incremental Functional Commitments

What are Functional Commitments? (FC)

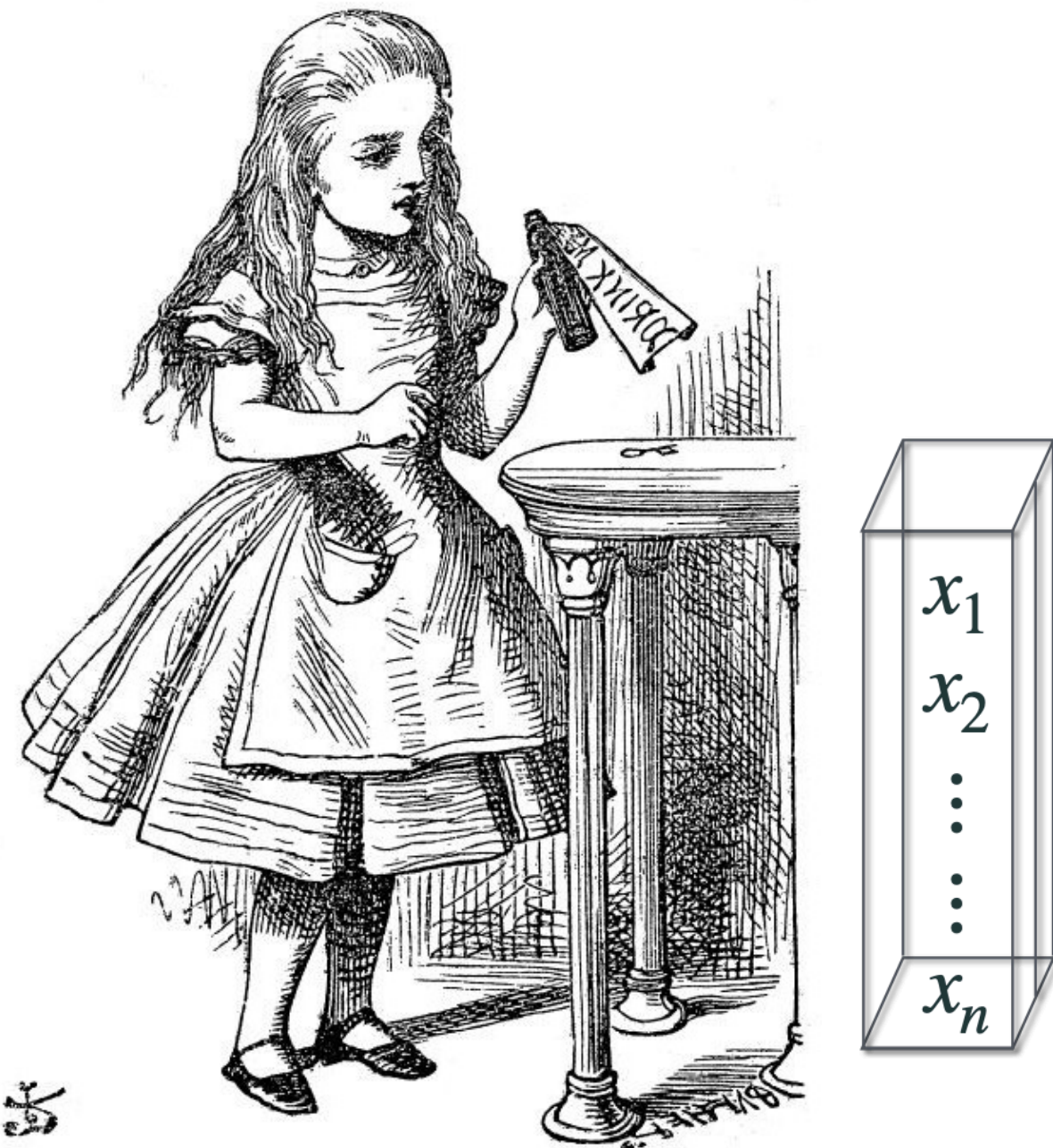


Server (Prover)



Client (Verifier)

What are Functional Commitments? (FC)



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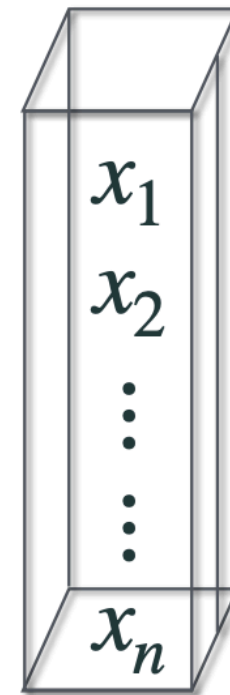


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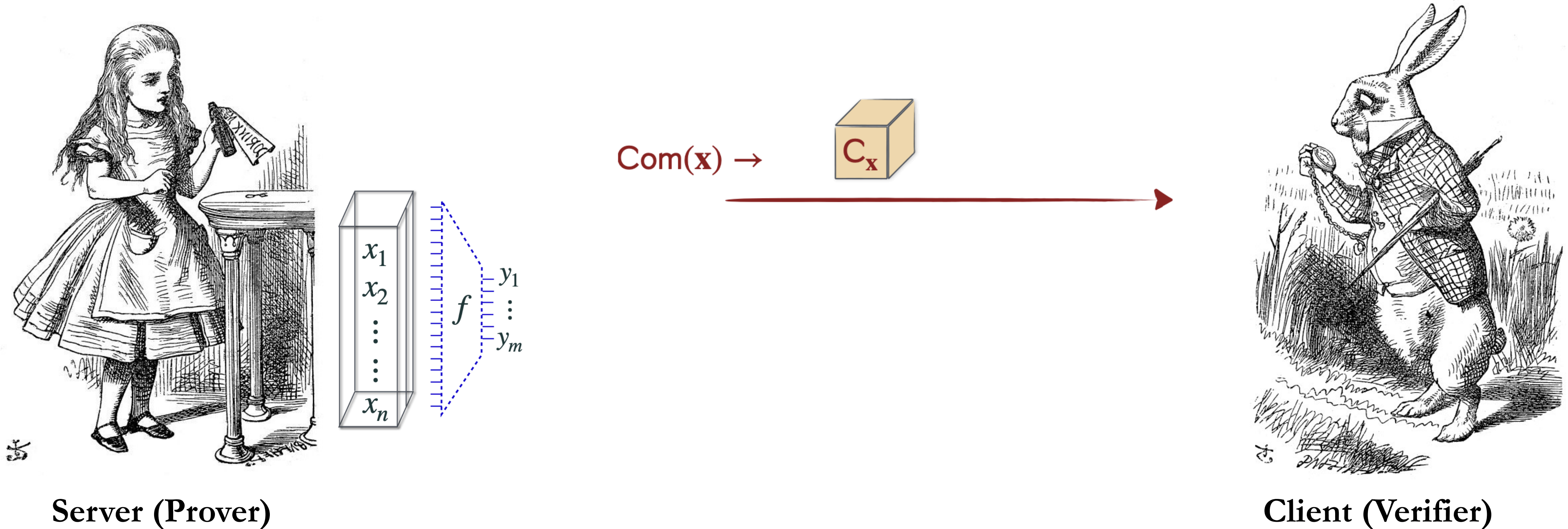


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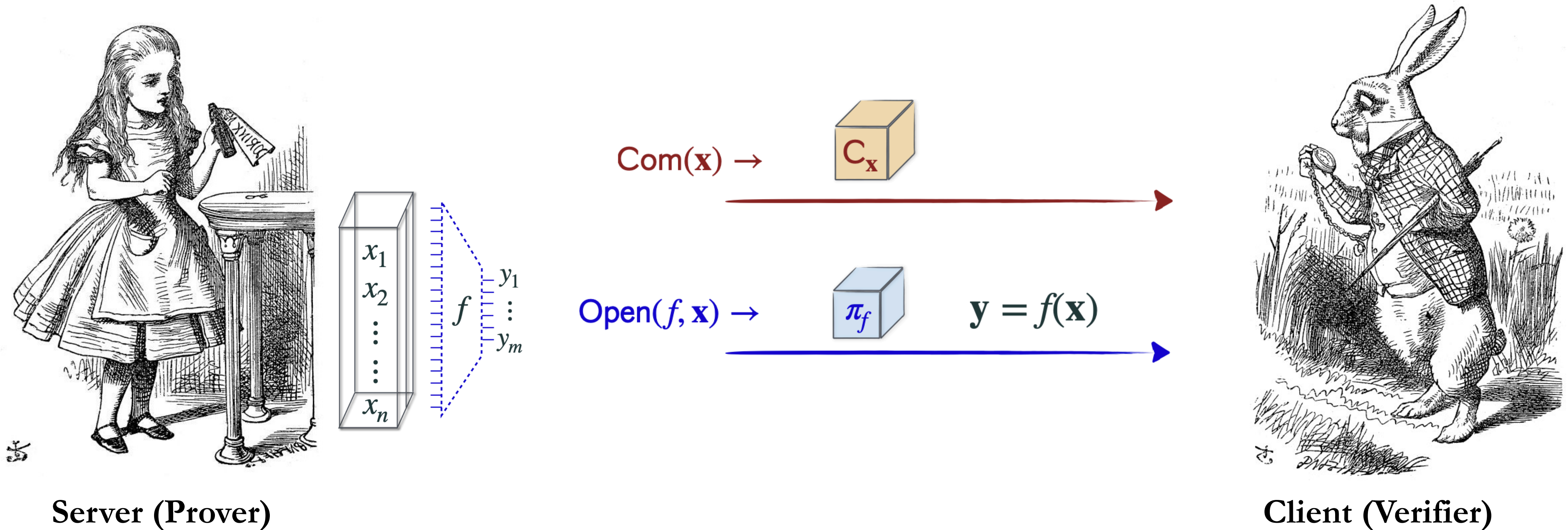


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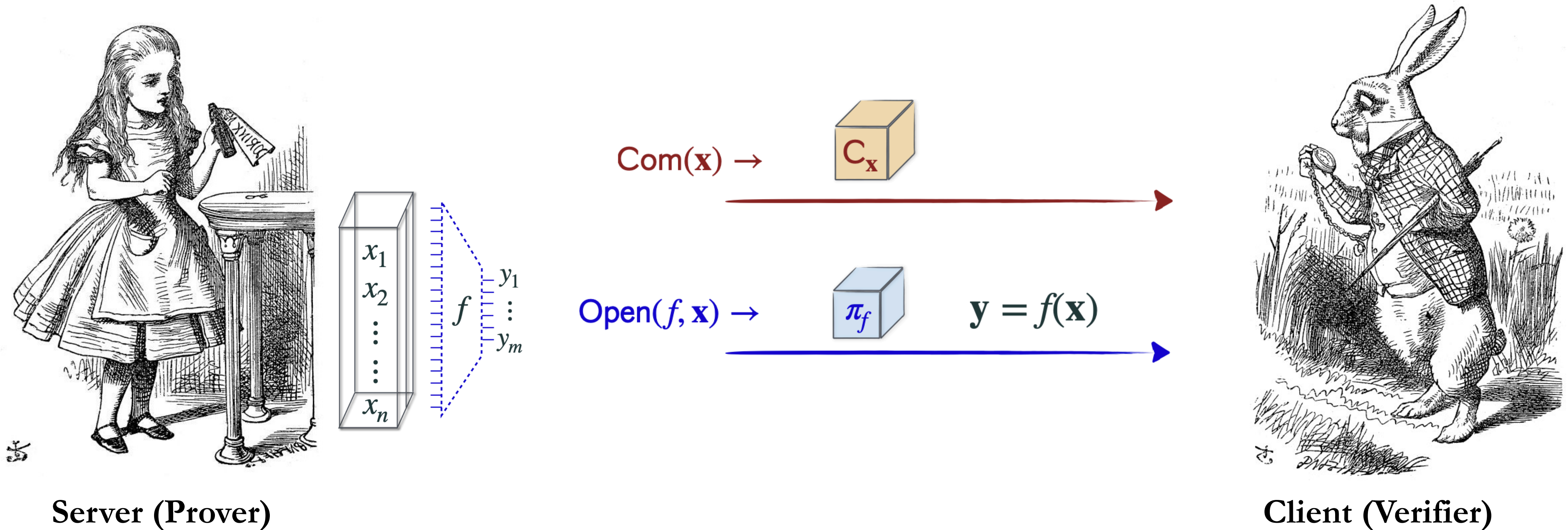
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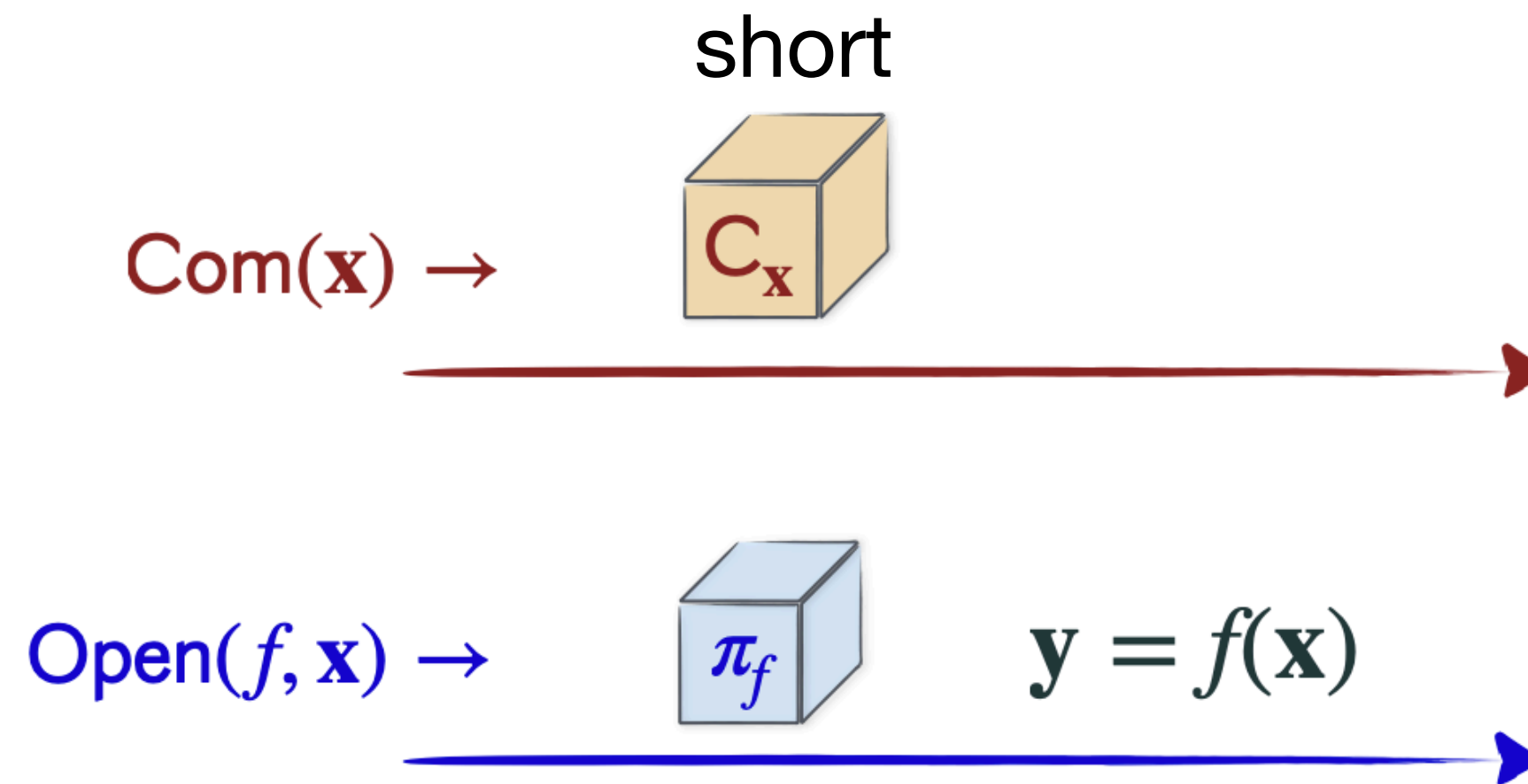
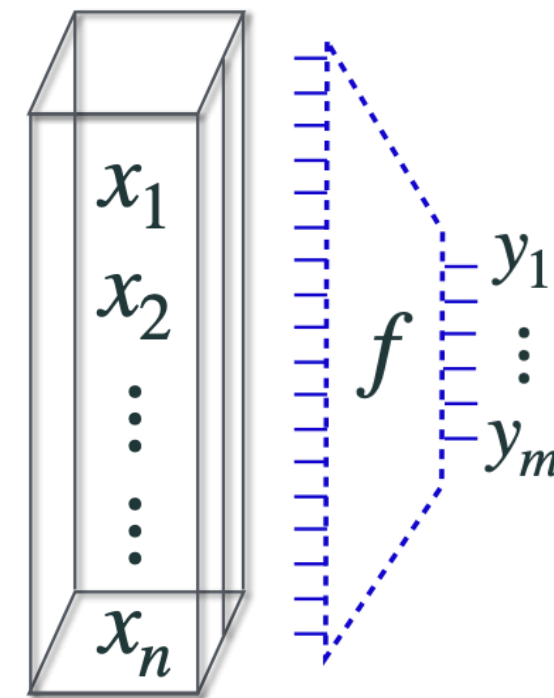


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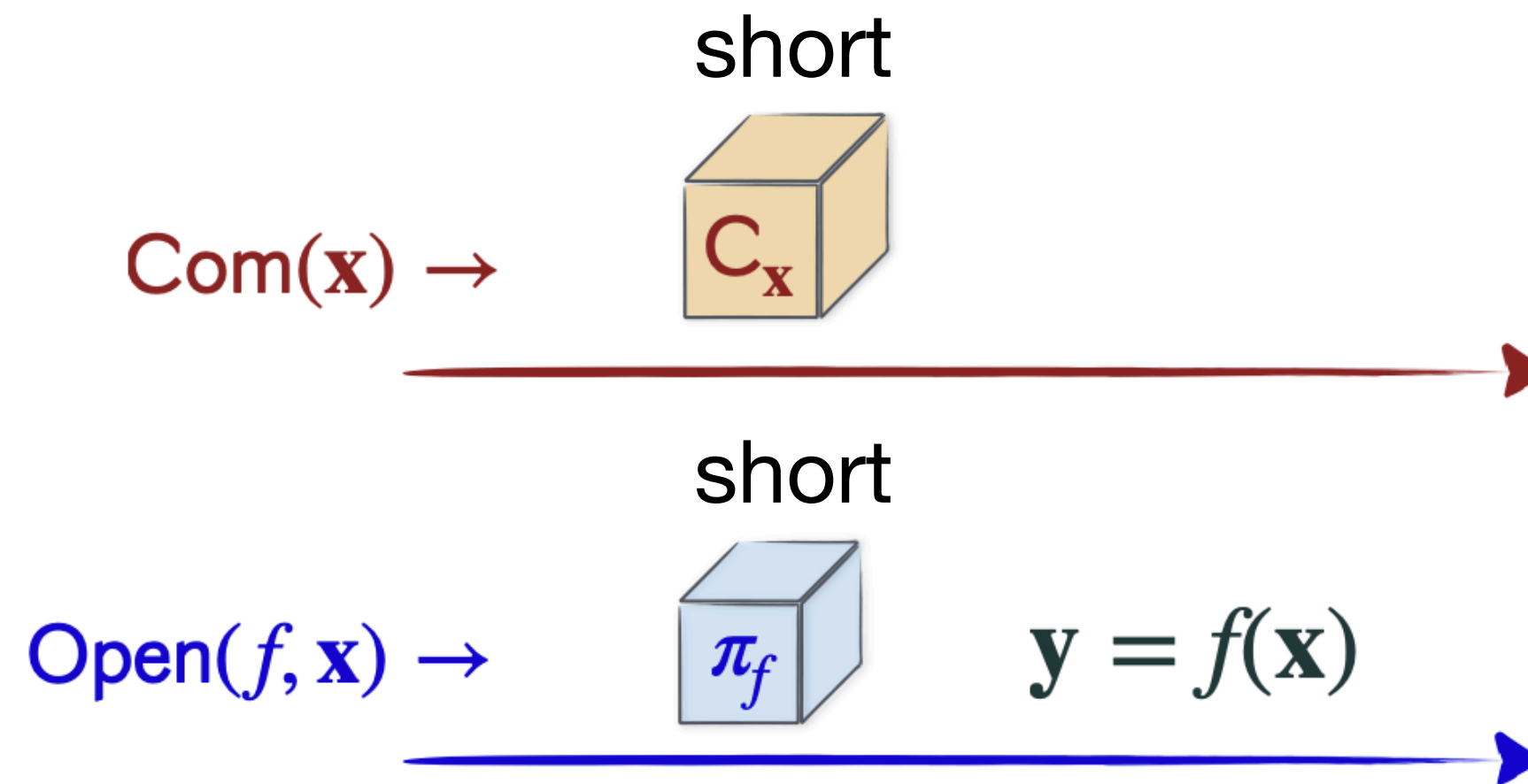
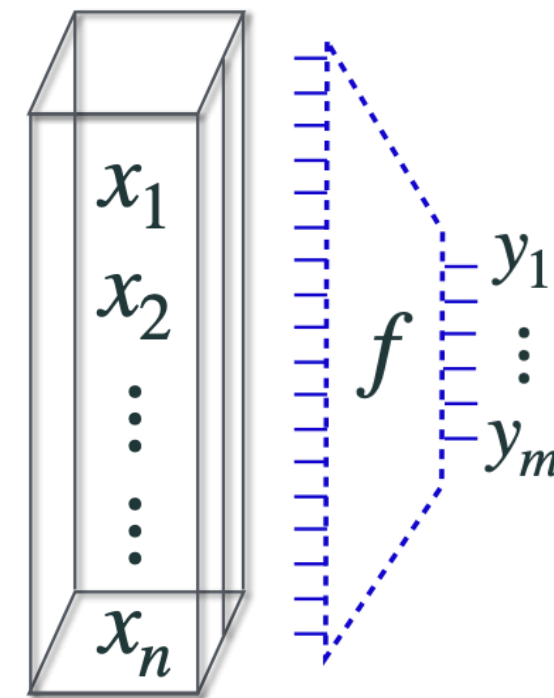
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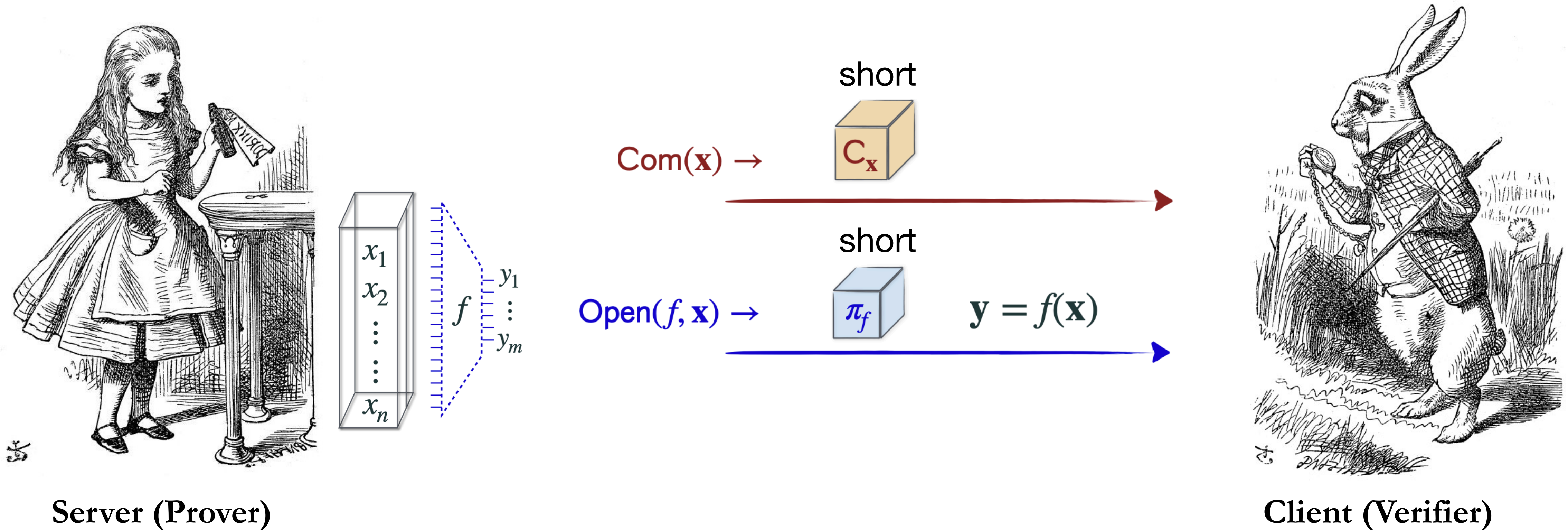
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Functional commitments generalize polynomial and vector commitments.

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Security of Functional Commitments

Evaluation Binding



Malicious Prover



Client (Verifier)

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Malicious Prover

No adversary can succeed in providing inconsistent valid-looking outputs.



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$$\xrightarrow{C_x \quad f, \pi_f, y, \pi'_f, y' \quad y \neq y'}$$



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NB: intuitively stronger than deterministic and non-deterministic soundness (but weaker than extractability).

Incrementality in Functional Commitments

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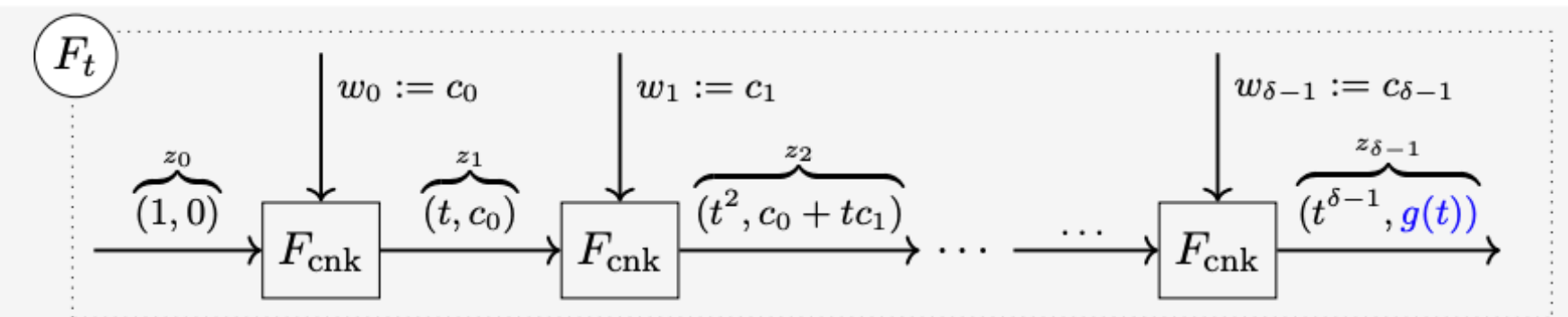
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“Incremental” polynomial

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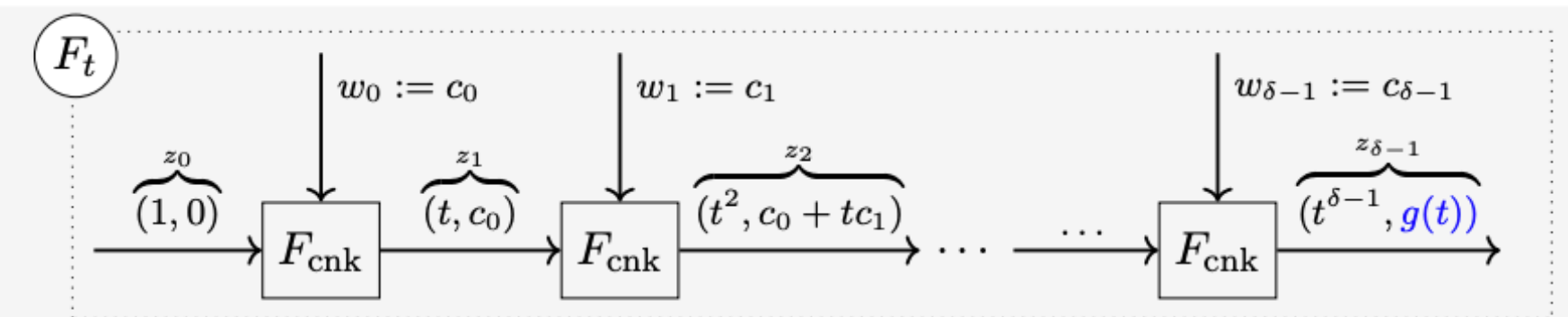
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“streaming” polynomial commitments and neural network evaluation.

Our contributions for IFC:

modeling, canonical construction,
security proofs, connections to other results.



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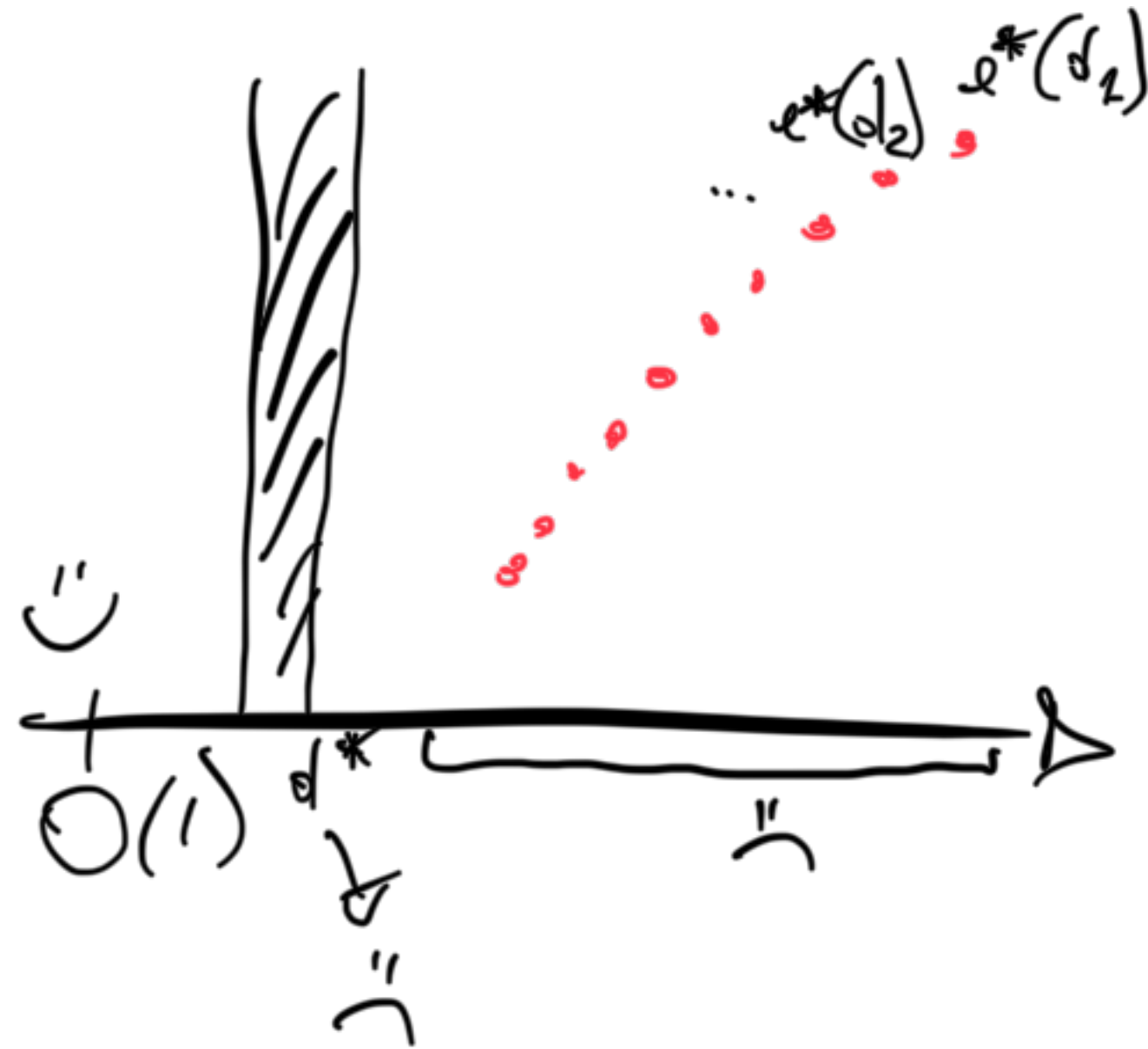
Thank you! Questions?

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Extra slide on graceful sec. degradation



Theorem 6. *Given any sequence of superconstant—i.e., $\omega(1)$ —depth bounds R_0, R_1, R_2, \dots , there always exists a superconstant depth bound L such that for all i , $R_i \succeq L$.*

$$\exists d_1, d_2, \dots: e^*(d_i) < e^*(d_{i-1})$$

$e^* := \approx \log$ of ADV. of
"BEST" d

Extra slide on io-soundness

Theorem (Informal statement of [Corollary 1](#)). Let Π be an IVC scheme and $D = \omega(1)$ be a depth bound. Let $E \subseteq \mathbb{N}$ be an infinite and “exponentially sparse” set of security parameters where Π achieves negligible soundness at depth bound D . Then there exists a depth bound $d = \omega(1)$ where Π achieves (standard) negligible soundness.

Theorem (Informal statement of [Corollary 2](#)). Let $\Pi, D = \omega(1)$, and E as in the previous theorem. Then:

- E exponentially sparse $\implies d = O(\log D)$.
- E sub-exponentially sparse $\implies d = O(\text{polylog} D)$.

Theorem 3. Let $E = \{\lambda_1 < \lambda_2 < \dots\} \subseteq \mathbb{N}$ be a constructible (2^{κ^T}) -sparse set for some T with $0 < T \leq 1$. Let Π be an IVC that is i.o-sound with respect to E for depth bound $D(\cdot)$. Let $d'(\cdot)$ be a depth bound. If for all $i \in \mathbb{N}$,

$$d'(\lambda_{i+1} - 1) \leq D(\lambda_i), \quad (\blacktriangle)$$

then Π is (almost-everywhere) sound for depth bound d' if appropriately parameterized ([Definition 3](#)). The resulting proving time, verification time and proof size are like those originally in Π (up to constant factors).